MATHEMATICAL MODEL AND EXPERIMENTAL PROCEEDINGS TO DETERMINE ROLL WAVES IN OPEN CHANNELS

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Abstract. *The goal of this paper is consolidate representative models previously developed by the team RMVP (Rheology of Viscous and Viscous-plastic Materials), from UNESP - Ilha Solteira, for typical phenomenon that occurs on spillways, river's bed, landslides, mudflows, blood flows, in Newtonian or non-Newtonian fluids, known as roll waves and also present an experimental project designed for capturing measurements (amplitude and wavelength) of these instabilities in open channels. At a mathematical viewpoint, the first-order analytical model is shown, based on Cauchy's equations system, once developed by the team, which provides a generation condition for roll waves through temporal linear stability analysis, in inclined open channels. This model follows the remarkable work of Dressler (1949) and it is able to generate roll waves for many rheological configurations such as Bingham and Herschel & Bulkley models, representing clean water up to muddy mixtures respectively. A program was developed in Matlab/Simulink, using Adams-Moulton-Bashforth multi-step algorithm, and is exhibited along with some results that illustrate roll waves pattern. Due to the lack of experimental data of amplitude and wavelength, the team started to focus on the experimental approach of the phenomenon, aiming to design a project that would be capable to reproduce roll waves in special conditions of the flow, isolated from external perturbations. This project is here presented along with a proposal of a photometric system for capturing measures of the fluid film through absorbance loss based on experiments found in the literature. The final execution of this experiment and the correct obtaining of amplitude and wavelength will contribute for the validation of the model here presented.*

Keywords: *roll waves, mudflows, hyper concentrated fluids, light absorption technique*

1. INTRODUCTION

Flows at open channels under certain steepness may be favorable to the generation of periodic instabilities in the flow surface depending on fluid and flow conditions. Usually found on artificial structures, such as spillways (Dressler, 1949), these waves may compromise the integrity of the surrounding structures since they might lead to an intermittent flow with occasional overflows, generation of debris flow, leaking of muddy and viscous materials, which can cause significant damage to the environment or the structure. Such incidents are frequently exemplified through catastrophic landslides that occur after heavy rains at places with no vegetation cover. One can cite the cases of Angra dos Reis and Ilha da Madeira last year, and the episodes of Rio de Janeiro's north region and Minas Gerais this year, as examples where landslides happened in places where the topography was very auspicious to the formation of roll waves. The picture shown in Fig. 1 shows the channel formed by the landslide of Angra dos Reis with a very steep slope and knowing that the fluid, composed by water from the rain and clay mixture from the soil, has non-Newtonian behavior, roll waves could have been formed and even amplified the impact. Besides, this kind of waves can also appear at natural structures like rivers (Fer *et al.*, 2002) and oceans (Swaters, 2003), and are even present in the human body, inside veins (Brook *et al.*, 1999; Pedley, 1980), contributing for its research matter. These typical waves, called roll waves, emerge in flows where the uniform flow, established by the balance between the body forces and the viscous forces, is then disturbed by natural frequencies. In the flow early stage, a random oscillatory pattern will be noticed at the surface and, after a finite time, the regimen will be established and a periodic wave, usually a sawtooth-like, will characterize the instability visualized.

Many theories were developed in order to explain the generation of these waves including stability analysis, aiming to determine conditions for the maintenance and generation for the waves. To represent the analytical models, numerical

Figure 1. Carioca's cliff in Angra dos Reis, Rio de Janeiro, after the landslide of January, 2010.

simulations are carried, reproducing qualitatively pattern and the generation and maintenance conditions. Nevertheless, experimental procedures are necessary to validate those models, which are not an easy job.

