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PIEZOELECTRIC OPTIMUM PLACEMENT SIMULATION USING LQR CONTROLLER IN A SIMPLY SUPPORTED BEAM

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Abstract. *The piezoelectric elements, that show the piezoelectricity property, have received a lot of attention from researchers. The reasons of this interest are because the piezoelectric materials are small, lightweight and resilient against adverse working environments. Moreover piezoelectric materials have been used as both actuators and sensors. Among these materials, there are the ceramics PZT, piezoelectric zirconate titanate lead, and polymers PVDF, piezoeletric vinylidede fluoride. Actuators and sensors placement study is a fundamental part to avoid undesirable effects in flexible structure under control, such as lack of observability and controllability system. It was used a singular analysis of input control matrix as a piezoelectric placement tool. After piezoelectric placement study, it was checked these positions through the piezoelectric elements placement in an optimum and no optimum positions and simulating the control through linear quadratic regulator technique in both positions. The flexible structure used as a model is a simply supported beam. The simulation has been developed in a Matlab ® environment.*

Keywords*: singular analysis, piezoelectric placement, linear quadratic regulator*

1. INTRODUCTION

The vibrations active control nowadays is real, its results are more effective than vibrations passive control. According to new emphasis, a structure could have its response minimized, using integrating active elements, such as: sensors, actuators and controllers. This integration would do the system answers the controlled mode of the outside excitation, compensating the effects that could move away its response of acceptable levels. Nowadays, these systems, integrating structures, sensors, actuators and controllers, are known as intelligent structures (Oliveira. 2003).

Several technologies and materials have been researched and proposed in the development of these intelligent structures. Among these materials, there are the piezoelectric materials, especially the ceramics, PZT ï piezoelectric zirconate titanate lead and polymer films, PVDF ï piezoelectric vinylidene fluoride (Lima Jr. & Arruda, 1999). The active control using piezoelectric materials is a topic of a lot of interest among the researchers. The reason of this interest is because the piezoelectric materials are small, lightweight and resilient against adverse working environments. Moreover piezoelectric materials have been used as both actuators and sensors, because they are owner of the ability to transform mechanical energy to electrical and vice versa (Wang, 2001). The ceramics have high stiffness, therefore are used as actuators, while that the polymer films are more handler than ceramics and can be produced in complex geometric shapes, for this reason, they are used as sensors, (Oliveira, 2008).

The intelligent structure design is dividing in three areas: Modeling in finite element method (FEM), Actuators and sensors placement and system controller. In a good intelligent structure design, actuators and sensors placement study is fundamental part to avoid undesirable effects in a structure under active control, such as: Lack of observability and controllability system and spillover, (Costa e Silva e Arruda, 1997). The purpose of this paper is to suggest, for optimum piezoelectric actuators and sensors placement in a flexible structure, measurements of the modal and spatial controllability. To quantify the controllability index, this work intends to use the singular value analysis through the [S] matrix. The system controller simulated in this work uses piezoelectric elements modal placement technique, where the optimum position is the maximum deformation modal for a specific mode. This placement and control technique is called IMSC (Independent Modal Space Control), it combines the modal decomposition with classical control law LQR (Linear Quadratic Regulator), (Carvalhal et al., 2005). The gain control for each mode can be found by solving the equation of second order of Ricatti.

2 MODELING OF BEAM WITH PIEZELETRIC ATTACHED

The Euler ï Bernoulli beam equation comes from shield model (Novozhilov, 1970) and the effects of piezoelectric actuator are introduced in the beam model by (Blanks & Wang, 1995). The Euler ï Bernoulli beam equation is:

$$\rho hb\frac{\partial^2 w}{\partial t^2}+YI\frac{\partial^4 w}{\partial x^4}=bF_z+b\frac{\partial m_x}{\partial x} \quad (1)$$

where: ρ is the material density (kg/m^3), Y is the Young module (Gpa), h is the thickness (m), b is the width (m), I is the inertia momentum (m^4), F_z is the force (N), m_x is the momentum (Nm/m) and *w* is the transversal displacement.

