INFLUENCE OF THE MODEL SIZE IN THE NUMERICAL DETERMINATION OF THE AVERAGE THERMAL STRESSES IN AN MMC COMPOSITE

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Abstract. *In previous works the authors discussed some issues related with a specific metallic matrix composites (MMC), the Aluminum matrix reinforced with SiC particles (Al+SiC) which has a metal matrix (powder) mixed with ceramic particles. These materials have some advantages when used as a structural material such as their high strength and good conformability. Their properties depend, among others, on the volumetric ratio, the particles size and distribution besides the matrix microstructure itself. Some of them are obtained at elevated temperature what produces a thermal stress state in the material. The Al+SiC is one of the later. The powder mix is extruded at 600 ^oC and it is used at 20 ^oC. Several numerical analyses were performed considering the random distribution of the particles and a non-linear behavior in the aluminum matrix. The results showed a strong influence of the aluminum elastic-plastic behavior in the composite thermal stress distribution due to its manufacturing process. However, one issue remained: the size of the model. It represents the central portion of a Al+SiC bar which is only about 10 times the size of a single particle (~10L). The present work investigates, always numerically, the influence of the model size on the thermal stress distribution. It considers 2 sets of non-linear analyses with random distributed particles: one with 20 models with 20L each one, the other set with another 20 models with 40L. This was done to allow a view of the results tendency compared with the previous ones. As done before, the modeled volumetric ratio has a very tight range of values with its average very near to the value in an actual Al+SiC composite. It is showed that the first model size was already enough to get good results without sacrificing neither the computer nor the analyst time.*

Keywords*: Composite; Metallic Matrix Particles; Thermal stresses.*

1. INTRODUCTION

Metal Matrix Composites (MMC) have several advantages when compared to 'pure' materials that make them very important in several structural applications. The composite can be molded and treated as any other metal alloy. When compared to polymers, the MMC presents high electrical and thermal conductivity and, yet, high mechanical strength.

The mechanical properties of MMC depend on the volumetric ratio, shape, size, and distribution of the disperse phase, and also depend on the matrix microstructure. There are composites formed by particles (big or small) and others formed by fibers (long or short) that interact in different ways, as described in the literature, e.g., Nardone and Prewo (1985), Humphreys, Basu and Djazeb (1991).

The fiber strength theory, considering the particles volumetric ratio, geometry and mechanical properties as well as the matrix characteristics, explains the mechanical properties of the composites reinforced with long fibers (Kelly and Mcmillan, 1986). The composite material reinforced with particles lacks of a well established formulation, and additionally, the microstructure itself, poses a great deal of complexity to the problem, modifying its properties and strength.

When the matrix of the MMC is Aluminum and the disperse phase are Silicon Carbide particles (SiC), herein referred as Al+SiC, the composite material is obtained by the mixture of the two materials, reducing fine particles as in a powder, heated at high temperatures, usually about 600º C. This heated mixture is extruded to form bars with the dimensions that fits their final usage, usually at room temperature (20°C), after the cooled down process. Therefore, besides the mechanical stress that acts on the material upon the specific application that it is intend for, there is an additional state of thermal stress due to the thermal gradient (-580º C). This thermal stress state is due to the distinct thermal expansion coefficients of its components, Aluminum and SiC, during the cooling process, and also their elasticity modules.

The evaluation of the thermal stress in the MMC was the subject of a previous work, as will be seen in detail next. Two cases were analyzed: first considering the material linear behavior (both, analytical and numerical analysis were performed, secondly considering the material non-linear behavior (only numerical analysis). In both cases the influence of the volumetric ratio besides size and shape of the particles of SiC were considered.

The results showed the strong influence of the aluminum matrix elastic-plastic behavior on the composite thermal stress distribution due to its manufacturing process and a minor influence of the volumetric ratio. However, one issue remained: the size of the models. Each one represents the central portion of an Al+SiC bar and it is only about 10 times the size of a single particle (~10L).

The present work investigates, always numerically, the influence of the model size on the thermal stress distribution. It considers 2 sets of non-linear analyses with random distributed particles of a generic quadrilateral shape and random size: one set with 20 models with 20L each and the other set also with another 20 models but with 40L. This was done to allow a view of the results tendency compared with the previous ones. As done before, the modeled volumetric ratio has a very tight range of values with its average very near the value in an actual Al+SiC composite.

1.1. Boari's analytical formulation

Details of the analytical formulation developed by Boari (2003) to obtain the most probable thermal stress in this kind of composite material, as well as more details of his numerical analysis are given in the references.