Several tests can be found in papers showing many types of configuration for measuring systems and reasonable results, with good comparison analysis such as Kapitza (1948) and Liu and Gollub (1994), but they are usually taken for Newtonian fluids, leaving the non-Newtonian's behind. Coussot (1994) developed experiments with water-clay mixtures, considering his test-fluid as Herschel-Bulkley type, and was able to validate his criterion of stability. Measures of wave amplitude and length are difficult to reach without interfering in the flow: intrusive methods, such as conductive and capacitive, are quite easy to construct and operate and have a good precision, but they require the use of a probe that disrupts the flow, what sometimes can be undesirable; non-intrusive ones don't interfere the flow, but the addition of a tracer to the test-fluid is usually needed, sometimes compromising the rheological behavior adopted, or they are limited by their nature characteristics, preventing from obtaining realistic values.

The presence of these instabilities on mudflows and debris flows have been the research subject of the RMVP team (Rheology of Viscous and Viscoplastics Material - FEIS/Unesp) and so analytical and numerical methods to predict measures of the waves in regimen, showing good results in agreement with those observed in the literature. The group now takes its efforts researching experimental methods to obtain measures of roll waves in inclined open channel, in a attempt to fill a lack of data that exists in the literature, validating the models and leading to the knowledge of the phenomenon's generation and spreading mechanisms for Bingham and Herschel-Bulkley fluids, that have wide representativity in the industrial sector.

2. MATHEMATICAL MODELING

The modeling here presented was once developed by Ferreira (2007) taking as guide the classic work of Dressler (1949), which falls upon discontinuous functions for roll waves. This mathematical work was carried through Cauchy's equations inserting the Herschel-Bulkley rheological model (3 parameters) at the viscous part of the stress tensor, resulting in the following system, in dimensionless variables:

Continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t} + \alpha u \frac{\partial u}{\partial x} - (\alpha - 1)\frac{u}{h}\frac{\partial h}{\partial t} + \frac{1}{Fr^2}\frac{\partial h}{\partial x} = h - C^* - (h - C^*)\left\{\left[\frac{u(1-C^*)}{h-C^*}\right]\left[\frac{((n+1)+nC^*)\,h}{(n+1)h+nC^*}\right]\right\}^n \tag{2}$$

where h is the fluid height, u, the mean flow velocity, x, longitudinal distance across the channel, t, the temporal variable and n is the flow index, Herschel-Bulkley parameter. The Froude number, Fr, and the dimensionless coefficient for critical stress, C^*, are calculated from uniform regimen flow:

$$C^* = \frac{\tau_c}{\rho g h_0 \sin\theta} \tag{3}$$

$$Fr = \frac{u_n}{\sqrt{g h_0 \sin\theta}} \tag{4}$$

where ρ is the fluid density, g, gravity acceleration, θ, channel steepness, h_0 and u_0 are the fluid height and mean velocity in uniform flow. Through this system, is possible to make an evaluation of it as a representative dynamical system of the phenomenon, therefore, stability analysis can be done.

2.1 Linear Stability Analysis

Linear stability theory is a tool that allows us to obtain important information from the dynamical system in subject, such as growth rate and propagation velocity of instabilities. Through this method, widely discussed by Briggs (1964), it is possible to characterize the necessary conditions to the formation of instabilities in the system. Di Cristo and Vacca (2005) carried this theory to evaluate the convective nature of roll waves instabilities, i.e. demonstrate how waves can appear and grow in time and space if the conditions of the flow are favorable. The objective is reach the dispersion equation for the system and analyze how a perturbation behave. Infinitesimal perturbations, $H(x,t)$ and $V(x,t)$, are added into the system in uniform regimen, Eq. 1 and 2, meaning that:

$$h = 1 + H(x,t) \tag{5}$$

$$u = 1 + V(x,t) \tag{6}$$

with $H \ll 1$ and $V \ll 1$. The system is now represented by

Continuity equation:

$$\frac{\partial(1+H)}{\partial t} + \frac{\partial(1+H)(1+V)}{\partial x} = 0 \tag{7}$$

Momentum equation:

$$\frac{\partial(1+V)}{\partial t} + \alpha(1+V)\frac{\partial(1+V)}{\partial x} - (\alpha-1)\frac{(1+V)}{(1+H)}\frac{\partial(1+H)}{\partial t} + \frac{1}{Fr^2}\frac{\partial(1+H)}{\partial x} = (1+H) - C^* - ((1+H)-C^*)\left\{\left[\frac{(1+U)(1-C^*)}{(1+H-C^*}\right]\left[\frac{((n+1)+nC^*)\,(1+H)}{(n+1)h+nC^*}\right]\right\}^n \tag{8}$$

