FLEXIBLE BEAM STRUCTURAL DYNAMIC ANALYSIS USING DYMORE MULTI-BODY ELEMENTS SOFTWARE

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Abstract. *Structural dynamic analyses have a wide application in industries such as automotive and aeronautical. The use of advanced tools warranties the optimization of the products. Currently, it is possible to model non-linear flexible multi-body systems with arbitrary topologies using these tools. This article studies fundamental dynamic analyses of flexible beam using the finite elements method. These dynamic analyses are executed using DYMORE multi-body elements software. Results obtained are compared with MSC/NASTRAN results. DYMORE uses the direct integration method to obtain the analysis responses. In this article, MSC/NASTRAN uses the mode superposition method to solve the equations of the structure. The focus of this article is to obtain the responses for the two methods and discuss the results. DYMORE has particular characteristics that improve the accuracy of responses for structural dynamic problems. These result's accuracy is fundamental to have DYMORE used on evaluating complex structural topologies.*

Keywords*: Flexible Beam elements, structural dynamic analysis.*

1. INTRODUCTION

This article aims to analyze fundamental problems of structural dynamics. Two programs are used to evaluate the results. The first software used in this article is DYMORE. This software was developed to analyze complex models of mechanisms such as non linear multi-body systems. In other words, the DYMORE can be used to simulate multi-body systems considering the flexibility of each body. The analysis elaborated in DYMORE uses the direct integration method to obtain the system responses. In this case, all vibration modes are considered in problem solution. The integration of the motion equations causes problems as undesirable high frequency oscillations of a purely numerical origin (BAUCHAU *et al*, 1994). DYMORE uses the finite element method and an internal algorithm called *Energy Decaying Scheme*. This algorithm eliminates the problems of numerical instability generated by high order frequencies. For this to occur, the algorithm maintains the system's energy constant, i.e., the energy does not increase (BAUCHAU *et al*, 1994). The demonstration of the energy preserving schemes that prove the solution methods stability is presented in (BAUCHAU, THERON, 1996).

The second software used in this article is MSC/NASTRAN. This software is a commercial program used in aeronautical and automotive industries. MSC/NASTRAN analysis was executed using the mode superposition in order to solve the motion differential equations. The main advantage of the mode superposition method utilization is the vibration modes number reduction that will be considered in the responses calculation. This artifice permits that the numerical stability problems generated by the high order modes can be eliminated. I.e. the method is based on the transformation of the matrices of mass, stiffness and damping of the system. This transformation produces the generalized matrices of mass, stiffness and damping of the system. Such system generalized matrices have lower bandwidth than the original system matrices. This process warranties that the equations become uncoupled. Then, it is possible to define how many vibration modes that will be considered in the analysis.

The mode superposition utilization presents advantages such as the vibration modes reduction that will be included in the responses calculation. However, the reduction process modifies the mode representation of the applied load, affecting the quality of the responses calculated. The complete demonstration of the mode superposition method is presented in (BATHE, 1976). MSC/NASTRAN analysis was made using mode superposition method purposely, in order to obtain the differences between the direct integration and mode superposition methods.

Three analyzes were performed and the results obtained are discussed herein. The beam models analyzed by both programs have the same properties, meshing, material, loading, and boundary conditions for each analysis.

2. STRUCTURAL DYNAMICS

2.1. Direct Integration (BATHE, 1976)

Considering a structural system with *n* degrees of freedom, subjected to forced vibration, the following matrix differential equation can be used to model the motion:

$$M\ddot{U} + C\dot{U} + KU = R, \qquad (1)$$

where *M*, *C* and *K* are matrices of mass, stiffness and damping of the system. *R* is the vector of the external forces, and $\ddot{U}$, $\dot{U}$, U are the vectors of acceleration, velocity and displacement of the system. The equation (1) can be written the following manner:

$$F_I(t) + F_D(t) + F_E(t) = R(t), \tag{2}$$

where: $F_I(t)$ are the inertial forces, $F_D(t)$ are the damping forces and $F_E(t)$ are the elastic forces of the system. $F_I(t)$, $F_D(t)$ and $F_E(t)$ are dependent of the time. First of all, the dynamic analysis is assumed that the static equilibrium at the time *t*, which includes that the inertial force is dependent of the acceleration. And the damping force is dependent of the velocity. On the other hand, on the static analysis, the inertial and damping effects are neglected. The choice of static or dynamic analysis (i.e. the inclusion or not the inertial and damping forces) is decided by the engineering examination. The objective of this examination is to reduce the necessary analysis effort. The equation (1) represents a second order differential equations linear system which solution can be obtained by the normal process of differential equation solution by constants coefficients. However, this solution can be very expensive, if the orders of matrices are very high, unless some special characteristics of the *K*, *C* and *M* matrices are utilized.

In the direct integration, the equation (1) is integrated by a numerical procedure step by step. The term "direct" means that no transformation is executed in the equations before the numerical integration. In essence, it is based on two ideas: The first idea is that, instead of satisfying the equation (1) at each time *t*, the solution is obtained at discrete time intervals Δt, separately. It means that the static equilibrium is calculated, including the inertial and damping forces effects, at discrete points of time within the solution interval. In this case, it is assumed that all the techniques applied in static analysis could be used in direct integration.

The second idea is that it is assumed a variation of the displacements, velocities and accelerations in each Δt time interval. The way in which this variation is assumed, determines the accuracy, the stability and the computational cost of solution. The direct integration can be divided into explicit and implicit methods.

In explicit methods, the motion equation solution is obtained at time $t+\Delta t$, considering the equilibrium conditions at time *t*. These integration schemes do not require the stiffness matrix inversion in the solution step by step. For this reason, the method does not require the matrices storage if the mass matrix (concentrated or generalized) is utilized. The explicit methods are conditionally stables and require the utilization of small steps of time to ensure the stability. For example, it can cite the method of central differences and the Runge-Kutta method (WIJKER, 2004).

In implicit methods, the displacement equations in this time step involve the velocities and accelerations at their own step $t+\Delta t$. Then, the determination of displacements at $t+\Delta t$ involves the structural stiffness matrix inversion in this time step. However, many implicit methods are unconditionally stable for linear analysis and the step size of integration only affects the accuracy of results. Among the most popular methods are: Houlbolt, Wilson-Θ, e Newmark (WIJKER, 2004).

2.1. Mode Superposition (BATHE, 1976)

The number of operations needed in the direct integration is directly proportional to the number of time steps used in the analysis. Therefore, in general, the use of direct integration can be effective when the desired response is relatively short (a few time steps, for example). However, if integration is performed for many time steps, can be more effective to transform the equations of equilibrium in a way that the solution step by step is less expensive.

In order to transform the equations of equilibrium (1) in a more efficient way of direct integration, the following transformation of the displacements *U* can be used.

$$U(x,t) = \Phi(x)q(t), \tag{3}$$

where Φ is a transformation matrix (square) to be determined, *q(t)* is the dependent vector of time of order *n*. The components of *q* are known as generalized displacements.

The transformation above can be used in equation (1), and then, the following expression can be written:

$$M\Phi\ddot{q} + C\Phi\dot{q} + K\Phi q = R, \tag{4}$$

and pre-multiplying the expression for Φ^T , it has:

$$\Phi^T M\Phi\ddot{q} + \Phi^T C\Phi\dot{q} + \Phi^T K\Phi q = \Phi^T R, \tag{5}$$

that results in:

$$\tilde{M}\Phi\ddot{q}(t) + \tilde{C}\Phi\dot{q}(t) + \tilde{K}\Phi q(t) = R, \tag{6}$$

where:

$$\tilde{M} = \Phi^T M \Phi,$$
$$\tilde{C} = \Phi^T C \Phi,$$
$$\tilde{K} = \Phi^T K \Phi,$$
$$\tilde{R} = \Phi^T R \Phi, \quad (7)$$

In essence, a transformation of the finite element displacement was made to generalized displacement. The objective of this transformation is the obtainment of a new system of stiffness, mass and damping matrix. These matrices have a lower bandwidth than the matrices of original system. The matrix [Φ] must be chosen properly.