2.1 Actuator equation

The contribution of piezoelectric material can be dividing in two classes, inside, material, and outside, forces and momentum. The inside contribution is due structure material propriety, such as: Mass, stiffness and damping and is present although no electric potential is apply, while that the outside contribution is due induced deformation when a potential electric is apply in PZT, (Tzou & Fu, 1994; Banks & Wang, 1995). The deformation amplitude induced in PZT, according to shown in Figure 1, is:

$$\varepsilon_{pe} = \left(\varepsilon_x\right)_{pe} = \frac{d_{31}}{h^a}\phi^a \tag{2}$$

where: Ů$_{pe}$ is the induced deformation, d_{31} the piezoelectric constant (m/V), ϕ^a electric potential applies in the actuator (V). The individual stress, ů$_x$ (Gpa), in PZT is:

$$\left(\sigma_x\right)_{pe} = -Y_{pe}\varepsilon_{pe} = -Y_{pe}\frac{d_{31}}{h^a}\phi^a \tag{3}$$

Integrating the voltage under element face, the results force and outside momentum, due PZT individual activation, can be writing like this:

$$\left(bN_x\right)_{pe} = -Y_{pe}bh_{pe}\varepsilon_{pe} = -Y_{pe}bd_{31}\phi^a \tag{4}$$

$$\left(bM_x\right)_{pe} = -\frac{1}{8}Y_{pe}b\left[4\left(\frac{h}{2}+h^a\right)^2 - h^2\right]\varepsilon_{pe} = -\frac{1}{2}Y_{pe}b\left(h+h^a\right)d_{31}\phi^a \tag{5}$$