Solving the system above for the variation of height, $H(x,t)$, the following partial differential equation is found:

$$\frac{\partial^2 H}{\partial t^2} + \left(\alpha - \frac{1}{Fr^2}\right)\frac{\partial^2 H}{\partial x^2} + 2\alpha\frac{\partial^2 H}{\partial x \partial t} + n\frac{\partial H}{\partial t} + (2n+1)\frac{\partial H}{\partial x} = 0 \tag{9}$$

From the linear theory, the surface wave problem is given by the solution of Laplace equation through the separation of variables method, applying the kinematic boundary conditions of the problem in the bottom and in the surface. Another way, knowing the solution periodicity and uniformity, one can consider the perturbation of the form:

$$H = \hat{H}e^{i(kx-\omega t)} \tag{10}$$

where $\hat{H}$ is a magnitude value (constant), k, wave number, and ω, frequency of the perturbation.

A wave is defined as unstable if there is a complex frequency $\omega = \omega_r + i\omega_i$ with $\omega_i > 0$, for a real wave number k, obtained from the dispersion equation of the system, meaning that there is an temporal growth of a infinitesimal spatial perturbation. Thus, manipulating Eq. 9 and 10, the following dispersion equation, Eq. 11 must be solved for ω.

$$D(\omega,k,n,Fr,\alpha) = \omega^2 + \left(\alpha - \frac{1}{Fr^2}\right)k^2 - 2\alpha k\omega + in\omega - i(2n+1)k \tag{11}$$

Evaluating the Eq. 11, a function for ω is found in dependence of the flow and fluid parameters. Considering that $Im\{\omega\} > 0$, one can reach a criterion for generating instabilities at the flow surface that will determine values of Froude number:

$$Fr > Fr_{min} = \frac{n\left[n(C^*+1)+1\right]}{\sqrt{p_1(n)C^{*2} + p_2(n)C^* + p_3(n)}} \tag{12}$$

where

$$\begin{aligned} p_1(n) &= 4n^4 + 4n^3 + n^2 \\ p_2(n) &= 6n^3 + 7n^2 + 2n \\ p_3(n) &= 2n^3 + 5n^2 + 4n + 1 \end{aligned}$$

2.2 First-order equation for Roll waves generation

The model here presented was developed by Ferreira (2007), member of RMVP/Unesp team, and uses the continuity and momentum equations, Eq. 1 and 2, to elaborate one unique differential equation that is capable of represent the phenomenon. The system is transferred to a mobile coordinate system and manipulated to reach a first-order differential equation (all mathematical steps are detailed in Ferreira (2007)):

$$\frac{\partial h}{\partial x} \equiv \frac{F(h)}{G(h)} = \frac{h - C^* - (1 - C^*)\left\{\left[\frac{(1+U(h-1))(1-C^*)}{h-C^*}\right]\left[\frac{(n+1)+nC^*}{(n+1)h+nC^*}\right]\right\}^n}{(\alpha-1)U^2 - \frac{\alpha(1-U)^2}{h^2} + \frac{h}{Fr^2}} \tag{13}$$

The wave celerity, U, and the velocity coefficient distribution, α, are function of flow parameters:

$$U = U(h_0, n, \alpha, C^*, Fr); \qquad \alpha = \alpha(h_0, n, C^*);$$

However, since this is a first-order model, it is prone to specifics peculiarities that must be treated, as well as the classic work of Dressler (1949). As the construction of the equation's solution arises, a discontinuity appears, exactly where $h = h_0$, causing a indetermination, $F(h)/G(h) = 0/0$. At that point, we call Rankine-Hugoniot's shock conditions to keep continuity in passing through, as long as it is known that exists a continuous solution at that very point. Once it's done, the continuous solution is given by:

$$h(x) = \int \frac{F(h)}{G(h)}\,\mathrm{d}x \tag{14}$$

bounded by the wave length λ, the interval between two successive shocks:

$$\lambda = \int_{h_1}^{h_2} \frac{G(h)}{F(h)}\,\mathrm{d}h \tag{15}$$

The value of h_2 is calculated through the Eq. 16, however h_1 must be estimated for real values of $F(h)/G(h)$.

