ESTIMATION OF BENDING STRAIN OF A BEAM USING HYBRID MODAL ANALYSIS

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Abstract. *In machines and equipments high strain/stress levels can, when occurring under a long period of time, cause failure from fatigue. Thus, is important the characterization of the spatial distribution of dynamic displacement/strain/stress fields. With this purpose, several methods of estimation of dynamics stress and strain using vibrational parameters have been developed. Basically, results from modal analysis are transformed from the displacement space to the strain space by use spatial differential operator. However in some situation, the displacement field can't be determined by measurements. So, the displacement field can be determined by Hybrid Modal Analysis (HMA). This technique is based on orthogonality of the eigenmodes of a system and the convergence properties by least squares generalized Fourier series. In this context, this paper presents the numerical and experimental of the estimation of bending strain at a point on the surface of an aluminum beam using HMA. In the first moment, a numerical simulation was done to determine the strain at a point on the surface of an aluminum beam using eigenmodes obtained by finite element method and the transverse displacements of the beam by simulation. After, a experimental analysis was done to determine the spatial distribution of strain using the eigenmodes anteriorly determined and the transverse displacement of the beam obtained by acceleration measurements. The cases examined showed promising results.*

Keywords*: Dynamic strain, Beam, Bending, Vibration.*

1. INTRODUCTION

In machines and equipments high strain/stress levels can, when occurring under a long period of time, cause failure from fatigue. Thus, the characterization of the distribution of stress / strain dynamics in machinery, equipment and structures is becoming increasingly important. This is due to the demands of these products are increasingly lighter, more flexible and stronger.

The dynamic strain, conventionally, are obtained experimentally by strain gage techniques. However, when it is necessary to measure strain in many parts of the structure, this technique becomes very costly. The transducers used in strain gage technique, for example, strain gages, are expensive, disposable and there is a need to clean the surface analyzed with the removal of paint and other standard procedures, which may require standing for long periods of equipment.

The contradictory demands for high spatial resolution and less extensive measurements may to some extent be solved using the Hybrid Modal Analysis (HMA) technique. The HMA technique utilizes a mix of experimental, measured vibration responses and good numerical approximations of well defined, three-dimensional, real, normal modes (Dovstam, 1998). The modes are assumed to be solutions to an elastic eigenvalue problem corresponding to the true geometry of the analyzed body or structure. Numerical approximations of the modes can be obtained by, for example, the Finite Element Method (FEM).

Having the modes and applying HMA to a vibrating structure, the spatial distribution of the dynamic strain field may be obtained transforming the displacement space to the strain space by use of finite difference schemes. This analysis of the dynamic strain analysis based on HMA can be called Hybrid Strain Analysis (HSA) (Sehlstedt, 1999).

In this sense, methods that use the results of modal analysis to predict or estimate the dynamic strain of structures were studied. These methods basically consist in the spatial differentiation of displacement obtained by modal analysis and using mostly the finite difference method for solving differential equations of the problem of elasticity.

In the work published in 1989, Bernasconi and Ewins shown as strain gages and displacement transducers can be used to determine the strain modes normalized by mass. Thus, the strain values in the time domain can be found by superposition in the same way that one can find the time response of displacement. Karczub and Koss (1995) proposed a method with respect to deformation in bending. The experimental evaluation using only two accelerometers in a Euler-Bernoulli beam, it is a one-dimensional method and therefore limited in its use. Okubo and Yamaguchi (1995) predicted the distribution of dynamic deformation under operating conditions, using the transformation matrix displacement - strain. Dovstam (1998) proposed the hybrid method of modal analysis to complement the conventional modal analysis to determine the displacement of three-dimensional structure and thereby determine the strain tensor. In Karczub and Norton (1999) the bending of an Euler-Bernoulli beam is studied in time domain and the approach was based on the finite difference schemes for the second order derivative of the transverse displacement of the beam, ie, analyzing the curvature of the beam. Measurements were made at equidistant points and distributed symmetrically around the point of analysis and the strain could not be predicted at boundaries. Lee and Kim (1999) studied the normal and shear strain in a plate with a viscoelastic core layer. The strain were calculated using the finite difference schemes on models obtained analytically bending vibration of the plate. Sehlstedt (1999) with the values of displacements obtained from hybrid modal analysis, made the analysis of dynamic strain tensor in a plate using the finite difference schemes and the eigenmodes of vibration by the finite element method. Lee (2007) proposed a method for estimating the strain responses from measurements using the displacement transformation matrix, obtained by the modal matrix of displacement and strain.