In theory, there may be different transformation matrices [Φ], however, a transformation matrix is established using the solution of displacements of equilibrium equations of free vibration, i.e., without damping, resulting in equation (8):

$$M\ddot{U} + KU = 0, \quad (8)$$

which solution can be considered as:

$$U = \phi sen[\omega(t - t_0)], \quad (9)$$

where ϕ is a vector of n order, t is the time variable, t_0 is the time constant and ω is a constant that represents the vibration frequency in (rad/s) associated with the vector ϕ. Substituting the equation (9) in (8), it has a problem of generalized eigenvalues which ϕ and ω must be determined, i.e.:

$$K\phi = \omega^2 M\phi, \quad (10)$$

this problem generates the n solutions: $(\omega_1{}^2, \phi_1), (\omega_2{}^2, \phi_2) \ldots (\omega_n{}^2, \phi_n)$, where the eigenvalues are normalized by the matrix of mass M, i.e.:

$$\phi_i^T M \phi_j = \begin{cases} 1, se\ i = j \\ 0, se\ i \neq j \end{cases} \quad (11)$$

and,

$$0 \leq \omega_1^2 \leq \omega_2^2 \leq \omega_3^2 \ldots \leq \omega_n^2. \quad (12)$$

The vector ϕ_i is called by modal vector of order i, and ω_i is the corresponding natural frequency. The equation (8) is satisfied using any one of the n displacement solutions.

$$U = \phi_i sen[\omega_i(t - t_0)], \quad i = 0,1,2,..n. \quad (13)$$

When the matrix Φ is defined, the columns are the eigenvectors ϕ. And a diagonal matrix Ω^2 that contains the eigenvalues ω_i^2 in the diagonal, as expressed below:

$$\Phi = [\phi_1, \phi_2, \ldots, \phi_n] \quad (14)$$

$$\Omega^2 = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ldots & \\ & & & \omega_n^2 \end{bmatrix}, \quad (15)$$

it is possible to write the n solutions for the equation (10), as the following expression:

$$K\Phi = \Omega^2 M\Phi. \quad (16)$$

The eigenvectors suffer an orthonormalization process by the matrix of mass M, then:

$$\Phi^T K\Phi = \Omega^2, \Phi^T M\Phi = I. \quad (17)$$

In this case, the matrix [Φ] could be the appropriate transformation matrix in equation (3). The equations of equilibrium that correspond to the generalized mode displacements can be obtained using the equation (3):

$$\ddot{q}(t) + 2\Xi\Omega\Phi\dot{q}(t) + \Omega^2 q(t) = \Phi^T R(t), \quad (18)$$

where Ξ is the diagonal matrix of the parameters of modal damping ξ_i.

The initial conditions of q(t) are obtained using the equation (18) at the time equal 0:

$$q_0 = \Phi^T MU_0, \dot{q}_0 = \Phi^T M\dot{U}_0, \quad (19)$$

3. NUMERICAL EXAMPLES

3.1. Modal analysis of a cantilever beam – Analysis 1

The first numerical example presented in this article is a modal analysis. The numerical examples presented in this article are fundamental problems of the structural dynamics. The objective of this topic is to obtain the results of these analyzes using the DYMORE multi-body elements software. These results are compared with the ones obtained using MSC/NASTRAN.

With the preliminary results of modal analysis, it is possible to adjust the input data cards in order to execute the analysis *2* and analysis *3*.

Figure 2 presents the sketch of the model analyzed in this item. The model shows a beam with two tips: one tip is clamped and the other is free. This structure is called *cantilever beam*. No load is applied in the structure. The modal analyzes considers only the stiffness and mass of the structure. The modal analysis can be elaborated solving the equation (8).

