CONJUGATE NATURAL CONVECTION-CONDUCTION HEAT TRANSFER IN A VERTICAL CHANNEL WITH NONUNIFORMLY HEATED PROTRUDING ELEMENTS

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Abstract. *An experimental analysis was performed to study conjugate convection-conduction heat transfer in a vertical parallel plate channel containing discrete and protruding two-dimensional heaters distributed uniformly on the surface of one of the plates. The objective is to predict the local temperature distribution in the protruding elements by nonuniform heating, based on measurements of the inverse discrete Green's function (DGF) matrix elements, called conjugate influence coefficients. The linear superposition method was used, assuming linear behavior of the energy conservation equation under the condition of negligible local buoyancy compared to the global flow induced throughout the channel. The channel is composed of an epoxy plate 0.0016 m thick, with seven protruding heat sources of polished aluminum mounted on its surface, and an adiabatic smooth plate on the opposite wall. The channel is 0.365 m long and 0.34 m wide, with channel wall to wall spacing of 0.02 m. The cross section of each protruding heat source is 0.0125 m long and 0.012 m high. In the experimental analysis, the global power was varied up to 45W over the entire channel. Initial experiments were performed by varying the power in one of the protruding elements while keeping it constant in the other elements to determine if the relationship between the power applied to the element and its temperature increase is linear. After this, each element was subjected to a no-power experiment and a uniform heating experiment to measure the overtemperature profiles and calculate the conjugate influence coefficients. These coefficients, at different power levels, were utilized to compare the experimental and predicted overtemperature profiles. The application of this methodology for the conjugate natural convection-conduction heat transfer problem provided a reasonable prediction of overtemperature distribution for arbitrary power dissipation in the protruding elements.*

Keywords: *Heat Transfer; Natural Convection-Conduction; Vertical Channel; Green's Function; Protruding Elements*

1. INTRODUCTION

This paper discusses the cooling of electronic components by conjugate natural convection-conduction in a vertical parallel plate channel. Protruding elements were mounted on one wall of the channel while the other wall was adiabatic. The purpose is to present a methodology applicable to an arbitrary thermal boundary condition (TBC), introducing an invariant descriptor of the heat transfer process that is independent of the TBC and is easily measured.

The general approach of Green's function is applicable to linear differential equations to which the linear superposition method can be applied. In the case of convective heat transfer, the energy equation is linear when the fluid velocity field and the fluid properties are independent of the temperature field. For the conjugate natural convection-conduction problem, this condition can be assumed provided that the local buoyancy effects are negligible compared to the global flow induced throughout the channel. Under this condition, the convection heat transfer at each individual element resembles a forced flow.

Ortega and Moffat (1985 and 1986) showed that, for buoyancy induced convection, an analytically exact linear superposition method can be implemented to predict element temperatures in a nonuniformly heated array of cubical elements on a vertical channel with closely spaced walls and good fluid mixing at any streamwise location. They examined the convective heat transfer from a single element using purely local descriptors and concluded that the heat transfer is driven primarily by globally induced channel flow, since the local buoyancy mechanisms are weak. A comparison of local heat transfer coefficient in both buoyancy induced forced flow and forced channel flow confirmed this conclusion. They calculated the element temperatures in buoyancy induced forced flow by linear superposition of the thermal contributions of all the upwind heated elements on the same vertical column that influences its temperature.

Arvizu and Moffat (1981) demonstrated that the superposition method can be used to describe the heat transfer in more complex flows on an array of heated rectangular elements in a vertical channel. They used the concept of the adiabatic temperature rise, which is defined as the temperature rise that would occur on a local adiabatic element due solely to the upstream heating effects. This rise is defined as the temperature reached when no-power is applied to the element, i.e., in the absence of conduction and radiation from other elements.

Anderson and Moffat (1990) extended the concept of adiabatic temperature by introducing the adiabatic heat transfer coefficient, an invariant descriptor of the convection heat transfer, which is independent of TBCs and is a function solely of the geometry, the flow field, and the fluid properties.

Hacker and Eaton (1997) introduced a concept of invariant descriptor for convection heat transfer in a backward-facing step flow obtained by the one-dimensional discrete Green's function method. In this new procedure, the convection coefficient concept is unnecessary, although it can be obtained knowing the discrete Green's function descriptor for a given condition.

Batchelder and Eaton (2001) summarized the development of superposition techniques in convective heat transfer, discussing heat transfer from a short uniform heat flux strip beneath a turbulent boundary layer with and without freestream turbulence. They showed that the one-dimensional discrete Green's function approach offers a convenient formulation of heat transfer problems in arbitrarily complex two-dimensional flows.