In view of these studies, this paper presents the methodology used by HMA and a numerical evaluation and experimental distribution of dynamic strain bending of an aluminum cantilever beam subjected to harmonic excitation, using data obtained by HMA.

2. HIBRID MODAL ANALYSIS

The hybrid modal analysis is used to supplement information obtained from conventional modal analysis. From the hybrid modal analysis is possible to estimate the displacement in points or directions of unmeasured structure. The hybrid modal analysis was proposed by Dovstan (1998). This technique is based on the orthogonality of the eigenmodes of a system and mean square convergence properties of generalised Fourier series.

With the displacements obtained by the displacement FRFs (Frequency Response Function), or measured by the ODS (Operating deflection shapes), it's possible to determine the spatial displacement not measured of points of the structure, by the HMA. For a successful hybrid modal analysis, some requirements are needed:

- Known of geometry and boundary conditions of the system.
- Good approximation of eigenmodes of the system, based on the actual distribution of the structure.
- High quality measurements of responses.

It is assumed that the material undergoes is isothermal and small deformations, and has a linear method is based on the elastic-static properties, ie, at zero frequency.

2.1. Development of Functions in Series orthonormal

In the same way that the three-dimensional vector $\{r\}$ can be write by a set of units vector can be written by a set of mutually orthogonal unit vectors in the form, $\{r\} = c_1 i + c_2 j + c_3 k$, according to Spiegel (1974) there is the possibility of developing a function $f(x)$ on a set of orthonormal functions, ie:

$$f(x) = \sum_{n=1}^{\infty} c_n \phi_n(x) \quad a \le x \le b \tag{1}$$

The series, called orthonormal series, are generalizations of Fourier series. Assuming that the right number of Eq. (1) converges to $f(x)$, we can formally multiply both sides by $\phi_m(x)$ and integrate them from a to b, obtaining:

$$c_m = \int_a^b f(x)\phi_m(x)dx \tag{2}$$

where, c_m are the generalized Fourier coefficients.

2.2. Approximation by Least Squares

Let $f(x)$ e $f'(x)$ piecewise continuous on $[a,b]$, with m = 1, 2, 3, ... M, orthonormal $[a,b]$. There is the finite sum:

$$S_M(x) = \sum_{m=1}^{M} \alpha_m \phi_m(x) \tag{3}$$

as an approximation of $f(x)$, where α_m is constant yet unknown. Then the mean square error of approximation is given by:

$$\text{Erro} = \frac{\int_a^b [f(x) - S_M(x)]^2 dx}{b-a} \tag{4}$$

And the root mean square error is given by:

$$\text{Erro}_{\text{rms}} = \sqrt{\frac{1}{b-a}\int_a^b [f(x) - S_M(x)]^2 dx} \tag{5}$$

By determining the constants α_m so as to minimize the root mean square error, Eq. (5), the mean square error is minimal when the coefficients are equal to the generalized Fourier coefficients, Eq. (2), ie, when:

$$\alpha_m = c_n = \int_a^b f(x)\phi_m(x)dx \tag{6}$$

It is often said that $S_M(x)$ with coefficients c_n is an approximation of $f(x)$ in the sense of least squares.