In accordance with the notation utilized in DYMORE, the axis that coincides with the neutral line is denoted axis *1*. The vertical axis is denoted axis *3* and the third, axis *2* (BAUCHAU, 2007). All units of the model are on the international system of units. The beam has length of $L = 1\ m$.

Figure 1. Cantilever beam sketch for modal analysis.

The beam cross section considered to the modal analysis has a rectangular cross section with *0.05 m* of width and *0.1 m* of height. The material of the beam defined has the following properties: Elastic Modulus: $E = 70\ GPa$, Poisson Ratio: $\nu = 0.3$, and Density: $\rho = 2780\ kg/m^3$.

For the DYMORE analysis, the beam properties are calculated using the material and cross section data defined previously. The values of the beam properties are calculated in accordance with DYMORE User's Manual (BAUCHAU, 2007):

Table 1. Beam property definition to DYMORE analysis.

SECTIONAL STIFFNESS PROPERTIES	
AXIAL STIFFNESS [N]	3.50E+08
BENDING STIFFNESS [N.m^2]	2.9167E+05, 7.2917E+04 , 0.0000E+00
TORSIONAL STIFFNESS [N.m^2]	7.7027E+04
SHEARING STIFFNESS [N]	1.3462E+08, 1.3462E+08 , 0.0000E+00
SECTIONAL MASS PROPERTIES	
MASS PER UNIT SPAN [kg/m]	13.9
MASS MOMENTS OF INERTIA [kg.m^2/m]	1.4479E-02, 1.1583E-02 , 2.8960E-03

The modal analysis results are obtained using Table 1 input data. Figure 3 presents the four vibration modes with their natural frequencies. Table 2 presents all vibration frequencies of the 10 modes for both programs: DYMORE and MSC/NASTRAN. The objective of this analysis is to compare results for both DYMORE and MSC/NASTRAN software.

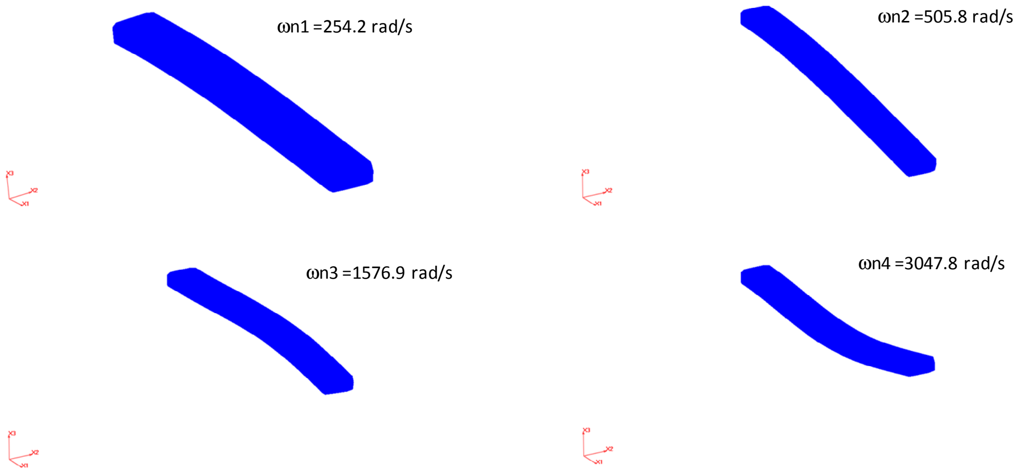

Figure 2. Modal analysis results of the cantilever beam using DYMORE.