Examples of application of the one-dimensional discrete Green's function method can be found in Booten and Eaton (2005 and 2007) for turbulent internal flow in serpentine cooling passages in turbine blades, and in Mukerji and Eaton (2005) for a single passage turbine model.

Alves (2010) investigated conjugate heat transfer by conduction and forced convection from three 2D heaters either flush mounted or protruding from the lower plate of a horizontal parallel plate channel. In his thesis, he introduced the term "conjugate influence coefficient," which he proposed as an extension of the inverse discrete Green's function descriptor for conjugate forced convection-conduction problems to predict the temperature profile due to an arbitrary distribution of the heat dissipated in the heaters.

This paper describes an application of the discrete inverse Green's function descriptor for conjugate conduction-natural convection heat transfer in a vertical parallel plate channel. The conjugate influence coefficients obtained here can be employed to calculate the distribution of temperature in each protruding element at any power distribution using matrix-vector multiplication.

2. DISCRETE GREEN'S FUNCTION TECHNIQUE

Applying the inverse discrete Green's function formulation to the conjugate natural convection-conduction heat transfer from a surface with N discrete heat sources, the temperature rise ΔT_{ij} in protruding element i due to the power q_j dissipated in element j can be calculated from the following relation, employing the same notation used by Hacker and Eaton (1997),

$$\Delta T_{ij} = g_{ij}^{-1} q_j \tag{1}$$

where g_{ij}^{-1} is a generic element of the inverse discrete Green's function matrix, which Alves (2010) calls the conjugate influence coefficient and which represents the influence of the heating of element j on the temperature rise in element i.

Employing the superposition method, the relation between the temperature rise, i.e., the element i overtemperature $\Delta T_i = T_i - T_{amb}$, and the power imposed on the other elements, including itself, can be expressed as a summation of the individual contributions, as:

$$\Delta T_i = \sum_{j}^{N} \Delta T_{ij} \tag{2}$$

This can be written in the terms of a matrix multiplication

$$\Delta T = G^{-1} q \tag{3}$$

or even in the extended form

$$\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \vdots \\ \Delta T_N \end{bmatrix} = \begin{bmatrix} g_{11}^{-1} & g_{12}^{-1} & \cdots & g_{1N}^{-1} \\ g_{21}^{-1} & g_{22}^{-1} & \cdots & g_{2N}^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N1}^{-1} & g_{N2}^{-1} & \cdots & g_{NN}^{-1} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix} \tag{4}$$

In the present study, $N=7$ and the elements in the channel were numbered #1 to #7 for the sake of convenience. Figure 1 indicates how these coefficients are calculated, as exemplified by an analysis of the influence of element #4. Two kinds of experiments were performed: 0W – called the no-power experiment, and the uniform heating experiment. The no-power experiment is performed by disconnecting the element under study and applying the same power to the other elements. Uniform heating is achieved by applying the same power to all the elements.

The conjugate influence coefficients g_{ij}^{-1} are calculated from the temperature rise, which is given by the temperature difference between the overtemperature of element *i* in a uniform heating experiment and its overtemperature in a no-power experiment, divided by the power applied to element *j*. For example, in Figure 1, the conjugate influence coefficient in element #6 due to heating of element #4 is given by

$$g_{64}{}^{-1} = \frac{\Delta T_{64}}{q_4} = \frac{\left(T_{6,\text{uniform heating}} - T_{amb}\right) - \left(T_{6,\text{no-power}} - T_{amb}\right)}{q_4} \tag{5}$$

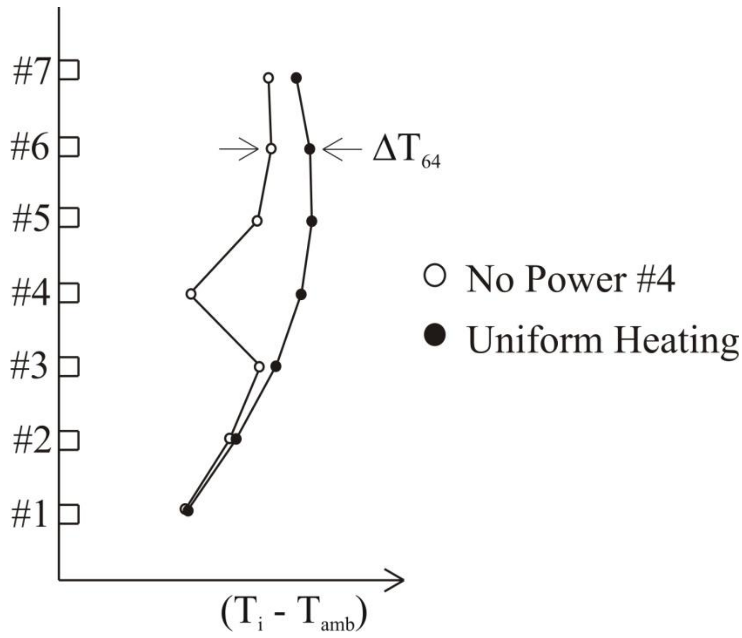

Figure 1 – Overtemperature profiles for uniform heating and no-power experiment

Repeating the methodology applied in the experiment of element #4 to all the other elements, which requires turning off the elements one by one, the conjugate influence coefficients matrix elements g_{ij}^{-1} can be calculated based on the temperature rise distribution and the power applied to all the elements.