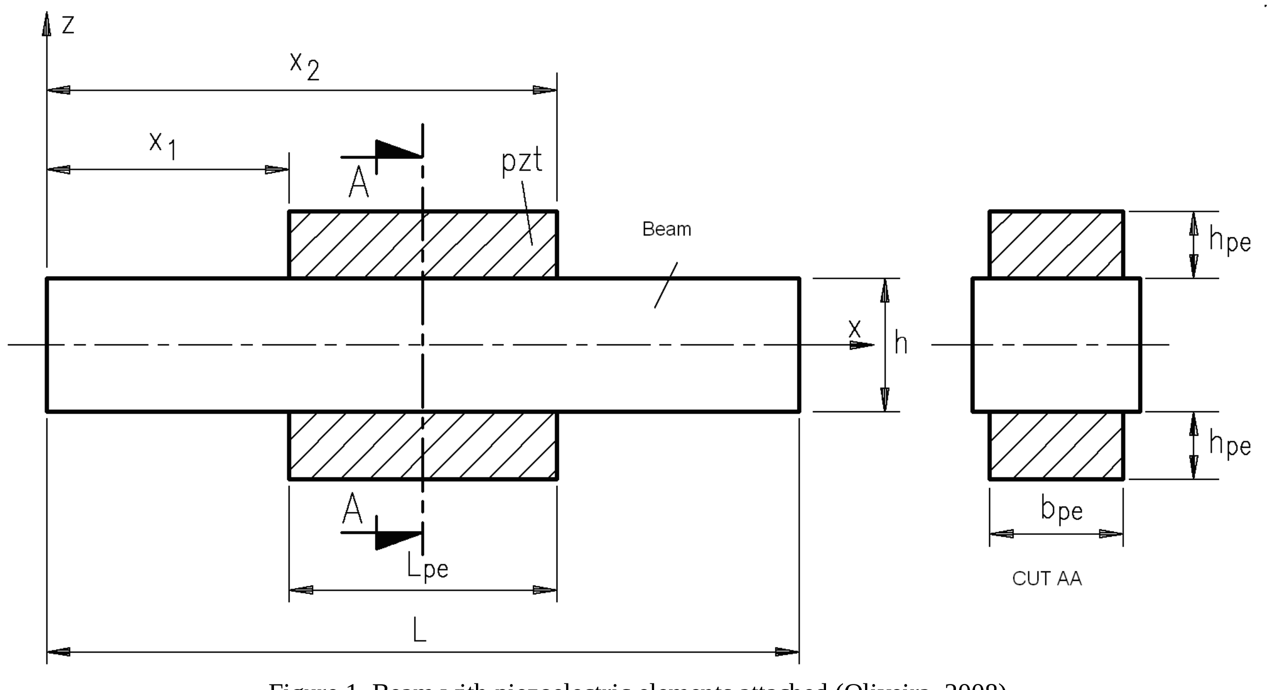

Figure 1. Beam with piezoelectric elements attached (Oliveira, 2008)

The Eq. (4) and Eq. (5) can be modified for finite piezoelectric, therefore for PZT with x_1 and x_2 length, the forces and momentumôs are:

$$bF_x = (bF_x)_{pe} = -\psi_{pe}(x)\chi_{pe}(x)\frac{\partial(bN_x)_{pe}}{\partial x} \tag{6}$$

$$bm_x = (bm_x)_{pe} = \chi_{pe}(x)\frac{\partial(bM_x)_{pe}}{\partial x} \tag{7}$$

$$\chi_{pe} = \begin{cases} 1 \; x_1 \le x \le x_2 \\ 0 \; otherwise \end{cases} and \;\; \psi_{pe} = \begin{cases} 1 \; x\langle(x_1 + x_2)/2 \\ 0 \; x = (x_1 + x_2)/2 \\ 1 \; x\rangle(x_1 + x_2)/2 \end{cases} \tag{8}$$

2.2 Sensor equation

The piezoelectric sensor equation comes from piezoelectricity and relation between stress and beam deformation. The piezoelectric material thickness is smaller than beam thickness, so the piezoelectric sensor deformation is constant, equal structure surface deformation. The voltage through the electrodes is:

$$\phi = -\int_{h^s} E_3 dz = h^s\left(h_{31}\varepsilon_x^s + \beta_{33}D_3\right) \tag{9}$$

where: β_{33} is the electric unpermeability (m/F). Rewrite the Eq. (9), yields:

$$D_3^s = \frac{1}{\beta_{33}}\left(h_{31}\varepsilon_x^s - \frac{\phi}{h^s}\right) \tag{10}$$

The D^s_3 is defined like load per area unit, integrating Eq. (10) under electrode surface, results the total surface load. The voltage open circuit can be obtained through zero load, then:

$$\phi^s = \frac{h^s}{S^e}\int_{S^e}\left(h_{31}\varepsilon_x^s\right)dS^e = \frac{h^s}{x_2 - x_1}\int_x\left(h_{31}\varepsilon_x^s\right)dx \tag{11}$$

So the Euler ï Bernoulli beam sensor equation, is:

$$\phi^s = -\frac{h^s}{L^s}\int_{x_1}^{x_2}\left(h_{31}h_r^s\frac{\partial^2 w}{\partial x^2}\right)dx \tag{12}$$

3 MATHEMATICS MODELING

The piezoelectric linear equation is given by (Cady, 1946):

$$\begin{aligned} \{\sigma\} &= \left[c^E\right]\{\varepsilon\} - [e]\{E\} \\ \{D\} &= [e]^T\{\varepsilon\} - \left[\xi^\varepsilon\right]\{E\} \end{aligned} \tag{13}$$

where:

$$\begin{aligned} [e] &= \left[c^E\right][d] \\ \left[\xi^\varepsilon\right] &= \left[\xi^\sigma\right] - [d]^T\left[c^E\right][d] \end{aligned} \tag{14}$$

where: {σ}- stress tensor; {ε}- deformation tensor; {E}- electric field vector; {D}- electric displacement vector; [C^E]- elasticity matrix for constant electric field; [e]- piezoelectric constants matrix; [ξ^ε]- dielectric constants tensor for constant deformation [ξ^σ]- dielectric constant matrix for constant stress; [d]- constant matrix of piezoelectric deformations. The variational principle equation for piezoelectric material is given by:

$$\begin{aligned}&\iiint_V \rho\{\delta u\}^T\{\ddot{u}\}dV + \iiint_V \{\delta\varepsilon\}^T\left[c^E\right]\{\varepsilon\}dV - \iiint_V \{\delta\varepsilon\}^T[e]^T\{E\}dV - \iiint_V \{\delta E\}^T[e]\{\varepsilon\}dV \\ &-\iiint_V \{\delta E\}^T[\xi]\{E\}dV = \iiint_V \{\delta u\}^T\{\bar{f}_V\}dV + \iint_{S_f}\{\delta u\}^T\{\bar{f}_S\}dS - \iint_{Sq}\delta\phi\sigma_q dS\end{aligned} \quad (15)$$

3.1 The finite element method

The structure discretization has been done with isoparametric beam element, therefore with four degree of freedom by node. The form function, $[N_i(x)]^T$, is a cubic polynomial. For the node the approximations are:

$$\begin{aligned}u'(y,t) &= \frac{d}{dx}[N_1(x)]^T\{q_u\} = [B_u]^T\{q_u\} \\ w'(z,t) &= \frac{d}{dx}[N_2(x)]^T\{q_w\} = [B_w]^T\{q_w\} \\ w''(z,t) &= \frac{d^2}{dx^2}[N_2(x)]^T\{q_w\} = [B'_w]^T\{q_w\}\end{aligned} \quad (16)$$

where:

$$\{q_u\} = \begin{bmatrix}u_i & \psi_i & u_j & \psi_j\end{bmatrix}^T, \{q_w\} = \begin{bmatrix}w_i & \theta_i & w_j & \theta_j\end{bmatrix}^T \quad (17)$$

3.2 Strain energy

The strain energy for piezoelectric materials in the matrix form is:

$$\delta u = \iiint_{V_{st}} \{\delta\varepsilon\}^T\{\sigma\}dV_{st} - \iiint_{V_{pe}} \{\delta E\}^T\{D\}dV_{pe} \quad (18)$$

In the beam model proposed, there are two domains. The first is structure material, V_{st} domain, and the second is piezoelectric material, V_{pe} domain. The model equation is:

$$\begin{aligned}&\{\varepsilon\} = \varepsilon_x \quad \left[c^E\right] = Y_{pe} \quad \{\sigma\} = \sigma_x, \\ &\{e\} = e_{31} \quad \left[\zeta^\varepsilon\right] = \zeta_{33}^\varepsilon \quad [D] = D_3 \ \{E\} = E_3\end{aligned} \quad (19)$$

The approaching by finite element method, the relation in matrix form is:

$$\{\varepsilon\} = \{[B_u] - z[B'_w]\}\{q_i\} \quad (21)$$

Replacing Eq. (20) and (19) in Eq. (18), results:

$$\begin{aligned}\delta U &= \{\delta q_i\}^T \iiint_{V_{pe}} ([B_u] - z[B'_w])^T Y_{pe} ([B_u] - z[B'_w]) d\Omega_{pe} \{q_i\} \\ &+ \{\delta q_i\}^T \iiint_{V_{pe}} ([B_u] - z[B'_w])^T e_{31} [B_\phi] d\Omega_{pe} \{\phi_i\} \\ &+ \{\delta \phi_i\}^T \iiint_{V_{pe}} [B_\phi]^T e_{31} ([B_u] - z[B'_w]) d\Omega_{pe} \{q_i\} \\ &- \{\delta \phi_i\}^T \iiint_{V_{pe}} [B_\phi]^T \xi_{33}^{\varepsilon} [B_\phi] d\Omega_{pe} \{\phi_i\}\end{aligned} \tag{22}$$