$$h_2 = \left[\left(\frac{h_1}{2} + (\alpha-1)U^2Fr^2\right)^2 + \frac{2\alpha(1-U)^2Fr^2}{h_1}\right]^{\frac{1}{2}} - \frac{h_1}{2} - (\alpha-1)U^2Fr^2 \tag{16}$$

Schematically, the phenomenon can be represented by the following Fig. 2:

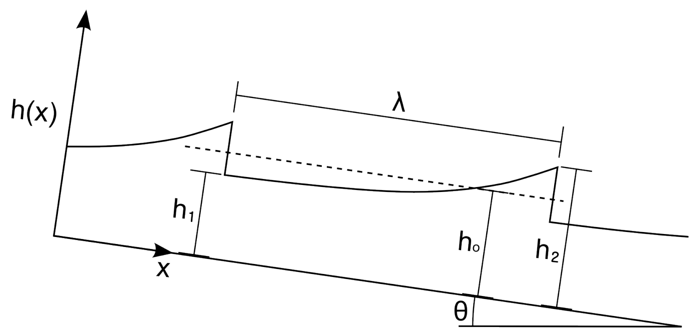

Figure 2. Sketch representing roll waves falling down an open channel and its typical dimensions.

2.3 Representativity Analysis

In order to prove the representativity of the model presented is last section, numerical simulations were done by testing the rheology to the fluid in motion. Matlab/Simulink tool was used for testing the model and plotting the results, since it is an excellent environment for dynamical systems and has a large function library.

For Newtonian fluids, the fluid flow begins starts to shear with the minimal force applied to it so the dimensionless coefficient for critical stress is minimal ($C^* = 0$) and, how the shearing stress dependence on the deformation is linear, the flow index must be 1 ($n = 1$). This kind of fluid represents common fluids usually observed in nature, such as water and air. Roll wave pattern may arise in flows with this kind of fluid and it is expected that the model would be capable of generating that pattern. Then, for $Fr = 1$, roll waves were numerically generated as Fig. 3 shows.

In a comparison between these results and those found by Dressler (1949), it's clear that the first-order model developed by the team was capable of reproducing qualitatively roll waves in open channels flow of Newtonian fluids. However,

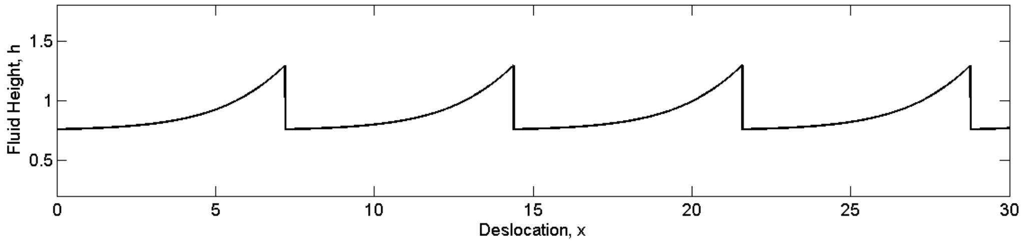

Figure 3. Numerical simulation of roll waves in Newtonian fluids ($n = 1$; $C^* = 0$; $Fr = 1$). Wave celerity $U_1 = 2,31$, wave amplitude $\Delta h_1 = 0,538$, and wavelength $\lambda_1 = 7,19$.

as said before, the proposal was to generate waves for non-Newtonian fluids flow because their presence in important natural and industrial phenomena, such as landslides, avalanches, etc. At first, instead of the linear relation between the stress and the deformation rate, it will be considered of another kind.