3. DESCRIPTION OF THE EXPERIMENTAL SETUP

The experimental setup consists of two symmetrically positioned channels, one of which acts as a guard heater to reproduce an adiabatic wall condition. Each channel is composed of a 0.0016 m thick epoxy plate (b) with seven protruding heat sources of polished aluminum mounted on its surface, and an adiabatic smooth plate on the opposite wall, composed of cork, epoxy and expanded polystyrene plates. The channel is 0.365 m long (l) and 0.34 m wide (w), with wall to wall spacing of 0.02 m (h). The channels were closed on both sides to prevent lateral airflow. Figure 2 shows a schematic diagram of the experimental apparatus.

A side view of the epoxy plate in Fig. 3 shows the points where the temperature of the protruding heat sources is measured. This measurement was taken with AWG 36 gauge Type J copper-constantan thermocouples with a diameter of 0.127mm. Small holes were drilled into the protruding elements to insert the thermocouples.

Figure 3 also shows a three-dimensional view of the protruding heat source and its installation. The protruding element consisted of two aluminum bars glued with a mixture of araldite adhesive and zinc oxide. A resistance wire composed of nickel-chrome alloy with a 0.3mm diameter and 10Ω was placed at the center of the protruding element. A rectangular groove was cut to hold the resistance wire between the bars. The surfaces of each protruding element were polished with diamond paste to minimize radiation losses. One of the surfaces was then coated with a thin layer of thermal paste to minimize contact resistance and was screwed onto the channel wall. Each protruding heat source had a 0.0125 m long, 0.012 m tall cross section. The space between the protruding heat sources on the epoxy plate was 0.035m.

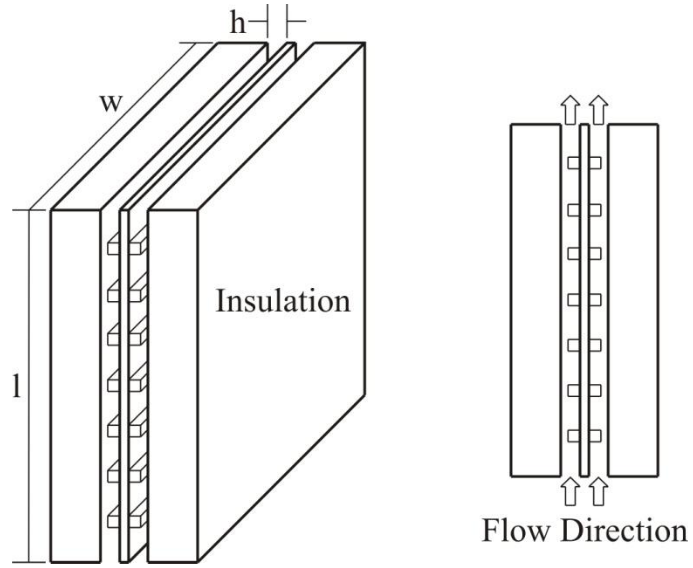

Figure 2 – Schematic diagram of the experimental apparatus in 3D and 2D views

Each symmetric couple of protruding heat sources was connected to a D.C. source. With this experimental setup, the power can be varied independently in each element because the electrical circuit of each couple is independent, so that discrete nonuniform heating can be applied to the channel.

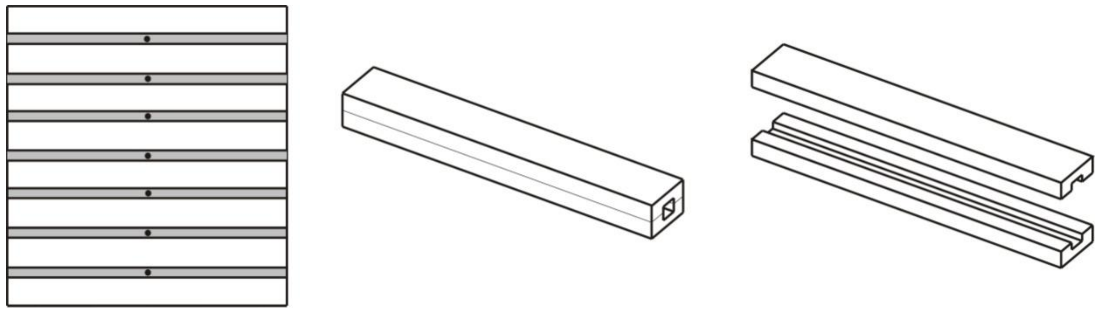

Figure 3 – Points of temperature measurement and a 3D view of the protruding heat source and its installation

The experimental apparatus was mounted on a structure standing one meter off the ground in a quiet room isolated from the external environment. The D.C. sources were placed in another room to prevent changes in the local room temperature.