2. PREVIOUS WORK

Due to the complex interaction between the particles and the metallic matrix, it is very difficult to develop an analytical model that allows predicting the thermal stress level in this composite material. Even more if we consider that locally at the interface between the particles and the metallic matrix high levels of stress will arise. Boari (2003) developed an analytical model to predict the most probable average thermal stress within a composite material of Al+SiC, considering the influence of the volumetric ratio of its disperse phase.

Typically, particles have dimensions about ~1 μm and act in the sense of restrain the matrix movement in their whereabouts absorbing, in this way, part of the load. The matrix carries most of the applied load while the particles oppose to the movement of discordances.

Therefore, the increase of mechanical strength relies (a) on the restrain that the particles impose to the discordance movement through the material and (b) on the discordance density around the particles. To validate his analytical results, Boari (2003) developed a set of numerical analysis (simulations using Finite Elements) in which the particles (of circular geometry) were generated on random positions and size. The materials (Aluminum and SiC) were considered to be linear, in agreement with the theoretical principles used in the analytical part of his work.

The statistical treatment of the set of results obtained by random distributions, allowed estimating the most probable average value of thermal stress in the composite material. This value was considered to be very close to the value that came out from the analytical model, showing that the Boari´s analytical approach was confirmed.

As the analyses have shown, in some points of the structure the stress values were found to exceed the aluminum yield strength. Both the analytical and the numerical results had a strong limitation in practical usages, once the Aluminum elastic-plastic behavior would significantly alter the level of average thermal stress in the material.

For this reason, and adopting a reverse procedure, assuming that the analytical result supported the numerical & statistical procedure established previously, new studies took place, based only on numerical analysis plus statistical treatment. In these studies, the influence of the particles geometry, circular vs quadrilateral, and material behavior, linear vs non-linear (stress-strain curve for the Aluminum), was systematically examined (Miranda, Libardi, Boari, 2009a).

2.1. Adopted material properties

For the Aluminum matrix the adopted properties were: Elasticity modulus, E_a = 73 GPa, specific mass, γ_a = 2800 Kg/m^3, thermal dilatation coefficient, α_a = 23.6 10^{-6} ºC^{-1}, Poisson's ratio, μ_a = 0.33, transversal elastic modulus, G_a = 27.4 GPa. For the SiC particles the adopted properties are: Elasticity modulus, E_S = 450 GPa, specific mass, γ_S = 3200 Kg/m^3, thermal dilatation coefficient, α_S = 4.0 10^{-6} ºC^{-1}, Poisson's ratio, μ_s = 0.17, transversal elastic modulus, G_S = 192 GPa. Table 1 shows the values of the stress x strain curve adopted for the aluminum matrix in all preceding and in the present analyses.

Table 1. Values of the stress x strain curve adopted in all non-linear analyses.

σ - stress (MPa)	146	175	195	205	215	220	225	228	229	230
ε - strain (%)	0.2	0.45	0.8	1.0	1.5	2.0	3.0	4.0	6.0	8.0

2.2. Previous results

The results of the numerical analysis, obtained through finite element method, carried out using the ANSYS (V11.0) software are presented as iso-stress curves associated to the last load-step of the analysis, when the temperature gradient reaches ΔT = -580 ºC. Each of these curves is univocally identified by a different color (RGB – Red, Green, Blue). For each analysis, with a random arrangement of particles, the figure generated at ΔT = -580 ºC is processed by a simple and efficient algorithm (Miranda, Libardi, Boari, 2009b) developed in Matlab (v7.6, 2008). The results, showed as iso-stress curves, obtained from the analyses using circular particles and those using quadrilateral particles are presented in Fig. 1, adapted from (Miranda, Libardi, Boari, 2009c).

The iso-stress curves can be weighted by the percentage that each of these colors (meaning iso-stress value) appears, in regard to the others, resulting in an average value for the composite material. On the other hand, as assumed in the previous works and also in this work, the value with the higher frequency (percentage) – the mode stress value – was chosen. The set of mode values is treated statistically, considering normal distribution, resulting in an average value (most probable average thermal stress value) and a standard deviation.

From results of previous work (Miranda, Libardi, Boari, 2009a, 2009b), it can be seen, as expected, the strong influence of the material non-linear behavior, therefore the average stress value decreases from ~500 MPa to ~180 MPa. This influence is easy to observe in the Fig. 2a (Miranda, Libardi, Boari, 2009a), in which the same arrangements of random round particles are submitted to linear and non-linear analyses.

The previous linear results, (Boari, 2003 – assuming the linear behavior of materials, mainly the Aluminum) were compared to the results of the non-linear analyses (where a typical stress-strain curve was used for the Aluminum). Both set of analyses (linear and non-linear) used the same round particles distribution and the same number of iso-stress curves in the pos-processing stage (9 curves / colors).