2.3. Displacements by the Hybrid Modal Analysis.

According Dovstan (1998) the displacement $\tilde{u}(p,\omega)$ of a point p in the frequency domain can be represented by generalized Fourier series, according to Eq. (7):

$$\tilde{u}(p,\omega) = \sum_{r=1}^{M} c_r(\tilde{u})\phi_{pr} + \tilde{u}_{res}(p,\omega) \tag{7}$$

where $\tilde{u}_{res}(p,\omega)$ is the pointwise error, or residual.

Since $\{\tilde{u}\mathbf{e}_i\}$ a vector of displacement in the direction i in the frequency domain, consisting of N responses of displacement:

$$\{\tilde{u}\mathbf{e}_i\} = \begin{Bmatrix} \tilde{u}_i(1,\omega) \\ \tilde{u}_i(2,\omega) \\ \vdots \\ \tilde{u}_i(N,\omega) \end{Bmatrix} \tag{8}$$

Since the modal displacement matrix $[\Phi]_{N\times M}$, containing information about the direction of the modes and points of the vector $\{\tilde{u}\mathbf{e}_i\}$, ie:

$$[\Phi\mathbf{e}_i] = \mathbf{e}_i \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1M} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2M} \\ \vdots & \vdots & \ddots & \\ \phi_{N1} & \phi_{N2} & \cdots & \phi_{NM} \end{bmatrix} \tag{9}$$

Then $\{\tilde{u}\mathbf{e}_i\}$ can be expressed by:

$$\{\tilde{u}\mathbf{e}_i\} = [\Phi\mathbf{e}_i]\{\tilde{c}\} + \{\tilde{u}_{res}\} \tag{10}$$

Since $\{\tilde{c}\}$ the vector of generalized Fourier coefficients that weigh the contribution of each mode in the response of the structure is given by:

$$\{\tilde{c}\} = \{\tilde{c}(\omega)\} = \{c_1(\tilde{u}) \quad c_2(\tilde{u}) \quad \dots \quad c_M(\tilde{u})\}^T \tag{11}$$

The residual vector $\{\tilde{U}_{res}\}$ is given by:

$$\{\tilde{u}_{res}\} = [\Phi_{res}]\{\tilde{c}_{res}\} \tag{12}$$

The matrix $[\Phi_{res}]$ and the vector $\{\tilde{c}_{res}\}$ of the Eq. (12) are defined with the Eq. (10) e Eq. (11), but refer to a large number *M* modes of analysis and are defined like $r = M+1, M+2, \dots\infty$, have infinite dimension, where $[\Phi_{res}]$ is a real array of dimensions N x ∞, while $\{\tilde{c}_{res}\}$ is a column vector of complex dimensions (∞ x 1).

Depending on the number of points *N*, the number of modes *M* and the damping of the structure, the $\{\tilde{u}_{res}\}$ can be neglect of the Eq. (10). In applications the number of modes, *M*, is chosen such that the angular frequency ω_M fulfils that $\omega < \omega_{\max} < \omega_M$, where ω_{max} is the maximum analysis frequency. The number of measurements, *N*, must then also be increased for the system of equations to be sufficiently overdetermined. Here means that the number of measurements, *N*, should be large compared to the number of modes, *M*, so that the estimated Fourier coefficient are smooth enough in the frequency interval of interest.

If $N = M$, the coefficient vector thus is estimated as:

$$\{\tilde{c}_{est}\} = [\Phi]^{-1}(\{\tilde{u}\} - \{\tilde{u}_{res}\}) \tag{13}$$

If $N > M$, the coefficient vector thus is estimated as:

$$\{\tilde{c}_{est}\} = [\Phi]^{+}(\{\tilde{u}\} - \{\tilde{u}_{res}\})^T \tag{14}$$

where $[\Phi]^{+}$ is the pseudo inverse matrix defined as:

$$[\Phi]^{+} = \left([\Phi]^T[\Phi]\right)^{-1}[\Phi]^T \tag{15}$$

Finally, the displacements not measured at specific points (*o*) or directions (*i*) can be predicted according to Eq. below:

$$\tilde{u}_i(o,\omega) = [\phi_{o1} \quad \phi_{o2} \quad \dots \quad \phi_{oM}]\{\tilde{c}_{est}\}(\omega) + \tilde{u}_i(o,\omega)_{res} \tag{16}$$