Figure 4 presents the finite element model considered in MSC/NASTRAN analysis. For both DYMORE and MSC/NASTRAN models, were considered *45* beam elements. The beam root is clamped. The beam element considered at MSC/NASTRAN was CBEAM. This element uses the Timoshenko Theory formulation. It's important to emphasize that the MSC/NASTRAN analysis was elaborated by mode superposition method. I.e., the frequency range of interest (final and initial) is defined as analysis input data (MSC.SOFTWARE CORPORATION, 2002). The value of the initial frequency is defined by *0 Hz* and the final frequency is defined as *3000 Hz*. Considering this frequency range input data, the *10* first modes of vibration are calculated. This artifice is utilized because the MSC/NASTRAN does not have a preserving energy algorithm. I.e., in this article, MSC/NASTRAN utilizes a mode superposition in order to avoid the numerical instabilities problems. All commands utilized to elaborate the MSC/NASTRAN card are defined in the Quick Reference Guide (MSC.NASTRAN, 2004).

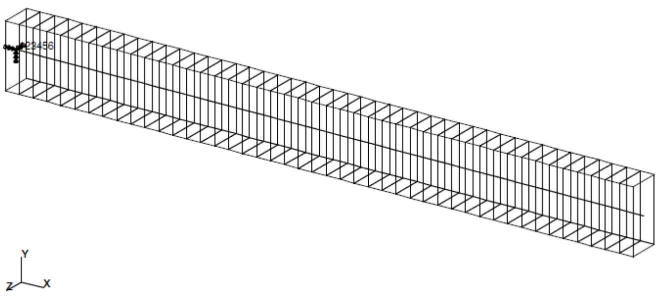

Figure 3. Finite element model of the cantilever beam elaborated in MSC/NASTRAN (model with 45 CBEAM elements).

Table 2. Modal analysis results of the cantilever beam using DYMORE and MSC/NASTRAN.

NATURAL FREQUENCIES OF THE CANTILEVER BEAM			
EIGENVALUES	DYMORE	MSC/NASTRAN	*Difference (%)*
	f (Hz)	*f (Hz)*	
1	40.460	40.460	0.000
2	80.502	80.556	-0.067
3	250.970	251.167	-0.079
4	485.068	487.439	-0.489
5	576.623	576.558	0.011
6	691.571	693.100	-0.221
7	1254.488	1254.425	0.005
8	1284.431	1296.452	-0.936
9	1325.379	1330.710	-0.402
10	1729.871	1728.970	0.052

The results presented in Table 2 certify that both of models (DYMORE and MSC/NASTRAN) are adjusted to elaborate the analysis 2 and analysis 3.

The differences between the two software (DYMORE and MSC/NASTRAN) are expected because the differences between the solution methods.

3.2. Dynamic analysis of the cantilever beam – Analysis 2

The beam dynamic analysis *2* considers the same DYMORE input data utilized in modal analysis, except for the loading applied. The dynamic loading utilized in analysis *2* is a time function such as showed in Figure 4. In this analysis, the load *P* has the same direction of the beam axis *2*, i.e. is an example of horizontal bending of the beam. The force varies linearly since *0* until *1* second. And the value of the load varies linearly from *0* to *100 N*. After the time reaches *1* second, the load keeps constant in *100 N*. Figure 4 shows that the time function is varying from 0 to 1. During the analysis solving, the time function is multiplied by *100N*.

The finite element model elaborated in MSC/NASTRAN for this analysis also has 45 beam elements. It is utilized the solution 112 for this analysis (Linear Transient Response Analysis, Modal Formulation). Figure 5 presents the DYMORE and MSC/NASTRAN analysis results.