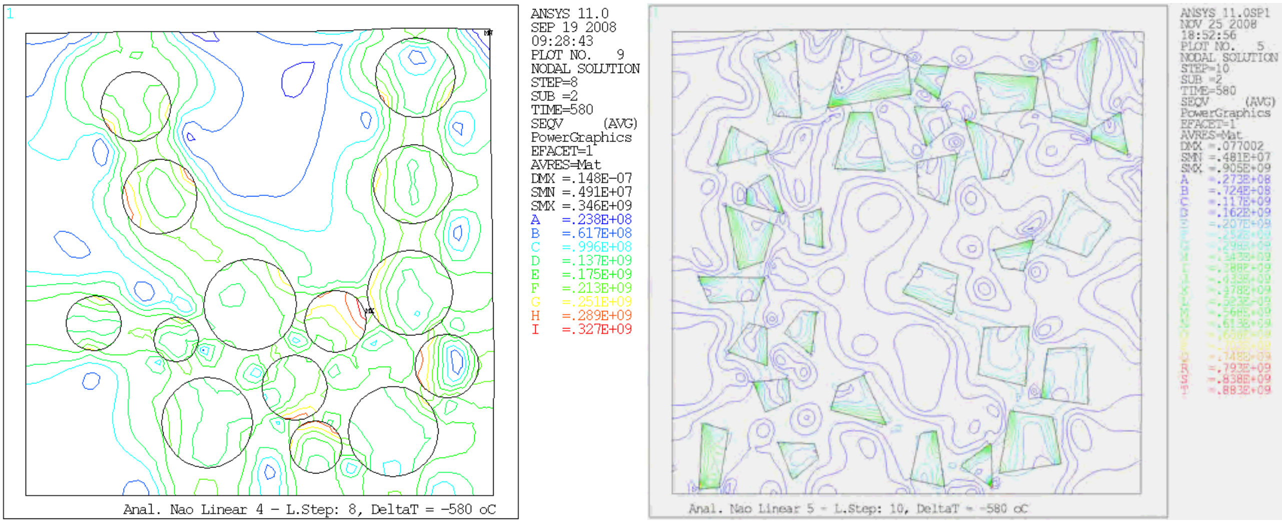

Figure 1. Typical (previous) results at ΔT = -580 ºC (Round and Quadrilateral particles)
(stress scales: from 23.8 MPa to 327 MPa and from 27.3 MPa to 883 MPa)

When using quadrilateral particles (in the non-linear analyses), the values of maximum stress were found much higher with respect to those obtained from round particles analyses. Due to this fact, when analyzing the stress resulting from the quadrilateral particles distributions, the iso-stresses were firstly generated using 9, 20 and 40 values (colors). The results using 20 or 40 curves gave almost the same values showing a convergence. So, all results were analyzed using 20 iso-stress curves. The Figure 2b, adapted from (Miranda, Libardi, Boari, 2009c), shows the mode and the averaged stress values for all analyses where the values obtained using 09 and 40 iso-stress curves values were suppressed.

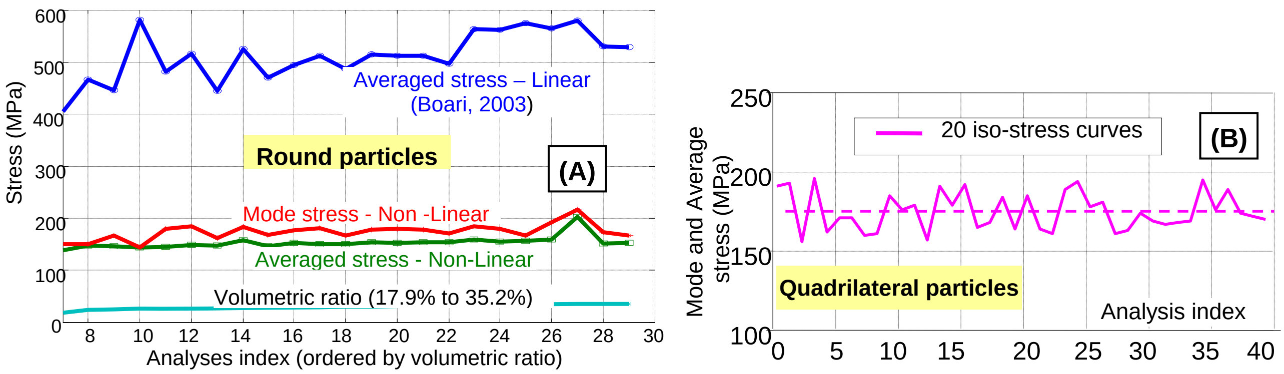

Figure 2. Comparison previous results: (A) Non-Linear X Linear numerical results (B) Mode stress and averaged values

From previous results, and considering that the particles of quadrilateral geometry should be more representative than the round geometry, we concluded that the average stress in the Al+SiC composite material, fabricated as defined previously, is between 155 MPa e 195 MPa with the confidence interval of 95%, with an average value of 175 MPa, and a standard deviation of 10 MPa (Miranda, Libardi, Boari, 2009c).