4. RESULTS AND DISCUSSION

The experiments were divided into three parts. First, experiments were performed to verify the linearity of the energy conservation equation that can be assumed for buoyancy induced flow. The second part consisted of experiments to calculate the conjugate influence coefficients of the inverse discrete Green's function matrix. The third part involved nonuniform heating tests to compare experimental data and predicted overtemperature distribution calculated from the conjugate influence coefficients.

4.1. Buoyancy induced forced flow

Ortega and Moffat (1985 and 1986) used linear superposition to predict the temperature profile of nonuniform heating under conditions of weak local buoyancy effects compared to the globally induced channel flow.

To verify this condition, the relation between the overtemperature (ΔT_i) and the power q_i applied to element i was determined experimentally. Element #4 was chosen for this study. The experiments were performed by varying the power in element #4 incrementally, and applying a constant power of 5W to the other elements in all the cases analyzed. The power applied to element #4 ranged from 0W to 15W, with increments of 1W. Figure 4 shows that in the range of power studied here, the overtemperature in element #4 (ΔT_4) increased linearly with the increase in power applied to element #4. This figure also illustrates the temperature rise profile, ΔT_{44}, resulting from the variation in power q_4, which is given by the difference between the overtemperature ΔT_4 and the overtemperature reached in the no-power experiment of element #4.

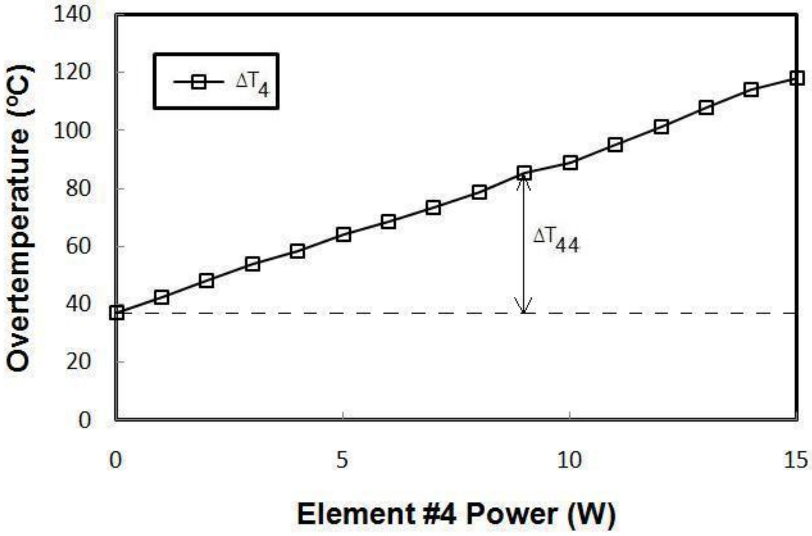

Figure 4 – Measured overtemperature profile in element #4

As expected from the previous result, Fig. 5 shows that the conjugate influence coefficient $g_{44}^{-1}=\Delta T_{44}/q_4$, i.e., the influence of self-heating in element #4, is almost constant, varying by no more than 5%. At the global power level of 45W, an extreme case in which 1/3 of the global power was applied to element #4, the conjugate influence coefficient still remained approximately constant, indicating that the linear superposition method is applicable up to this extreme case.