For a Power-Law fluid, with $n = 0,7$, for example, the numerical model was simulated and the result is shown at Fig. 4.

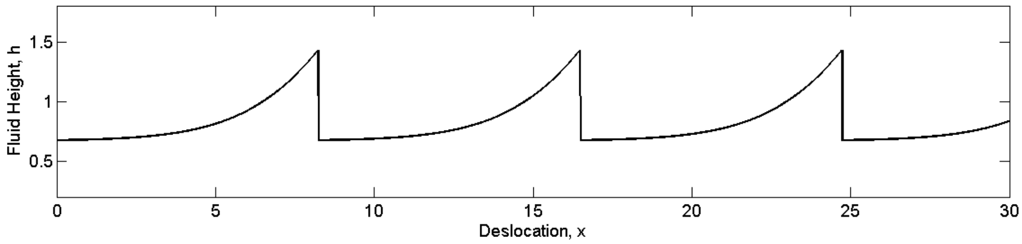

Figure 4. Numerical simulation of roll waves in Power-Law fluids ($n = 0,7$; $C^* = 0$; $Fr = 1$). Wave celerity $U_2 = 2,27$, wave amplitude $\Delta h_2 = 0,756$, and wavelength $\lambda_2 = 8,25$.

A simple analysis based on the results can evidence some characteristics of the roll-wave phenomenon. The non-Newtonian effect of the index flow contribute to the "enlargement" of the wave, increasing the amplitude ($\Delta h_2 > \Delta h_1$) and the wavelength ($\lambda_2 > \lambda_1$) (Ng and Mei, 1994). Adding complexity to the rheological behavior, the Herschel-Bulkley model for fluids is reached. Besides the non-linear relation between the shearing stress and the deformation, the presence of a critical shearing stress makes necessary that the body forces surpass a critical value so the fluid can begin its movement. This effect is summarized by the dimensionless coefficient for critical stress C^*. Adopting $C^* = 0,4$, numerical simulation were carried to observe the resulting waves on Fig. 5.

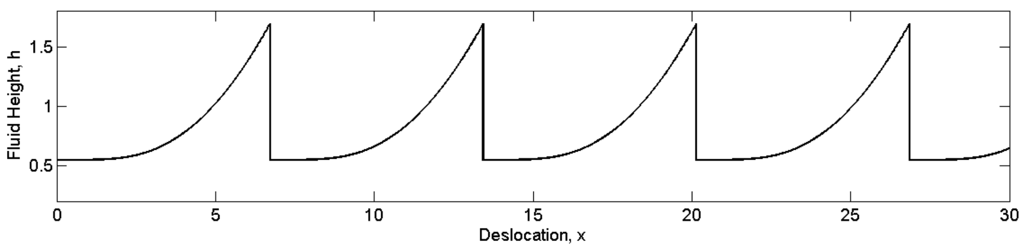

Figure 5. Numerical simulation of roll waves in Herschel-Bulkley fluids ($n = 0,7$; $C^* = 0,4$; $Fr = 1$). Wave celerity $U = 2,16$, wave amplitude $\Delta h = 1,146$, and wavelength $\lambda = 6,71$.

The coherence effect due to the fluid rheological behavior contributes to the increase of the wave amplitude ($\Delta h_3 > \Delta h_2$) and to the decrease of the wavelength ($\lambda_3 < \lambda_2$). The last effect is equivalent to the anticipation of the wave's generation reported by Maciel *et al.* (1997) when increasing the coherence effect in his second-order model for roll waves in Bingham fluids.

These effects has been well explored mathematically and numerically by many researchers all over the world and the results found by RMVP team show good agreement with those. As will be shown in section 3, the work that is now being developed the team is also related to the physical experimentation of these waves, in a attempt to evaluate quantitatively the models developed and make them usable.