In a general case, different mode coefficients $c_m(\tilde{u})$ and $c_n(\tilde{u})$ in the modal expansion Eq. (10) are not independent. The modes are said to be coupled. Thus, when the modal coupling cannot be neglect, it is necessary to evaluate the residue. The residual $\{\tilde{u}_{res}\}$ may be approximated using the uncoupled modal receptance model derived in Dovstam (1998).

An approximation is obtained for the displacement response (receptance) due to a point force with unit spectrum $F_j(\omega) = 1$ at point *p*:

$$\tilde{u}_i(p,\omega)_{res} \approx \alpha_{pj} \approx \sum_{r=1}^{M} c_r(\tilde{u})\phi_{pr} + \sum_{r=M+1}^{r_{max}} \frac{\psi_{pr}\psi_{kr}}{m_r(\omega_r^2 - \omega^2 + i\eta_r\omega_r^2)} \quad (17)$$

where ω_r is the natural frequency, η_r is the damping loss factor and m_r is the modal mass defined by:

$$\{\psi_r\}^T[M]\{\psi_r\} \quad (18)$$

where $\{\psi_r\}$ is the modal vector and $[M]$ is the mass matrix.

When the approximation (Eq. (17)) is not good enough, in the general case, the residuals then have to be modeled and approximated in another way or, alternatively neglected. Neglecting the residuals of course requires a large number, M, of modes and thus also a larger number, $N > M$, of measured responses compared to cases when good residual approximations are known. Approximation of residuals therefore is a very important task in application of the HMA technique in practical vibroacoustics (Dovstam, 1998).

3. STRAIN ANALYSIS

The idea behind the method is to obtain the strain tensor in a vibrating structure without the use of conventional strain gauges. Vibration (displacement, velocity or acceleration) measurements are carried out on the structure (Sehlstedt, 1999). Thereafter, the complete displacement field of the body in question, or a part of it, can be obtained by means of HMA. With the use of finite difference schemes the strain tensor can be calculated for the whole body, or just certain parts/points of interest.

The Eulerian finite strain tensor in Cartesian co-ordinates is defined as:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{r,i}u_{r,j}\right) \quad (19)$$

where the summation convention is used; i.e., sum on index k. In most applications the strain tensor, as in Eq. (19) is approximated as linear; i.e., the last term of Eq. (19) is neglected. Calculating the strain tensor field from the displacement field, u, in a three-dimensional body thus involves calculations of nine different first order derivatives.

Since u, v e w the displacement components along the axes x, y and z, respectively, the strain tensor can be expressed in matrix form, by:

$$\varepsilon_{ij} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z} \end{bmatrix} \quad (20)$$

For beams subjected to pure bending, initially straight, and with constant cross section along the length of the longitudinal normal strain specific ε_x can be found by the second derivative of the deflection function related to the distance y' between the neutral axis and the point of analysis , ie by analyzing the curvature of the beam. Eq. (21) shows is about:

$$\varepsilon_x(x) = -y'\frac{d^2y(x)}{dx^2} \quad (21)$$

By Eq. (21) the strain ε_x may be determined for any point x, since, knowing the deflection y of this point and the distance y'.

4. NUMERICAL SIMULATION

In this work, will be made to estimate the bending strain of a surface point of a beam, using the first derivative of the longitudinal displacements obtained by the Hybrid Modal Analysis. The size of the beam was 25.4 mm x 420mm x 3mm. The beam will be considered a conservative system with zero damping. The strain estimated by modal analysis will be compared with the hybrid strain obtained by analyzing the curvature of the beam. The beam used in the simulations is depicted in Fig. (1).