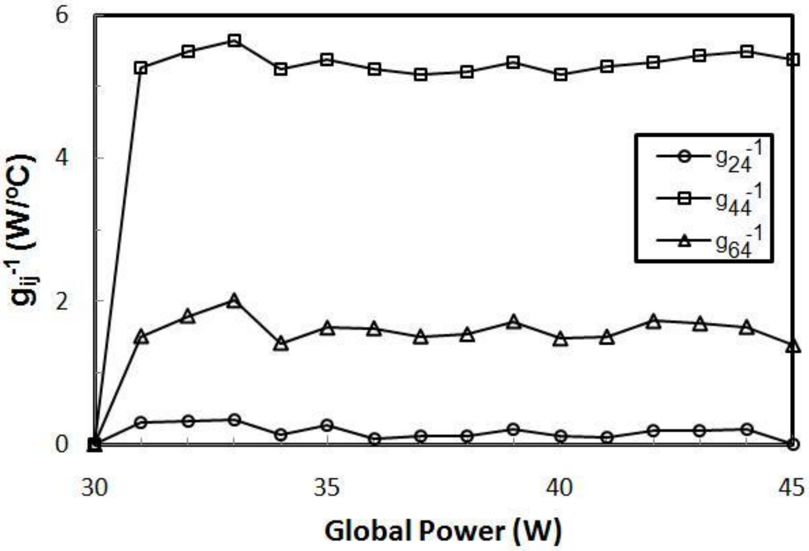

Figure 5 – Conjugate influence coefficients

Figure 5 also shows the influence of the heating of element #4 on the overtemperatures of elements #2 and #6, which are expressed by g_{24}^{-1} and g_{64}^{-1}, respectively. These conjugate influence coefficients follow the same trend as that observed in g_{44}^{-1}, characterized by a linear relation between the temperature rises, ΔT_{24} and ΔT_{64}, and the power applied, q_4. Note that the influence of element #4 heating on the upstream elements, due to conduction through the substrate and the possible flow recirculation between the protruding elements, is less marked than the influence of element #4 heating on the downstream elements, due to the thermal wake effect and substrate conduction.

Another experiment involved varying the power applied to element #4 up to 3W and applying a power of 1W/element to the other six elements. The conjugate influence coefficient was also found to vary within 5 percent. This suggests that, for the geometric arrangement under study, a value of up to 1/3 of the global power per element can be adopted as a criterion for the linear behavior of the energy conservation equation, as evidenced by the constant value of the conjugate influence coefficient. The same result was assumed to be valid for all the other elements mounted on the wall channel. The six remaining elements were also subjected to the above described experiments, using the same power. Similar experiments, but applying the same total power distributed nonuniformly to the other six elements, presented practically identical results to those reported.

4.2. Temperature measurement and calculation of the conjugate influence coefficients

Experiments were performed with a global power of 1W/element and of 5W/element, following the methodology described in Section 2, to identify the elements of the conjugate influence coefficients matrix. Each element was subjected to a uniform heating experiment, followed by the no-power experiment.

Figure 6 shows the overtemperature profiles at the power level of 1W/element, while Fig. 7 shows the overtemperature profiles at the power level of 5W/element. The profiles in these figures correspond to the following experiments: uniform heating and the no-power experiment of elements #1, #4 and #7, respectively.