3. PRESENT WORK

The conclusion presented above, however, doesn´t solves one remaining doubt: the appropriate size of the model. As mentioned before, the model used in the previous studies assumes a slice of square cross section geometry, whose side length is about 10 times (10L) the particles average size (L). As the thermal stress depends on the thermal gradient, it is not the absolute value of the particles and models that matters, but its relative size.

For this reason, the present work analyzed two sets of 20 models each, assuming the quadrilateral geometry to represent the particles whose shape, position and size are random. The size (cross section square's side length) of the first set of models is 20L and for the second set it is 40L. Typical models/meshes are presented in Fig. 3. For each model, in both sets, the properties of the Aluminum and SiC particles, boundary conditions and thermal loads are specified as indicated in section 3.1.

Each model was analyzed using the ANSYS default convergence parameters of a static analysis with non-linear material properties. The results were pos-processed to obtain images with the iso-stress curves at ΔT = -580 ºC. These images were analyzed with a Matlab algorithm to identify the percentage of each color (iso-stress value).

3.1. Numerical Analyses

Material properties. The modeled volumetric ratio has a very tight range of values with its average near the value in an actual Al+SiC composite (between 20% and 23%). This will allow a direct comparison with the previous results when the volumetric ratio was between 20% and 25%. The material properties are the same already adopted in the previous analyses and showed in section 2.1 and, particularly for the Aluminum, in Table 1.

Boundary Conditions. The nodes on the X axis (Y = 0) were restrained in the Y direction while the nodes on the Y axis (X = 0) had their X direction restrained. When doing so, the adopted model represents the central portion of a bar and we are assuming that it represents the entire bar cross-section.

Loading. The only applied load is the uniform temperature field imposed in several steps to arrive to the end of the cooling process, from 600 °C to 20 ºC. So, the last load step is associated with the ΔT=-580 °C. This means it is assumed a controlled slow cooling process to assure an almost uniform temperature over the entire model. The temperature gradient during the cooling process will be present even if the cooling time is long, however its influence is considered negligible once the uniform final temperature state (20 ^{o}C) is reached.

3.2. Finite Element Results – Models 20L and 40L

Two typical results from the models 20L, obtained from the numerical analyses at ΔT=-580 °C, are shown in Fig. 4.

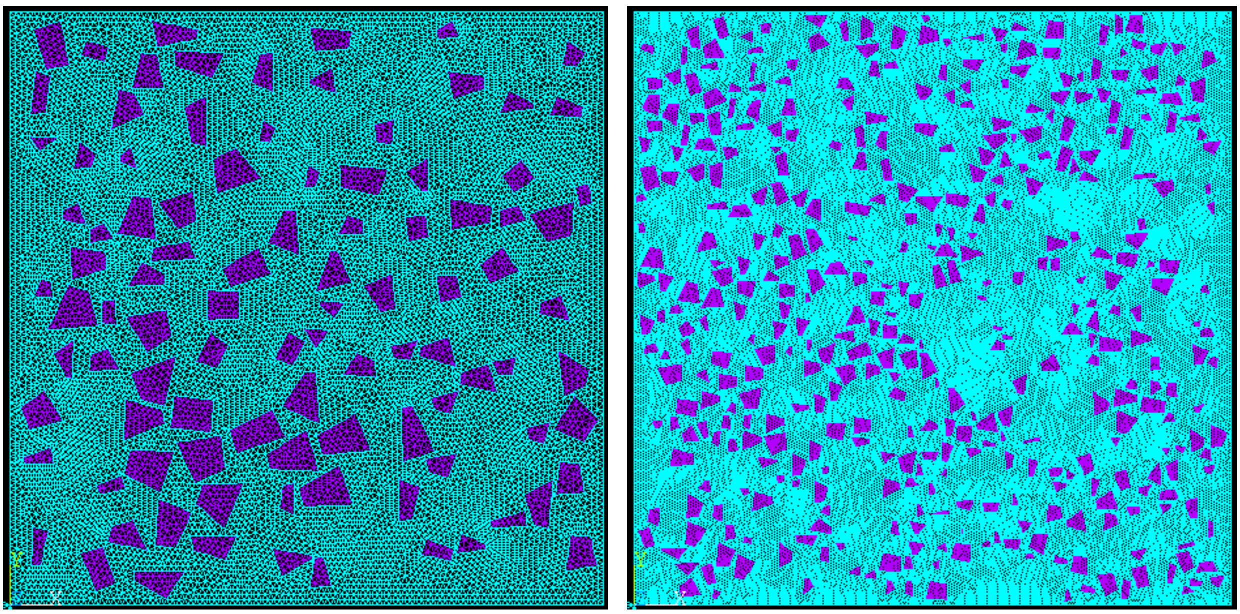

Figure 3. Typical meshes (model 20L #19 and model 40L #10)