3. EXPERIMENTAL PROCEEDINGS

The first-order model presented in the Section 2 could be used to represent others fluids with different rheological behavior, since the model inserted has 3 parameters that can express parameters from others models, such as Bingham, Power-law, and even Newtonian. From this model, numerical methods were exploited to verify how and when instabilities appears in open channel flows using many environments, like Python, Fortran and Matlab/Simulink. The programs developed were able to generate wave-like patterns just like presumed by the analytical modeling, but to assume that the data acquired from numerical simulations are realistic, it is necessary to validate the numerical and analytical models adopted. At this point the major problem emerges, where experimental data for roll waves has to acquired by empirical tests.

Taking measures of amplitude and wavelength are delicate because if one wishes to make it while disturbing the flow it can be done by inserting resistive probes or capacitive systems, which are easily manipulated, but may contribute to misevaluate the data acquired, besides having no good precision. In other hand, non-intrusive systems, such as laser-based, have much better precision but are more complicated to manipulate. An option has to be made depending on how it is desired the flow to be: if large and not so fine, intrusive system could be a easy and cheap choice; for sensitive and delicate flows, non-intrusive are a wiser option, and must be carefully studied for adapting to the flow configuration. The experimentation in roll waves initialized with Cornish (1934) that registered for the first time roll waves, and described their behavior in prismatic and artificial channels. After that, the experiments carried by Brock (1969) were very useful to comprehend in what circumstances the waves would appear, however the apparatus used, pressure transducers and resistive probe (called by him as wire gage), had low precision and were intrusive to the flow. Later on, once of the most impressive works found, an apparatus composed by a channel capable of resist to external perturbations were developed by the research of Gollub (Liu and Gollub, 1994; Liu *et al.*, 1993). The measures were taken by fluorescence imaging technique which consists in adding a fluorescent dye to the fluid and illuminate the flow with a special light, giving the flow his own bright and, assisted by a camera, all can be recorded in images that, after treated, will have the wave data, as shown in Fig. 6.

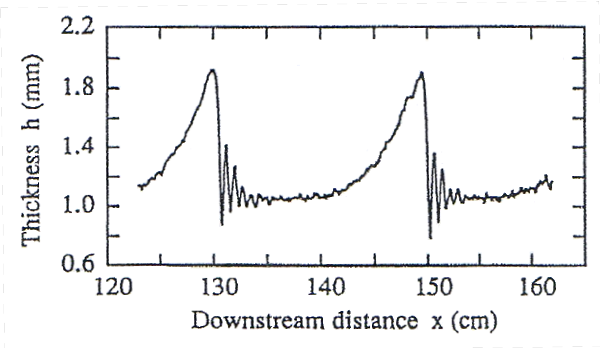

Figure 6. Profile of a wave train obtained from a disturbed uniform flow; $\omega = 1.5$ Hz (disturbing frequency), $Re = 29$ (Reynolds number) and 6.4^o of steepness. (Liu and Gollub, 1994)

Recent works use light techniques such as total external or internal reflection, light absorption, interferometry, to get more reliable and precise data for different flow configurations. Mouza *et al.* (1999) were capable of acquiring a precise data, compared with a conductive probe, for a uniform flow of a thin layer fluid, Fig. 7. Thin layer fluid, or film flow, is a phenomenon explored by chemists, physicists and engineers for its presence in many industry process such as electrochemical plating, crystal growth, condensers, etc. The measure techniques adopted by this experts for measuring films flows are useful because they are quite precise and have a rapid time response. Moreover, instabilities can be observed in film flows and even roll wave-type (Fig. 6).

In measuring film flows by non-intrusive systems, many photometrics methods are able in acquiring information about the flow height, but the choose depends on how precise will be the measure and how difficult (and expensive) it will be. In the next section, the method adopted by the group is described.