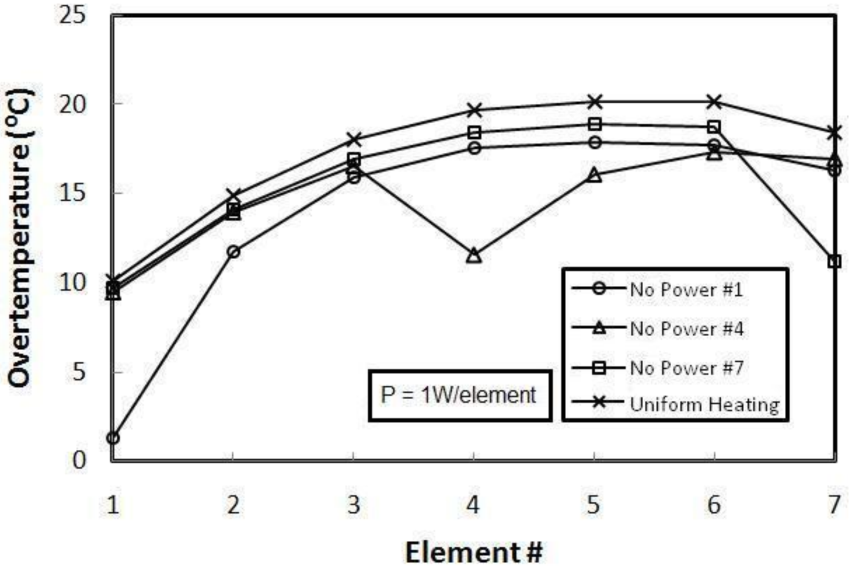

Figure 6 – Overtemperature profiles at a power level of 1W/element

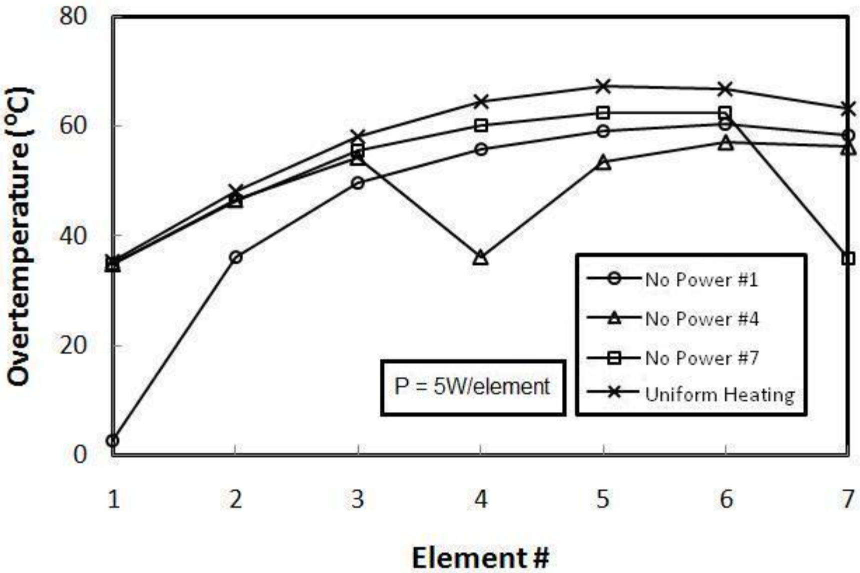

Figure 7 – Overtemperature profiles at a power level of 5W/element

In Figures 6 and 7, the thermal wake effect of element *j* on the downstream elements is clearly visible by comparing the no-power experiment overtemperature curve of element #1 to the uniform heating overtemperature profile. All the overtemperatures in downstream elements are higher than those achieved in the no-power experiment. The effect of the conduction through the substrate and the flow recirculation between the upstream elements is illustrated by the curve of the no-power experiment of element #7. All the overtemperatures in upstream elements are lower than the overtemperatures achieved in uniform heating, but their influence is minor compared to the thermal wake effect.

The conjugate influence coefficients were calculated, as exemplified in Eq. 5, based on the overtemperature profiles for a given heating imposed on the elements in the no-power and uniform heating experiments.

Figure 8 shows the conjugate influence coefficients profiles, g_{ij}^{-1}, for the power level of 1W/element, while Fig. 9 shows the g_{ij}^{-1} for the power level of 5W/element. These figures depict the profiles of elements #1, #2, #4 and #7, respectively.

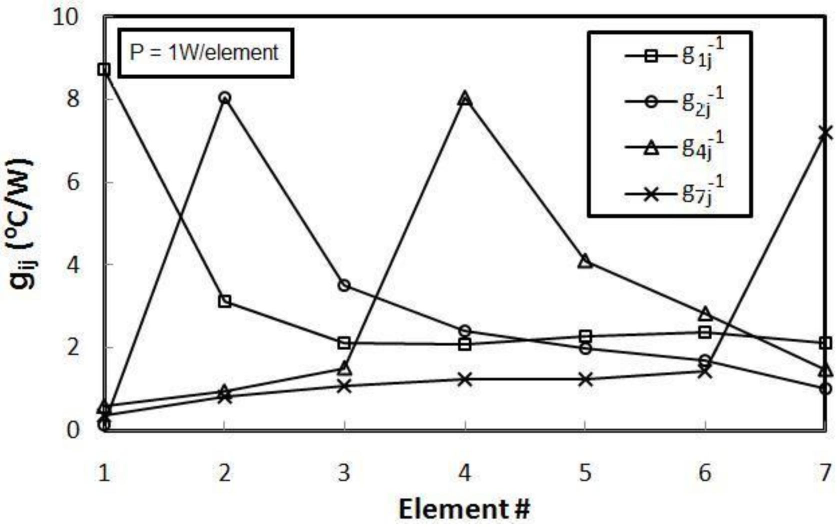

Figure 8 – Conjugate influence coefficient profiles at the power level of 1W/element

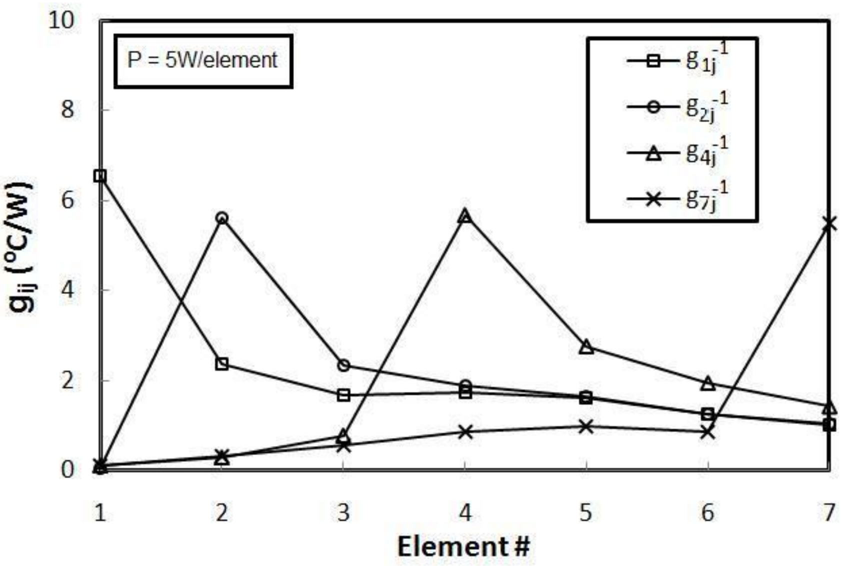

Figure 9 – Conjugate influence coefficient profiles at the power level of 5W/element