Resulting:

$$[k_{q\phi}] = Y_{pe} A_{pe} d_{31} \int_0^1 [B_u]^T [B_u] dx - \left(h + \frac{h_{pe}}{2} \right) Y_{pe} A_{pe} d_{31} L \int_0^1 [B'_w]^T [B_w] dx \tag{23}$$

$$[k_{\phi\phi}]_e = \frac{\xi_{33}^{\varepsilon} A_{pe} L}{h_{pe}^2} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{24}$$

$$[k_{qq}] = L(Y_{st} A_{st} + Y_{pe} A_{pe}) \int_0^1 [B_u]^T [B_u] d\xi + L(Y_{st} I_{st} + Y_{pe} I_{pe}) \int_0^1 [B'_w]^T [B'_w] d\xi \tag{25}$$

The index st refers structure material and the index pe refers piezoelectric material.

3.3 Kinetic energy

The kinetic energy variational equation, applied for proposed beam element, results:

$$[m_{st}] = \rho_{st} A_{st} L \int_0^1 [N_q]^T [N_q] d\xi, \quad [m_{pe}] = \rho_{pe} A_{pe} L \int_0^1 [N_q]^T [N_q] d\xi \tag{26}$$

where: $[m_{st}]$ is the structure mass matrix and $[m_{pe}]$ is the piezoelectric mass matrix.

3.4 The work

Applying the work variational, realized by outside loads and forces, results:

$$\{f_q\} = \int_0^1 [N_w]^T \{\bar{f}_s\} L d\xi, \{q_s\} = -\int_0^1 [N_\phi]^T \sigma_q L d\xi \tag{27}$$

3.5 Equation global system

The equations global system is given by:

$$\begin{cases} [M_{qq}]\{\ddot{q}_i\} + [k_{qq}]\{q_i\} + [k_{q\phi}]\{\phi_i\} = \{F_S\} \\ [k_{\phi q}]\{q_i\} + [k_{\phi\phi}]\{\phi_i\} = \{Q_s\} \end{cases} \tag{28}$$

In the piezoelectric sensor there isnȇt voltage applied (Qs = 0). Then the electric potential yielded by sensor is:

$$\{\phi_s\} = -[K_{\phi\phi}]^{-1}[K_{\phi q}]\{q_i\} \tag{29}$$

Replacing the Eq. (29) in the Eq. (28), yields the equation global system for a beam with actuator attached, that is:

$$[\mathrm{M}_{qq}]\{\ddot{\mathrm{q}}_i\} + [\mathrm{K}^*_{qq}]\{\mathrm{q}_i\} = \{\mathrm{F}_s\} + \{\mathrm{F}_{el}\} \tag{30}$$

where:

$$[K^*_{qq}] = [K_{qq}] - [K_{q\phi}][K_{\phi\phi}]^{-1}[K_{\phi q}] \tag{31}$$

$$\{F_{el}\} = -[K_{q\phi}]\{\phi_a\} \tag{32}$$

3.6 Controllability index

The system controllability has origin in the modern control theory. It is used to determine if a system can be controlled there being a controller. The decomposition of singular matrix [S] yields a measurement to quantity the system controllability. This index shows the energy that is need in the actuator to control a given input. The Eq. (30) can be writing in the state space:

$$\begin{aligned} \{\dot{x}\} &= [A]\{x\} + [B]\{u\} \\ \{y\} &= [C]\{x\} \end{aligned} \tag{33}$$

where:

$$[B] = \begin{bmatrix} [0] \\ [M_{qq}]^{-1}\{F_{el}\} \end{bmatrix}, \quad \{y\} = \{\phi_S\} \quad e \quad [C] = \left[\left([0]_i - [K_{\phi\phi}]_S^{-1}[K_{\phi q}]_S\right) \quad [0]\right] \tag{34}$$

The rank of state space matrixes, depend numbers of modes that are considered and the numbers of actuators put in the structure. The control force applied can be writing, such as:

$$\{f_c\} = [B]\{u\} \tag{35}$$

where {u} is the electric potential vector, then:

$$\{\mathrm{f}_c\}^T\{\mathrm{f}_c\} = \{\mathrm{u}\}^T[\mathrm{B}]^T[\mathrm{B}]\{\mathrm{u}\} \tag{36}$$

Writing:

$$[B] = [M][S][N]^T \tag{37}$$

Using singular analysis value, where:

$$[M],[N] \in R^n e [M]^T[M] = [I] e [N]^T[N] = [I] \tag{38}$$

where:

$$[S]^2 = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \sigma_p^2 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \tag{39}$$

The biggest value, σ^2_p, is the optimum position to place the actuators in the flexible structure.