The iso-stress lines within the composite can be seen together with their scale and, also, the particle distribution for that specific model which is one out of twenty. Figure 5 shows equivalent results from models 40L. Each iso-stress curve in the last load step, at ΔT=-580 °C, has its own color defined by the R(ed), G(reen) and B(lue) values used previously in the ANSYS POST1 pos-processor. This color map was used in the developed a Matlab program to analyze the images/figures.

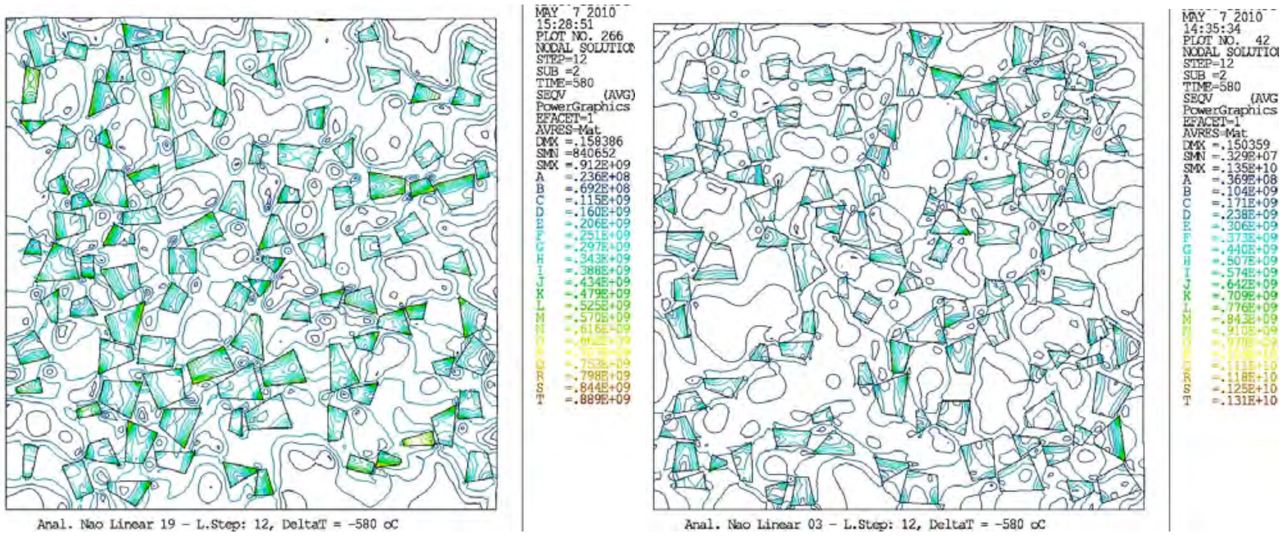

Figure 4. Typical results at ΔT = -580 ºC (from models 20L)
(stress scales: from 23.6 MPa to 889 MPa and from 36.9 MPa to 1310 MPa)