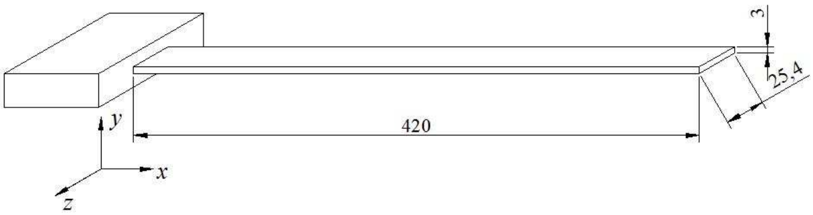

Figure 1. Beam used in the simulations

The first 10 vibration modes of the beam were obtained from the finite element model using ANSYS ® 11.0. In Tab. 1 are the parameters used in the analysis in finite elements.

Table 1. Parameters used in finite element simulation

Element type	SOLID186
Material Properties	Linear Elastic Isotropic E = 74 Gpa ν = 0,33 ρ = 2680 kg/m^3
Geometric model	Dimensions of the beam studies previously
Mesh Definition	56 elements Size: 15mm
Modal Analysis	10 modes analyzed Modes normalized by the mass Block Lanczos Algorithm

The vibration modes of displacements of all points of the beam in the *x* and *y* directions were extracted and saved for later use in the program developed in MATLAB ® 7.3. To simulate the use of hybrid modal analysis, the transverse displacements (*v*) to a harmonic exciting force, with frequency *f* = 70 Hz and *F* = 10 N, at the free end of the beam, were determined to simulate modal harmonic analysis in ANSYS ® 11.0. It was selected 29 points on the surface of the beam, spaced at 15 mm to observe the operating mode and use the simulation by HMA. Fig (2) illustrates the determined mode:

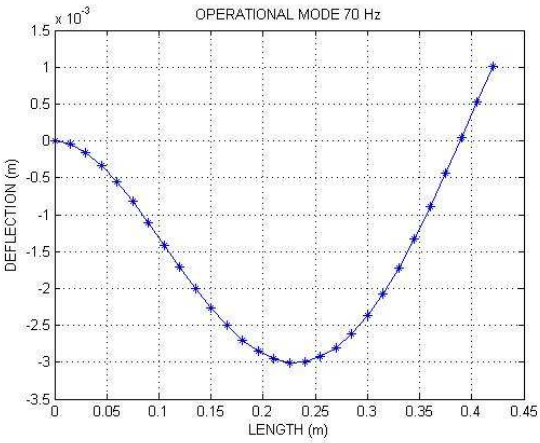

Figure 2. Operating mode 70 Hz

The displacements $\tilde{v}$ of the beam deflection, at a frequency of 70 Hz, were used to estimate the generalized Fourier coefficients for this frequency through modal analysis hybrid. Modal relations of these points were grouped into modal matrices. The 10 eigenmodes related to the 29 selected points and the *x* direction (longitudinal) were conditioned in a modal matrix $[\Phi\mathbf{e}_x]$ of dimensions 29×10, and the 10 eigenmodes related to 29 points and the *y* direction (transverse) were conditioned in a modal matrix $[\Phi\mathbf{e}_y]$.

Since $\{\tilde{u}_y\}_{29\times1}$ is the vector of displacements for the operating mode, and based on the concepts of hybrid modal analysis, the generalized Fourier coefficients for the frequency of 70 Hz, were estimated by Eq. (22).

$$\{\tilde{c}_{est}\} = [\Phi\mathbf{e}_y]^{+}\{\tilde{u}_y\} \tag{22}$$

From the generalized Fourier coefficients was possible to predict the displacements of the beam points in the direction *x*. The vector of these displacements have been determined by Eq. (23).

$$\{\tilde{u}_x\} = [\Phi\mathbf{e}_x]\{\tilde{c}_{est}\} \tag{23}$$

A point of analysis has been chosen and the predicted displacements have been spatially derivatives to estimative of the strain at this point. The beam in bending and displacements $\tilde{u}$ in the *x* direction are depicted in Fig (3).