3.1 Photometric Measuring System: Light Absorption method

Whenever light passes through any medium, solid, liquid or gas, the electromagnetic waves that composes it will suffer from interference, scattering or being absorbed, by the medium. Also, depending on how the light focus the medium it will reflect and refract at some degree. If, for example, a light beam is directed through a liquid sample of a solution and aligned with a photodetector (positioned after the sample) the light intensity perceived by the detector is proportional to

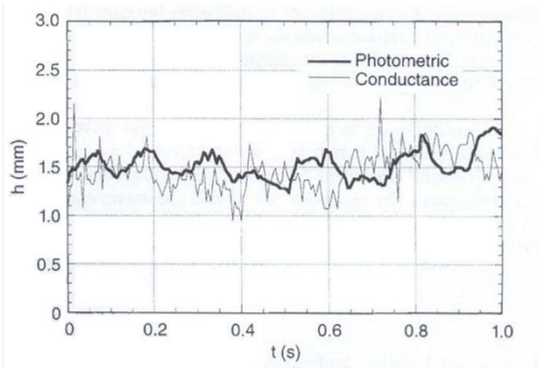

Figure 7. Measures of a film fluid (water + methylene blue solution) obtained by photometric system (dark line) and by conductive probe (light line), for the same flow configuration. (Mouza *et al.*, 1999)

the concentration of the substance in solution and to the height of that liquid, obeying to the Beer-Lambert's Law:

$$I = I_0 e^{-\varepsilon C h} \tag{17}$$

where I_0 is the incident light intensity, ε is the absorptivity of the substance, C is the concentration of that substance in solution and h is the pathlength of the light (liquid height). No light scattering is considered. Then, the transmittance, denoted by T, will be

$$T \equiv \frac{I}{I_0} = e^{-\varepsilon C h} \tag{18}$$

and the absorbance, A, is:

$$A \equiv -\log(T) = -\log\left(e^{-\varepsilon C h}\right) \Rightarrow A = \varepsilon C h \tag{19}$$

which express a linear relation between the light intensity absorbed and the light pathlength.

In the literature can be found some experiments for measuring film flows that exhibits good results of precision. Barter *et al.* (1993) were able to get height and slope information from a wave pattern generated inside a water tank with an pneumatic oscillator. The certainty reported is about 0.005 mm which is a very precise measure for fluid height.

Some difficulties appears when trying to develop this experiment which lays on how will light reflection and refraction disturb the final result. Most of the tests are made considering the flow as uniform, maintaining the flow undisturbed from undesired sources. The uniform flow allows the apparatus' user to keep approximately 90^o angle of the incident light, minimizing the effects of the reflection. Refraction effect do not bother as much as reflection and a well-chosen photodetector, with a wide effective area, will minimize its effects.

3.2 Experimental results

Aware of the studies here referenced, our team initiated the project of calibration set of the photometric system following the work of Mouza *et al.* (1999). A BPX65 photodetector was coupled with a He-Ne laser, 633 nm, to mount the elementary part of the system. Three different test solution were elaborated using pure water and methylene blue in different concentrations to make the first effort of representing the absorption phenomenon.

The first results for calibration tests exhibited at Fig. 8 shows good linearity response of the apparatus but not so good correlation. The calibration coefficient found, that correspond to the molar absorptivity coefficient of the methylene blue, had a value that does not fit the data acquired very well. The Fig. 9 shows that a major problem exists for low heights of the sample.

Below 2 mm of solution, for all 3 tests, the molar absorptivity had a non-constant behavior what suggests that the substance diluted, methylene blue, had it natural characteristics changed with the fluid height, which it is not true. Statistical analysis showed a mean error of $15,57\%$ for the system, which is considerable value. It is evident that the mean value found, $\varepsilon_{medio} = 2.78 \cdot 10^4$ Lmol^{-1}cm^{-1}, is lower than the mean values of each test because of the low values for the absorptivity below 2 mm. Thus, the tabulated values for a light source of 633 nm are between $4.29 \cdot 10^4$ and $4.42 \cdot 10^4$ Lmol^{-1}cm^{-1} (OMLC, 2010). Test were carried at the laboratory using a spectrofotometric equipment DR 5000 UV/VIS at wavelength 633 nm and showed that the molar absorptivity of the methylene blue is $4.37 \cdot 10^4$ Lmol^{-1}cm^{-1} which is between the range specified by the reference.