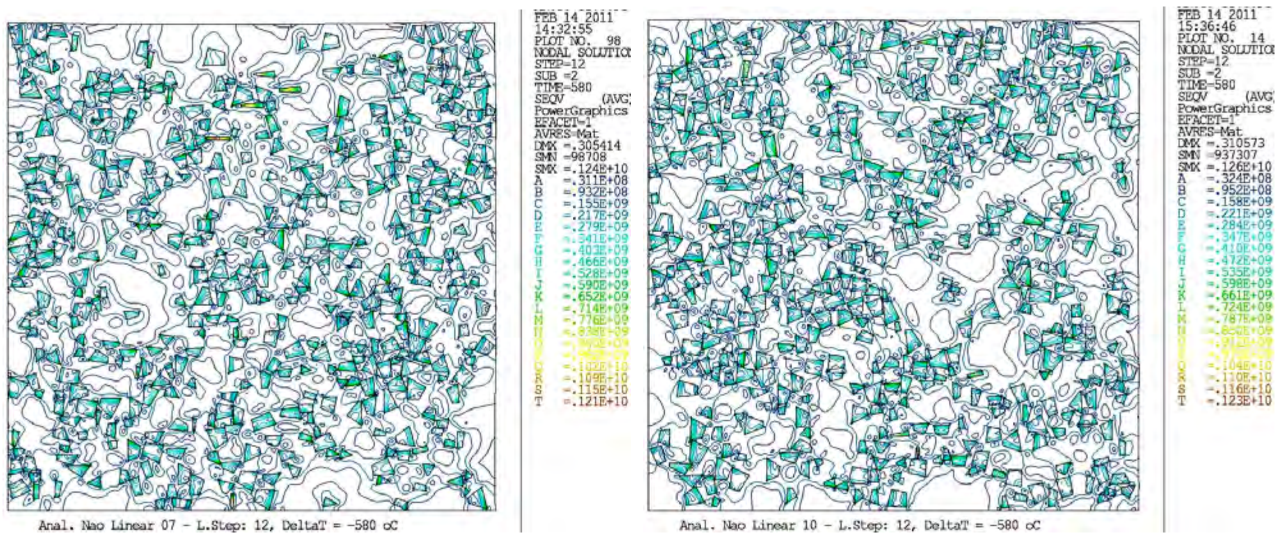

Figure 5. Typical results at ΔT = -580 ºC (from models 40L)
(stress scales: from 31.8 MPa to 1210 MPa and from 32.4 MPa to 1230 MPa)

Before the preliminary analyses of the results it should be pointed out the adoption of the Tresca equivalent stress as the parameter to show and compare the results from all analyzes in this work as well as in the previous ones. Other point to keep in mind: the interest is to obtain the final stress state in the composite, after the end of the cooling process. During the composite cooling there is a temperature transient in which the material properties vary with the temperature. So, the thermal stresses field will vary non-linearly. However, the final stress state can be assessed with the procedure described in the item 3.1 ("loading") of this work (it was also adopted in the previous works).

Preliminary result analyses. Fig. 6 presents a typical distribution of percentages associated to each iso-stress curve of a typical analysis that is valid to the models '20L' as well as to the models '40L'. It is clear, from this figure and also from figures 4 and 5, that the percentages associated to the high levels of stress are very low, and can be neglected (~0%). Setting the number of iso-stress curves to 20 and considering that the maximum values of stress are too high (although less representative) implies a wide stress range associated to each curve/value of iso-stress.

Therefore, after a preliminary analysis of the results it was found appropriated to re-process the analyses generating new figures of iso-stress cutting the tail of the asymmetric percentages distribution.

In a first approach, the maximum stress value was restrained to 650 MPa. This doesn't have any practical influence on the average stress and also in the mode stress values, since the accumulated percentage that is lost is less than 1%. This is equivalent, roughly, to double the number of iso-stress curves.

In the second approach, the cutting value was set to 330 MPa. This represents less than 5% of accumulated lost percentage which has a small influence on the average stress (not used in this work) and an even lesser influence on the mode stress. This means, again and roughly, to double the number of iso-stress curves.

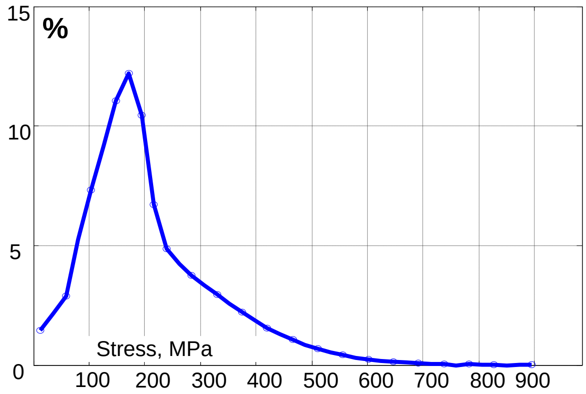

Figure 6. Typical iso-stress percentage distribution for one single analysis (model ‘20L’)

New post-processing result analyses. Figure 7 and Fig. 8 present two typical stress distributions (one for the set of models '20L' and one for the set of models '40L') at ΔT=-580 °C obtained in the second phase of pos-processing when the maximum stress (σ_{max}) was limited to 650 MPa (Fig. 7) and when it was limited to 330 MPa (Fig. 8).