4 NUMERIC SIMULATION

It was simulated a simply supported beam with piezoelectric attached, whose dimension and properties of beam and piezoelectric element are shown in Tab. 1 and Tab. 2.

Table 1. Beam Dimension and Properties

Properties	*Value*	*Unit*
Length L	1,5	m
Width b	0,075	m
Thickness h	0,075	m
Specify Mass ɟ	7800	kg/m^3
Young Module Y	$210x10^9$	N/m^2
Poisson Coefficient з	0,3	-
Shear Coefficient ə	0,833	-
Transversal Elasticity Module G	$80x10^9$	N/m^2
Area A	$5,625x10^{-3}$	m^2
Inertia Momentum I	$2,6367x10^{-6}$	m^4

Table 2. Piezoelectric Element Dimension and Properties

Properties	Value	Unit
Young Module Y_{pe}	$130x10^9$	N/m^2
Length L_{pe}	0,150	m
Width b_{pe}	0,0750	m
Thickness h_{pe}	0,010	m
Piezoelectric Coefficient d_{31}	$390x10^{-11}$	m/V
Specify Mass ɟ	7800	kg/m^3

According the graphics of Figure 2, the optimum place to put piezoelectric element in the first mode of simply supported beam is showed in Tab. 3.

Table 3. Optimum Position for Simply Supported Beam

(x/L)	(m)
0,50	0,750

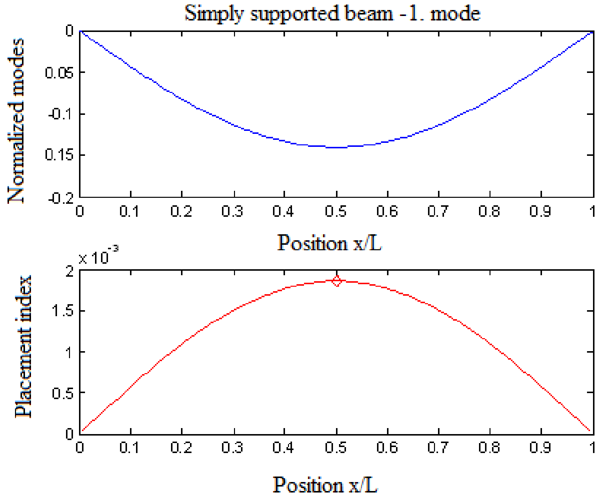

Figure 2. a) First mode of vibration of simply supported beam b) Placement index

In this simulation it was considered an excitation of the type step unit, where it excitation was applied in the middle of the length of the beam. The actuator piezoelectric was positioned, considering the first mode of vibration of the simply supported beam, the element 6 to the optimum position and the element 1 for the no optimum position, according to shown in Figure 3.

The actuator piezoelectric applied momentum in the structure, ie, momentum in the angular degrees of freedom.

The Figures 4 and 5 shown simulations with the actuator placed in the optimum position of the first mode of vibration of the simply supported beam, for the open and closed loop in time and the frequency domain.

The Figures 6 and 7 shown simulations with the actuator placed in the no optimum position of the first mode of vibration of the simply supported beam, for the open and closed loop in time and the frequency domain.