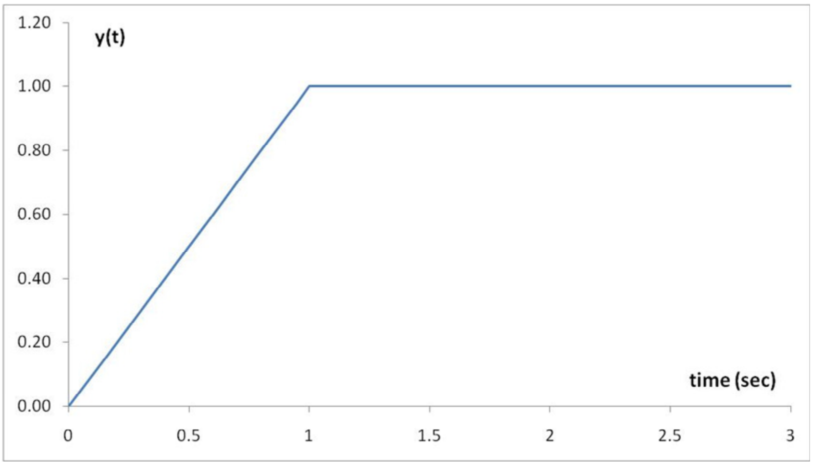

Figure 4. Loading time function of the dynamic analysis 2 (DYMORE).

The dynamic response of the displacement *u2* in meters of DYMORE and *T3* in meters of MSC/NASTRAN for the cantilever beam considering a type of loading presented in Figure 4 is showed in Figure 5:

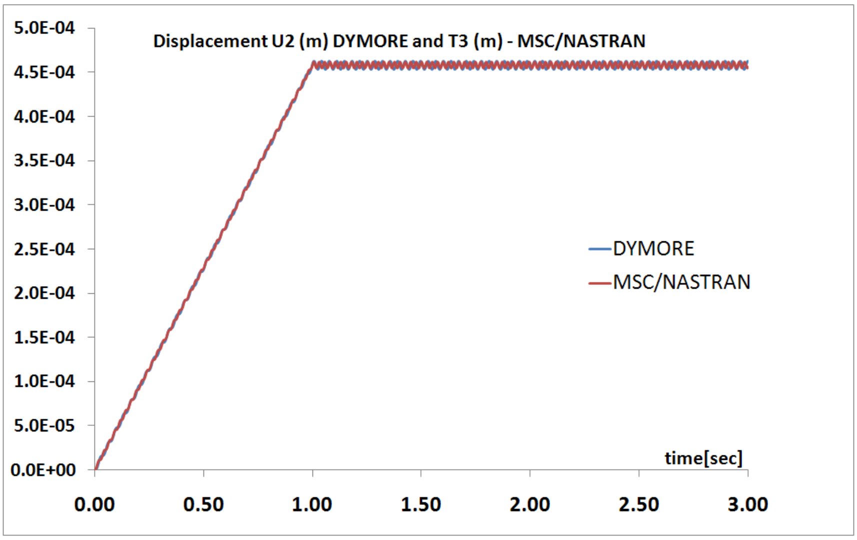

Figure 5. Comparison between DYMORE and MSC/NASTRAN responses.

These results demonstrated that the responses obtained from the both of software DYMORE and MSC/NASTRAN are very close. Table 3 presents the numerical results. The differences can be explained by the solution methods utilized.

Table 3. Results of analysis *2* of the cantilever beam using DYMORE and MSC/NASTRAN.

TIME	DYMORE	MSC/NASTRAN	*Difference*
(s)	*U2 (mm)*	*T3 (mm)*	*%*
2.990	0.4575	0.4581	-0.146%
2.991	0.4583	0.4573	0.231%
2.992	0.4592	0.4564	0.589%
2.993	0.4599	0.4557	0.914%
2.994	0.4606	0.4551	1.191%
2.995	0.4612	0.4547	1.409%
2.996	0.4618	0.4546	1.558%
2.997	0.4622	0.4546	1.637%
2.998	0.4624	0.4549	1.637%
2.999	0.4625	0.4553	1.555%
3.000	0.4625	0.4560	1.403%

3.3. Cantilever beam dynamic analysis – Analysis 3

The dynamic analysis 3 considers the same DYMORE input data of the modal analysis, except for the loading and the boundary conditions. In this case, the rotation around the axes *1*, *2* and *3* are released. The dynamic loading is a time function as the equation (20).

$$F(t) = 100.y(t) \quad Where: \; y(t) = \begin{cases} 5.t, & for \;\; 0 \le t \le 0.2 \text{ sec} \\ 0, & for \; t > 0.2 \text{ sec} \end{cases} . \tag{20}$$