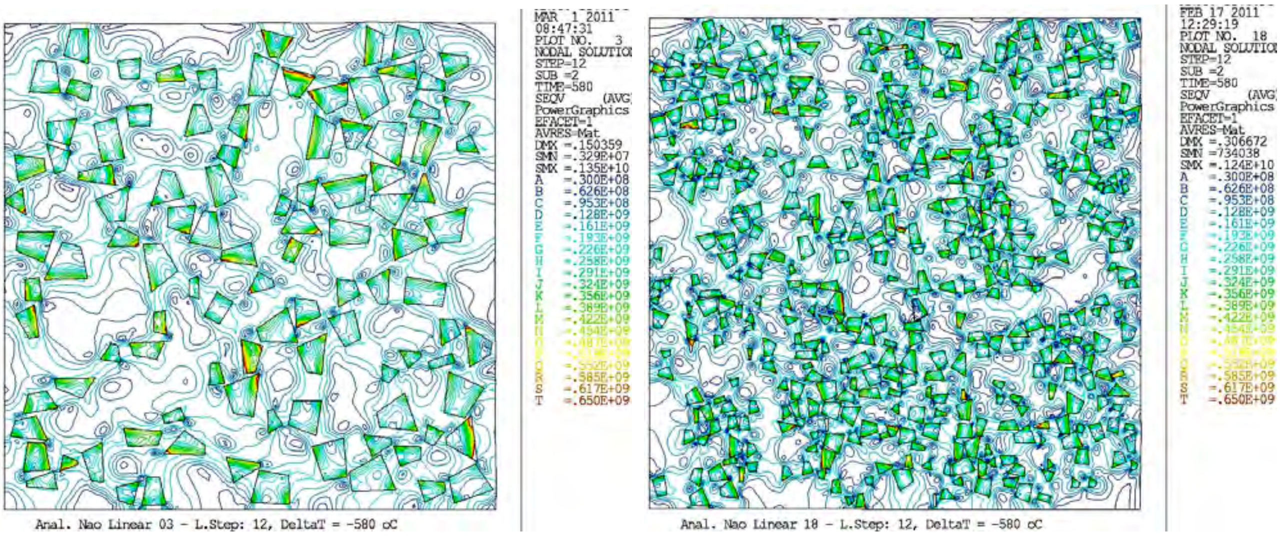

Figure 7. Typical results at ΔT = -580 ºC with σ_{max} = 650 MPa (respect. from models 20L and 40L)

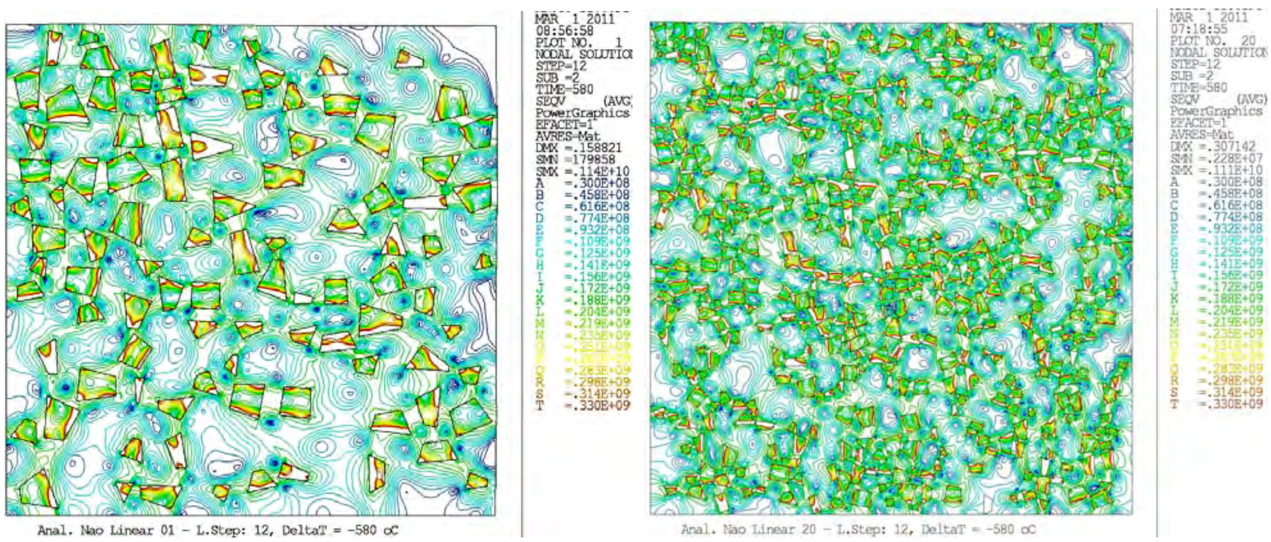

Figure 8. Typical results at ΔT = -580 ºC with σ_{max} = 330 MPa (respect. from models 20L and 40L)

4. RESULTS ANALYSIS AND DISCUSSION

To assess the influence of the model size on the most probable thermal stress value developed in the MMC composite (Al+SiC) due to its fabrication process, three sets of results with 20 models each one were compared. These sets have the size of 10L (previous work), 20L and 40L (present work), where L is the medium size of the particles. One should remember that, in each doubled model (20L) there are, roughly, four times more particles than in a 10L model, and in a 40L model there are 16 times more particles than in a 10L model (again, roughly speaking).