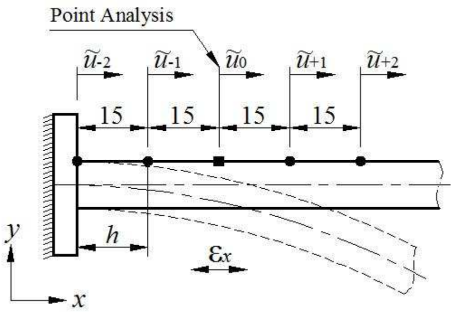

Figure 3. Displacements $\tilde{u}$ in the *x* direction

The strain ε_x at the point of analysis has been estimated using the finite central differences and first derivative of order $O(h^4)$. The finite difference equation may be written as Eq (24) (Mathews et al. 1999).

$$\varepsilon_x = \frac{-\tilde{u}_2 + 8\tilde{u}_1 - 8\tilde{u}_{-1} + \tilde{u}_{-2}}{12h} \tag{24}$$

where *h* is the distance between points, illustrated in Fig (3).

The value determined by Eq. (24) was compared with the strain predicted by the second derivative of displacement $\tilde{v}$ in the *y* direction, perpendicular to the direction of analysis of strain. The beam in bending and the displacement $\tilde{v}$ in the *y* direction are depicted in Fig (4).

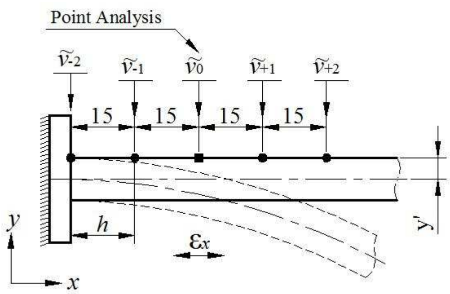

Figure 4. Displacement in the *y* direction

The finite difference equation of the second derivative may be written by Eq. below:

$$\frac{d^2\tilde{v}}{dx^2} = \frac{-\tilde{v}_2 + 16\tilde{v}_1 - 30\tilde{v}_0 + 16\tilde{v}_{-1} + \tilde{v}_{-2}}{12h^2} \tag{25}$$

According to Eq. (21) the strain can be predicted by multiplying the second derivative, Eq. (25), the distance from neutral axis to the surface of the beam, y' = 1,5mm. The strain values estimated by the derivative of the first and second order of the longitudinal and transverse displacements are listed in Tab. (2).

Table 2. Strain estimated by the first and second derivative

ε_x First derivative	$4{,}3993\ .10^{-4}$
ε_x Second derivative	$4{,}1895\ .10^{-4}$
Value Percentage	≈ 1%

The strain value estimated by the first derivative of the displacements obtained by modal analysis hybrid approached the value of the strain estimated by the derivative of transverse displacements obtained by the superposition of finite elements.

5. EXPERIMENT TEST CASE

In order to predict experimentally the dynamic strain distribution by HMA was realized an experimental evaluation. The analysis was performed on a cantilever aluminum beam. The beam has the same dimensions are provided in Section 2. To perform the experiment was mounting and connecting all the equipment as shown in Fig (5).