The time of simulation for this analysis is defined in *3* seconds. Figure 6 presents the time function (20) graphically:

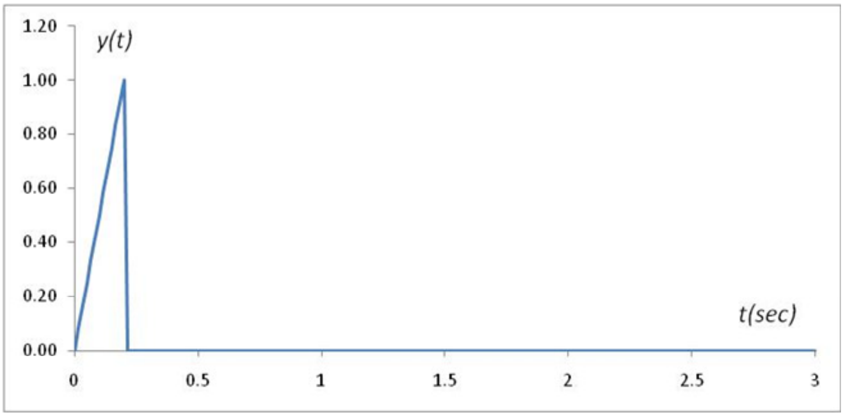

Figure 6. Loading time function of the dynamic analysis 3.

This analysis was performed considering both programs: DYMORE and MSC/NASTRAN. The dynamic responses of the angular velocity $\omega 3$ in *rad/sec* and *R2 rad/sec* obtained using DYMORE and MSC/NASTRAN respectively, are showed in Figure 7:

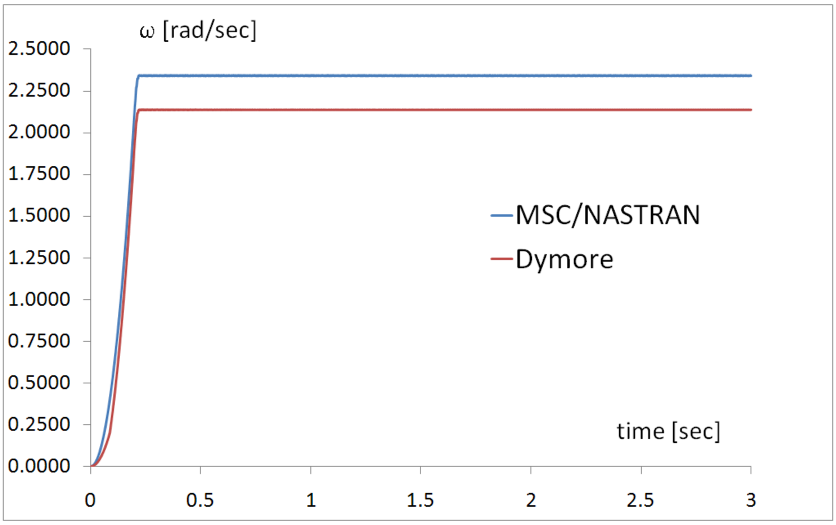

Figure 7. Angular velocity dynamic response $\omega 3$ (DYMORE) and *R2* (MSC/NASTRAN) for the cantilever beam.

Figure 7 shows that the beam keeps a uniform circular motion around axis *3* after the loading finished. The difference obtained is *8.697%*. The differences can be explained by the solution methods utilized. The DYMORE analysis uses the direct integration method and the MSC/NASTRAN uses the mode superposition.