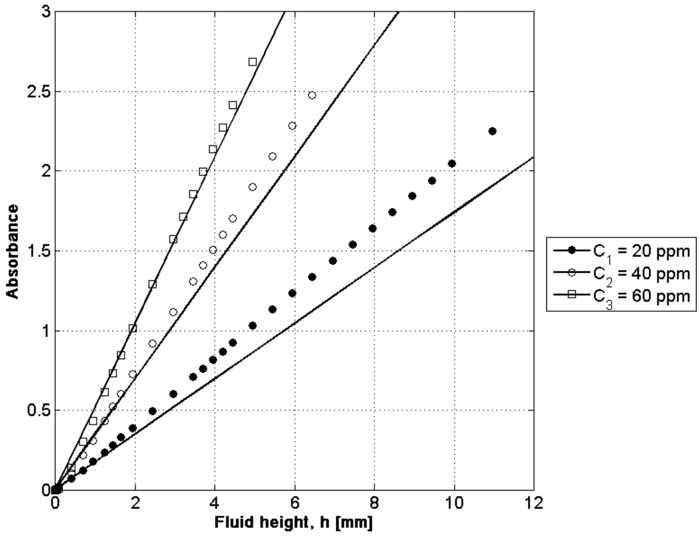

Figure 8. Results for calibration tests for water + methylene solutions in different concentrations.

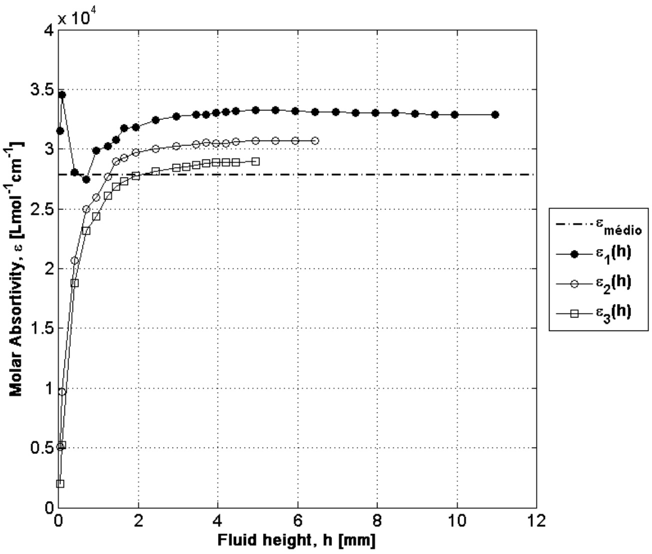

Figure 9. Molar absorptivity, ε, plotted against fluid height, h. The indicial number refers to respective concentration (1, 2 and 3).

4. CONCLUSION

Shock waves presents discontinuities that must be taken seriously and with caution when developing effective models and realistic representation of the phenomenon. In other words, modeling roll waves is not a easy job to be done. However, based on solid theories and past models, the mathematical model presented by the team shows itself as congruent as the classic ones. The results acquired by the numerical modeling through Matlab/Simulink, an effective tool for solving dynamical systems, illustrated the results expected with fidelity and assured models such as Ng and Mei (1994), Maciel *et al.* (1997) and Dressler (1949). At this point, it's relevant to note that the reduced model here presented can generate roll-waves solutions in a few seconds for a specific parameters' group.

A brief conceptualization about the experimental method adopted by team were made based on well-known experiments in fluids flow. The initial attempt to measure thin layer fluid was acceptable showing good behavior of the system, in agreement with the light absorption theory. In order to reduce the error detected, more tests should be conducted, intensifying the efforts to minimize the random errors of the system and making easier to detect systematic errors that could be infecting the tests. With the calibration apparatus complete and functional, its adaptation for the channel will be made in order to use the photometric system based on absorption technique to measure film flows in motion and with oscillatory pattern. The data acquired will make possible to validate the numerical and analytical methods adopted for estimating roll waves formation in Newtonian and non-Newtonian fluids flow at open channels, and will allow the prediction for what circumstances roll waves could contribute to intensify hazards and accidents.

5. ACKNOWLEDGMENTS

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7. Responsibility notice

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