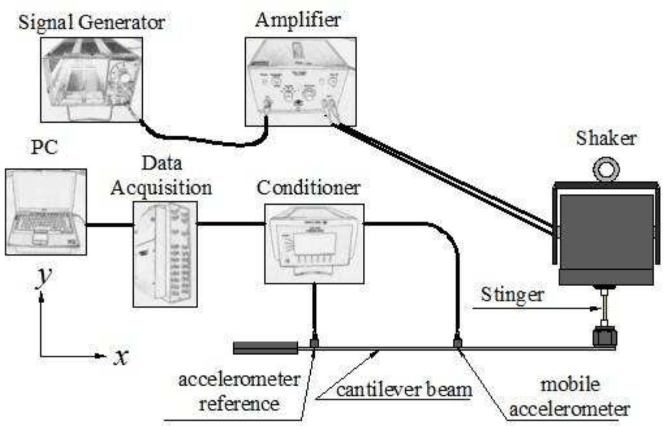

Figure 5. Arrangement of the experiment

The analyzed beam was subjected to a harmonic sinusoidal excitation with a frequency of approximately 70 Hz, generated by the signal generator. The accelerations was measured in the y direction of 29 points equally spaced on the surface of the beam using piezoelectric accelerometers Delta Tron ® brand type 4508 Brüel & Kjaer.

In the measurements, an accelerometer has remained fixed at the same point p (reference accelerometer) and one accelerometer (mobile accelerometer) was used for measurements of all other points j. The operating mode was determined by transmission T_{jp}, as follows:

$$T_{jp}(\omega) = \frac{S_{j,p}(\omega)}{S_{p,p}(\omega)} \tag{26}$$

where $S_{j,p}$ is the cross spectral density between the reference signal and the signal of each measured point and $S_{p,p}$ is the power spectral density of the reference point.

With information on the operating mode in the y direction of the beam was constructed a vector $\{\tilde{u}_y\}_{29\times1}$. Thus, through the displacement modal matrix $[\Phi \mathbf{e}_y]$, found earlier, and Eq. (22) were unable to estimate the coefficients. Later using Eq. (23), the displacements $\{\tilde{u}_x\}$ of 29 points in the longitudinal direction of the beam was estimated. With these displacements, and by equation (24) was possible to estimate the longitudinal strain on the surface points of the beam. Fig (6) illustrates the strain mode or the dynamic strain distribution of the beam predicted by hybrid modal analysis and simulated numerically by analyzing the curvature of the beam.

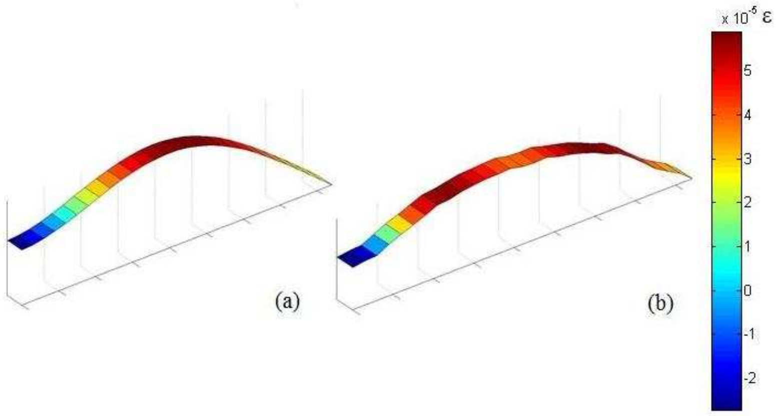

Figure 6. Operation Strain mode (a) numerical; (b) experimental

The distribution of longitudinal strain along the beam could be identified by hybrid modal analysis, as shown in Fig (6), when comparing the deformation modes obtained numerically and by experimental data using the HMA. Regions with dark blue colors show areas of higher deformation. This region is close to the crimp region of the beam, where the operational mode analyzed, it really is the region of greatest strain.

6. CONCLUSION

The hybrid modal analysis is a technique based on the estimation of weighting coefficients of modal contribution and can be used to supplement information not measured in the structure. In this work, the hybrid modal analysis to estimate the distribution of dynamic strain in a cantilever beam was analized . In the qualitative comparison of the distribution of strain predicted using data obtained from HMA and strain simulated by analyzing the curvature of the beam, one can observe good consistency of the results. Thus, the use of HMA to estimate the dynamic strain shows to be promising and opens a very wide range of issues that need to be studied in future.

7. ACKNOWLEDGEMENTS

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