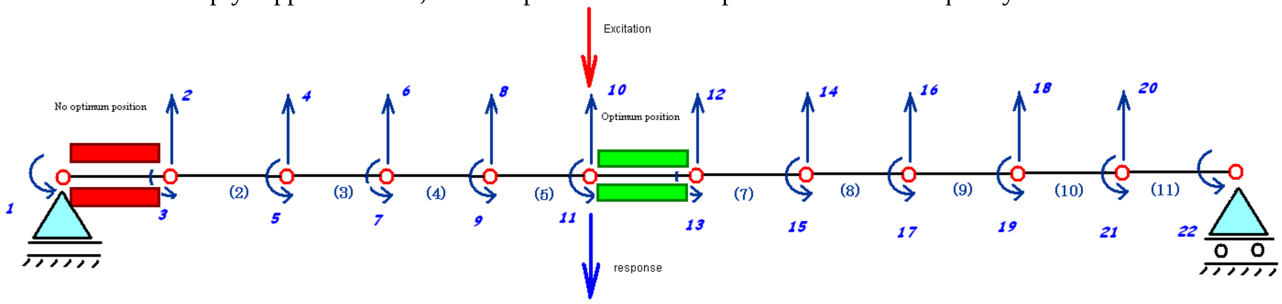

Figure 3 Elements of simulated beam

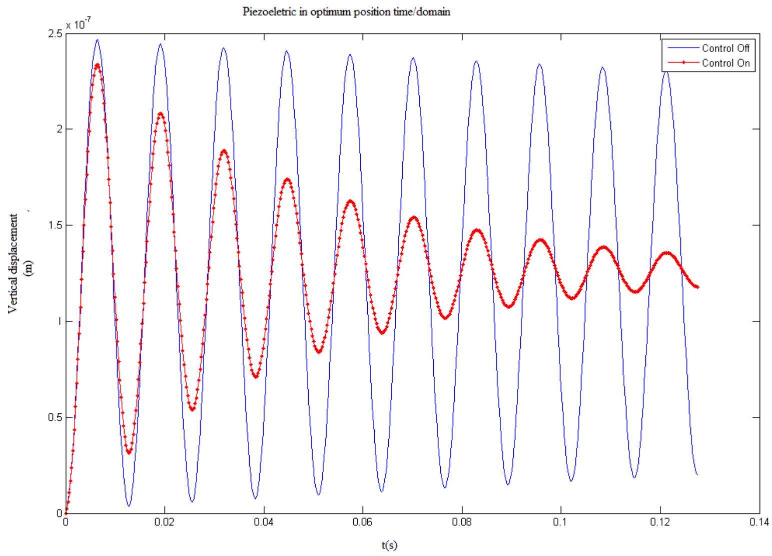

Figure 4 Open and closed loop of optimum position for the time domain

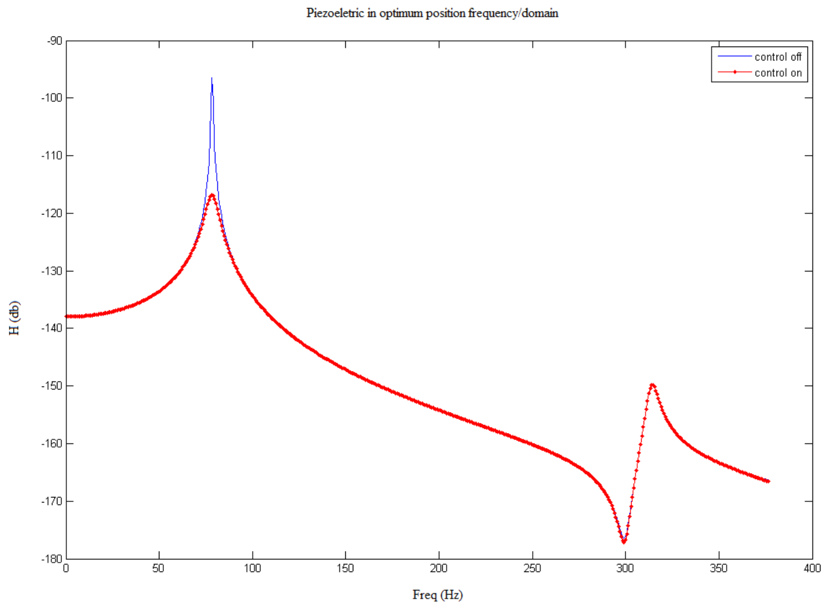

Figure 5 Open and closed loop of optimum position for the frequency domain

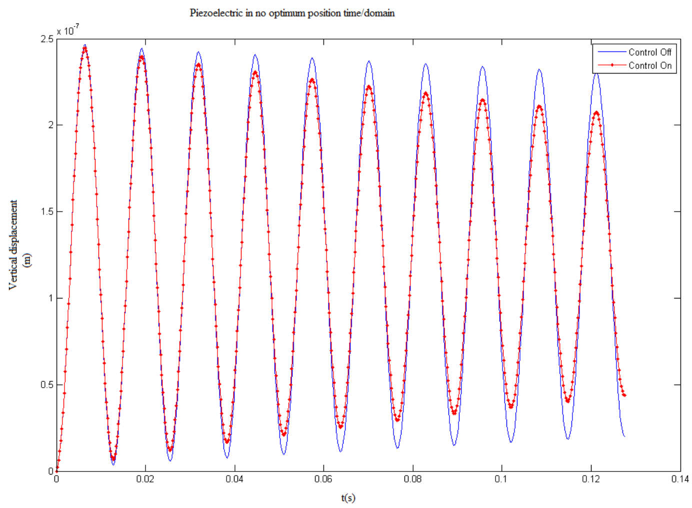

Figure 6 Open and closed loop of no optimum position for the time domain

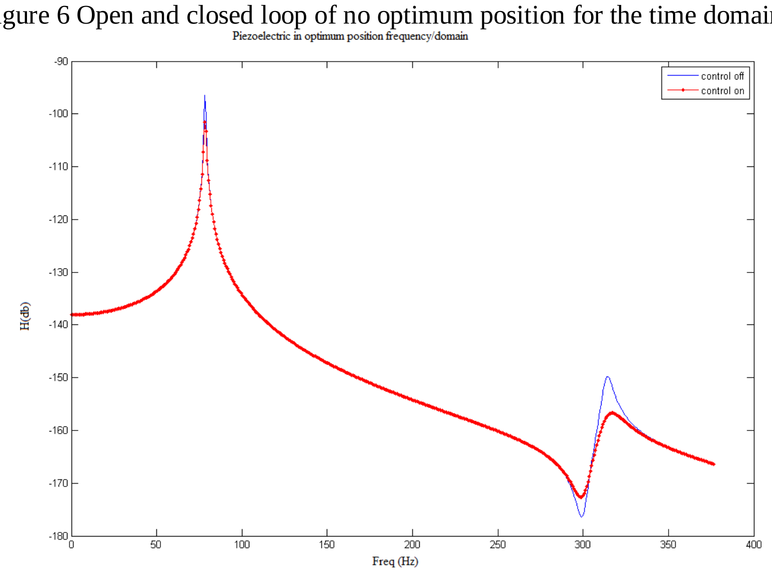

Figure 7 Open and closed loop of no optimum position for the frequency domain

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5. CONCLUSION

In this paper, was showed an index to quantify controllability system of the simply supported beam with piezoelectric attached. This index enables to determine the optimum position to place piezoelectric actuators in the flexible structure, minimizing the controller effort. It was showed that the singular value decomposition, of the control matrix, could be used as a measurement to quantify the energy supplied to actuators. The control simulation of the graphics of the Figure 4, Figure 5, Figure 6 and Figure 7 show that the optimum position to place a piezoelectric actuator to control the first mode of vibration of simply supported beam is the middle of the beam. This optimum position is the according to the placement index graphic of the Figure 2 and Tab. 3. Then, the performance of the proposed index to piezoelectric actuator placement in the simply supported beam is satisfactory.

6. ACKNOWLEDGEMENTS

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