A comparison of the conjugate influence coefficients illustrated in Figs. 8 and 9 indicates that the g_{ij}^{-1} at the power level of 1W/element is higher than the g_{ij}^{-1} at the power level of 5W/element. This behavior is explained by the increase of the global power, which raises the overtemperatures throughout the channel. This rise increases the cumulative buoyancy effects, as well as the mass flow rate across the channel and the convection heat transfer mechanism. By definition, the conjugate influence coefficient must vary contrarily to the convection heat transfer coefficient. Hence, the conjugate influence coefficient decreases as the global power increases, as indicated in the comparison of Figs. 8

and 9. Therefore, the influence of the heating of element *j* on the rise in temperature of element *i* is greater at the power level of 1W/element than at that 5W/element.

The g_{4j} curves shown in Figs. 8 and 9 also demonstrate that the influence of element *j* heating on the upstream elements caused by the effects of conduction through the substrate and flow recirculation between the elements is weaker than its influence on the downstream elements, due primarily to the thermal wake effect.

4.3. Overtemperature prediction in nonuniform heating test cases

Experimental nonuniform heating test cases were carried out to compare the experimental data with the predicted overtemperature profiles, ΔT_i, calculated from the conjugate influence coefficients. Table 1 lists the power applied to each element in the experimental test cases. The global power of the test cases was 7W for test case 01, 30W for test case 02 and 45W for test case 03. A sharp alternate turns profile was chosen intentionally to test the prediction procedure.

Table 1 – Power applied to each element in the experimental test cases

Power Applied	Element #1	Element #2	Element #3	Element #4	Element #5	Element #6	Element #7
Test Case 01	2W	0W	2W	0W	2W	0W	1W
Test Case 02	7W	3W	7W	0W	3W	7W	3W
Test Case 03	10W	5W	10W	0W	5W	10W	5W

Figure 10 compares the experimental and predicted overtemperature profiles, ΔT_i, of the test cases. The predicted overtemperature profiles were calculated from Eq. 03, using the conjugate influence coefficients, g_{ij}^{-1}, calculated from the experimental results presented in Section 4.2.

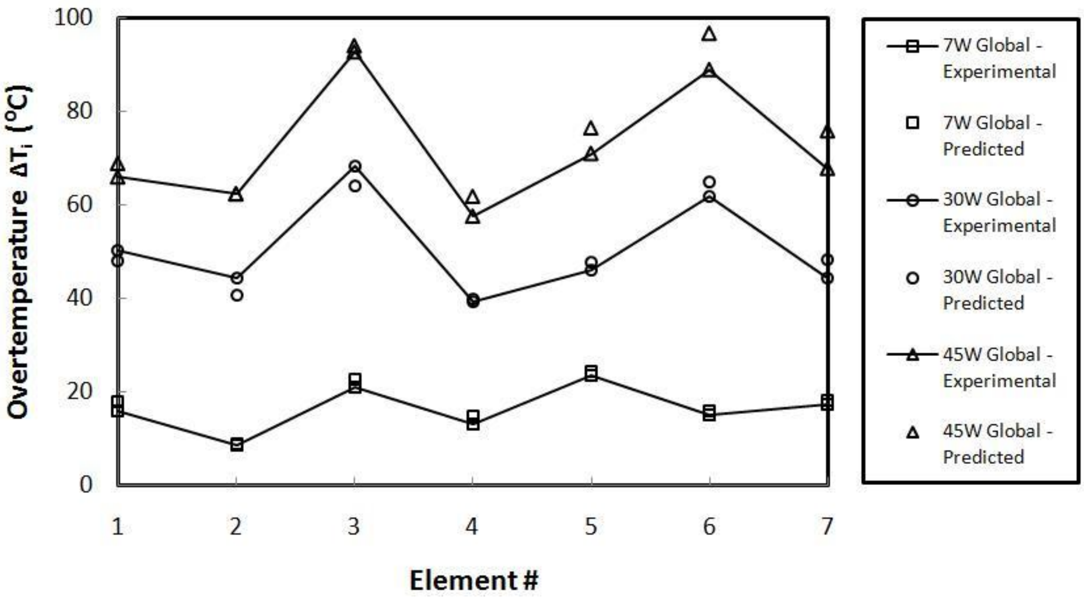

Figure 10 – Comparison of experimental and predicted overtemperature profiles of the test cases

In Figure 10, which illustrates test case 01, the conjugate influence coefficients overpredict the experimental data with a maximum deviation of 13 percent, as indicated by the overtemperature of element #1. In test case 02, the maximum deviation of overtemperature was 9 percent, which occurred in element #7. Finally, maximum deviation in test case 03 was 12 percent in element #7.