Considering the cantilever beam as a rigid body, it is possible to obtain the exact response of the angular velocity using the following expression:

$$J_m \ddot{\theta} = M_3(t). \tag{21}$$

Where J_m is mass moment of inertia of the beam, $\ddot{\theta}$ is the angular acceleration of the beam and $M_3(t)$ is a time function for the moment applied in the tip. The J_m mass moment of inertia can be written according to expression (22):

$$J_m = \frac{mL^3}{3}. \tag{22}$$

The angular velocity can be obtained making a time integration of the equation (10) for $t = 0$ *sec* to $t = 0.2$ *sec*. Considering that the moment $M_3(t) = F(t).L$, resulting:

$$\dot{\theta} = \frac{L}{J_m}\int_0^{0.2} F(t)dt. \tag{23}$$

Substituting the values, the angular velocity obtained for the time of *0.2 sec* is *2.158 rad/sec*. Table 4 below presents the results of both programs and the exact solution for the rigid body.

Table 4. Dynamic structural analysis results of the cantilever beam.

TIME	DYMORE *ω3 (rad/s)*	Exact solution *ω (rad/s)*	*Difference* *%*	MSC/NASTRAN *R2 (rad/s)*	Exact solution *ω (rad/s)*	*Difference* *%*
2.990	2.136	2.158	1.022	2.343	2.158	8.557
2.991	2.136	2.158	1.048	2.341	2.158	8.464
2.992	2.135	2.158	1.055	2.342	2.158	8.528
2.993	2.136	2.158	1.035	2.341	2.158	8.487
2.994	2.136	2.158	1.011	2.340	2.158	8.399
2.995	2.137	2.158	1.008	2.342	2.158	8.531
2.996	2.136	2.158	1.031	2.340	2.158	8.420
2.997	2.136	2.158	1.053	2.344	2.158	8.596
2.998	2.136	2.158	1.051	2.340	2.158	8.431
2.999	2.136	2.158	1.026	2.341	2.158	8.472
3.000	2.137	2.158	1.006	2.340	2.158	8.423

4. CONCLUSIONS

The analysis 1 results showed a difference between DYMORE and MSC/NASTRAN responses. The difference calculated was smaller than *1%*, according to data presented in table 2.

The results of analysis 2 also showed a small difference between DYMORE and MSC/NASTRAN. In the moment of time of *2.997s*, the difference calculated reached *1.6%*. MSC/NASTRAN was adjusted to obtain the responses using the mode superposition method. This artifice was defined purposively. On the other hand, DYMORE uses the direct integration method in order to obtain the responses. I.e., DYMORE considers all modes of vibration to calculate the dynamic responses. As expected, the results were different, but very close.

The analysis 3 results showed the highest differences between DYMORE and MSC/NASTRAN. As mentioned before, MSC/NASTRAN used the mode superposition method to obtain the dynamic responses. In this analysis, the difference calculated was *9.499%*. I.e., in case that the structure has velocity and acceleration, the inertial and damping forces have more influence in the results. So, the differences between direct integration and mode superposition methods also are big in analysis 3. This fact is expected because MSC/NASTRAN was adjusted to consider *10* modes of vibration in the responses calculation. DYMORE considered all modes of vibration to calculate the dynamic responses, i.e., *135* modes of vibration. The analysis of the beam as a rigid body also was elaborated in section 3.3 of this article, i.e., without effects of damping and elastic forces. As expected, the highest differences were obtained when the comparison between rigid body beam and MSC/NASTRAN analysis was made. This fact confirms that: If the influence of damping and elastic forces is big, the difference between results of different solution methods also is high.

The main cause of the differences of the responses calculated is the difference between two methods of solution: mode superposition and direct integration. The mode superposition method utilization presents advantages as the reduction of the number of vibration modes to be included in the responses calculation. However, the reduction process modifies the modal representation of the applied load, affecting the quality of the results calculated. The direct integration method is more precise because uses all modes of vibration to obtain the responses. But, instability problems of purely numerical origin can occur. Then, DYMORE is more accurate to solve structural dynamic problems. I.e., DYMORE uses the direct integration method to solve the differential equations of the system. Besides, DYMORE intern algorithm of the energy preserving maintains the energy level constant, i.e., the energy level does not increase. This fact eliminates the problems of numerical instabilities generated by high order modes.

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