In each model of these sets, the thermal stress field that arises due to the MMC fabrication process was represented by 20 iso-stress curves equally spaced obtained when ΔT = -580 ºC. For each one of these curves its relative frequency, in percentage terms, was determined from an image analyzer algorithm developed in Matlab language. The mode stress value was considered to be representative of the central tendency for the thermal stress estimate of a given model, and the 20 values in each set were statistically analyzed to get the mode average value and its standard deviation that represents the data dispersion in the set.

In the first approach to analyze the results, the raw values were adopted, i.e.: no limitation was imposed on the maximum stress value. It was observed a large dispersion in all of the three sets (10L, 20L and 40L). It occurs due to the high level reached by the maximum stress values associated with the adopted fixed number of iso-stress curve (20 curves). However, this elevated stress level are very little significant for the central tendency value once they occur with very low frequency (~0%).

The second approach was to restrain the maximum stress value to overcome this issue. Analyzing the distribution of the stress frequency in each model, the stress level of 650 MPa was chosen as a cutoff. This accounts for less than 1% of loss in the accumulated percentage, what is negligible in a practical sense and it is equivalent, roughly, to double the number of iso-stress curves. This approach was applied only to the 20L and 40L models, developed specifically for the present work, and for both sets it was found the value of 161 MPa as the most probable thermal stress value, with null dispersion.

In a third approach, for both sets (20L and 40L models), the stress level cutoff was reduced to 330 MPa, which represents a loss of accumulated percentage below 5% that is still negligible in a practical purpose and it is equivalent, roughly, to double again the number of iso-stress curves. A value of 172MPa was found as the most probable thermal stress value, still with null dispersion.

These results are summarized in Fig. 9 while Table 2 shows the complete set of results obtained in the previous works as well as in the present work.

The refinement in the analyses results, as presented in this work, performing a cut in the tail of the stress percentage distribution in each model, was not performed in the previous work (10L). So, a direct comparison is not possible. However, no practical difference was observed between the results in the 20L and 40L models when considering the stress cutting limit of 650 MPa and when the cutting value was 330 MPa (respectively, 161 MPa and 172 MPa for the estimated most probable thermal stress value in the composite). These values can be considered within the stress range established in the previous work considering a confidence interval of 95% (average value ±2 standard deviation).

However, due to the lack of direct comparison with the previous work (10L) and, so, considering only the present results, one can say that the model 20L is a good approach to establish the most probable thermal stress value in an metal-matrix composite due to its fabrication process.

As the results have shown, due to their random size and shape some particles can experience strong stress gradients. This can lead them to brittle fracture. However, most of them are confined by the ductile surrounding material which turns more unlikely their fracture. Even so, the overall results remain once the stress redistribution will be localized.

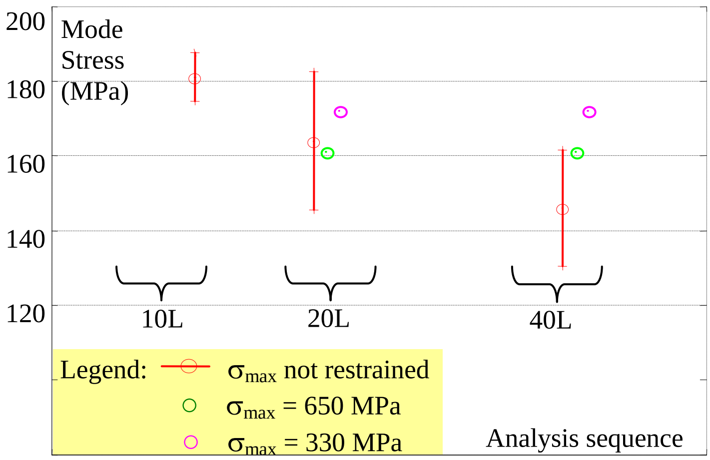

Figure 9. Summary of results from models 10L, 20L and 40L.

Table 2. Results Summary (including the Circular Linear and Non-Linear previous results).

Model Size		Material behavior & Particle shape	σ_{max}	Most Probable Thermal Stress
10L (Previous work)		Linear / Circular	Not restrained	~500 MPa Std Dev: not calc.
		Non-Linear / Circular		150.0 MPa Std Dev: not calc.
		Non-Linear / Quadrilateral		180.8 MPa Std Dev: 6.5 MPa
Present work		Non-Linear / Quadrilateral	Not restrained	163.6 MPa Std Dev: 18.5 MPa
	20L		650 MPa	161.0 MPa Std Dev: 0.0 MPa
			330 MPa	172.0 MPa Std Dev: 0.0 MPa
	40L	Non-Linear / Quadrilateral	Not restrained	146.1 MPa Std Dev: 15.7 MPa
			650 MPa	161.0 MPa Std Dev: 0.0 MPa
			330 MPa	172.0 MPa Std Dev: 0.0 MPa

5. ACKNOWLEDGEMENTS

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