5. CONCLUSIONS

This paper discussed the discrete inverse Green's function applied to conjugate natural convection-conduction heat transfer in a vertical parallel plate channel. The linear superposition method used here was only applicable because the linearity of the energy conservation equation for natural convection problems, indicated by the linear relation between the temperature rise and the applied power, proved to be valid for the conditions of this study.

Test cases of nonuniform heating were performed to compare the experimental results with the predicted profiles obtained from the conjugate influence coefficients of the matrix G^{-1}. The comparison of experimental and predicted overtemperature profiles revealed a small deviation. Therefore, the methodology is valid for the cases analyzed here, and the inverse discrete Green's function approach proved to be a practical and viable methodology to calculate the overtemperature distribution. It is worth noting that the conjugate influence coefficients used in this work are applicable only to the geometric arrangement described here, defined by the channel geometry and the protruding heat sources distribution and size, as well as the materials and coolant fluid.

Because the present study is a work in progress, it would be necessary to interpolate the results to various power levels and fit a curve that correlated the conjugate influence coefficients to any power level within the power range studied. In addition, a proper criterion based on dimensionless parameters must be determined to generalize the conditions in which linear superposition methods can be applied to natural convection problems. This will enable the general inverse discrete Green's function to be employed to predict overtemperature profiles of arbitrary nonuniform heating in arrangements of discrete heat sources cooled by natural convection.

6. NOMENCLATURE

A = surface area of heat exchange
G^{-1} = inverse discrete Green's function matrix
g_{ij}^{-1} = conjugate influence coefficient of the inverse discrete Green's function matrix
h = wall to wall spacing
l = channel vertical length
q_j = power applied to element *j*
T_{amb} = ambient temperature
T_i = experimentally measured local temperature in element *i*
ΔT_i = overtemperature in element *i*
ΔT_{ij} = temperature rise in element *i* due to element *j*
w = channel width

7. REFERENCES

Alves, T. A., 2010, "Resfriamento Conjugado de Aquecedores Discretos em Canais", Campinas: Mechanical Engineering Faculty, State University of Campinas, 121p, Thesis.

Anderson, A., and Moffat, R. J., 1990, "Convective Heat Transfer From Arrays of Modules With Non-Uniform Heating: Experiments and Models", Report Nº. HMT-43, Thermosciences Division, Department of Mechanical Engineering, Stanford University, Stanford , CA.

Arvizu, D. E., and Moffat, R. J., 1981, "Experimental Heat Transfer from an Array of Heated Cubical Elements on an Adiabatic Channel Wall", Report Nº. HMT-33, Thermosciences Division, Department of Mechanical Engineering, Stanford University, Stanford , CA.

Batchelder, K. A., and Eaton, J. K., 2001, "Practical Experience With the Discrete Green´s Function Approach to Convective Heat Transfer". ASME, Journal of Heat Transfer, Vol. 123, pp. 70-76.

Booten, C., and Eaton, J. K., 2005, "Discrete Green´s Function Measurements in Internal Flows". ASME, Journal of Heat Transfer, Vol. 127, pp. 692-698.

Booten, C., and Eaton, J. K., 2007, "Discrete Green´s Function Measurements in a Serpentine Cooling Passage". ASME, Journal of Heat Transfer, Vol. 129, pp. 1686-1696.

Hacker, J. M., and Eaton, J. K., 1997, "Measurements of Heat Transfer in a Separated and Reattaching Flow With Spatially Varying Thermal Boundary Conditions", Int. Journal of Heat and Fluid Flow, Vol. 18, pp.131-141.

Mukerji, D., and Eaton, J. K., 2005, "Discrete Green´s Function Measurements in a Single Passage Turbine Model". ASME, Journal of Heat Transfer, Vol. 127, pp. 366-376.

Ortega, A., and Moffat, R. J., 1986, "Buoyancy induced convection in a non-uniformly heated array of cubical elements on a vertical channel wall", Heat Transfer in Electronic Equipment, Vol. 57, pp. 123-134.

Ortega, A., and Moffat, R. J., 1985, "Heat Transfer from an Array of Simulated Electronic Components: Experimental Results for Free Convection with and Without a Shrouding Wall", Heat Transfer in Electronic Equipment, Vol. 48, pp. 5-15.

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