

## EXPERIMENTAL AND NUMERICAL ANALYSIS OF END MILLING TOOLS DYNAMIC PARAMETERS

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**Abstract.** *Chatter is a situation that must be avoided on machining due to the bad surface generated by the tool with extreme conditions of vibrations. It is possible to calculate optimal machining parameters configurations to avoid chatter, as the maximum depth of cut for a defined spindle speed that does not produce chatter in end milling processes, using dynamic models based on the dynamic behavior of the tool and its fixture to the machine-tool. Nevertheless, dynamic models need experimental data to determine dynamic parameters as stiffness, damping and natural frequency of the system. This work presents a methodology to assess tool dynamic parameters using a nonlinear numerical model based on the Finite Element Method (FEM). To calibrate the proposed model the dynamic parameters acquired experimentally are compared with the ones obtained through FEM model for an end milling process. Using the Altintas model for predicting chatter, the two sets of dynamic parameters (taken from experiments and from simulation) are computed separately and lobe diagrams for each set are constructed and compared. Results indicate that dynamic parameters obtained from FEM model can be used with literature models, as the Altintas model, to predict chatter in end milling.*

**Keywords:** *End Milling, Machining Models, FEM, Chatter.*

### 1. INTRODUCTION

Several machining operations are executed by a large variety of machine-tools in order to produce all needed shapes and tolerances. Being under the desired tolerances is a target for the machine tools to produce non rejected pieces. Machined pieces presents dimensional or geometry error as a function of the inherent process and also due to vibrations and misplacing of tool and workpiece. Controlling the workpiece is a problem when chatter appears due to low stiffness of tool-machine-tool system or workpiece specially when using high cutting velocities values, as in HSM (Patel, Mann and Young, 2008). Stability affects directly the precision and quality of workpiece and wear and breakage of the tool. Large tool displacement is one of the causes of damage on the mechanical system, due to lower stiffness. One very important factor to avoid the appearance of chatter is controlling the stiffness of the machine-tool system (Zhao and Balachanfran, 2001).

Several papers has been published to model and predict the dynamic behavior of the system, for example the articles on milling process from Mann et al (2008), Inspegerger et al (2003) and Albrecht et al (2005), nevertheless it is necessary to run impact tests to estimate stiffness and damping coefficients and model the machine and the tool holder to avoid unexpected instability. Altintas (2000) built a model for chatter prediction on milling that uses the impact test procedure. For each tool-holder condition, it is necessary to run a set of experiments to input on the model the dynamic parameters and the lobes diagram can be used to predict chatter.

Araujo, Savi and Calas (2010) and Araujo and Savi (2008) described the dynamic of milling using a mathematical model that assumes the end milling tool as a non smooth mass-spring-dashpot system. The process of cutting in that is related to a stick-slip behavior that defines whether the chip is being removed from the workpiece. The tool holder displacement is prescribed and the local force that allows the slip motion is related to the workpiece shear stress and the equation of motion is represented by a differential equation that is solved employing the Runge-Kutta method. The dynamic parameters used on this article were estimated based on mechanical assumptions. It is needed an experimental ou numerical procedure in order to approximate the assumptions to the real parameters.

This article presents a comparison between a numerical study and experimental procedure of the dynamic parameters of an end milling tool and its tool holder. The results were used as input parameter on a known chatter model proposed by Altintas (2000) as a first step of the research (Castilho e Braga, 2010). The methodology implemented on this article is applied for that specific chatter model but can be used as input data in other dynamic modeling for end milling (Araujo, Savi and Calas, 2010).

## 2. END MILLING TOOL DYNAMICS

### 2.1 Fundamentals of forced vibrations

The end milling tool is similar to a cylindrical beam fixed in one extremity by the tool-holder and forced on the other side by the contact with the workpiece under cutting process. The dynamic of the system composed by the end milling tool, tool-holder, machine-tool and the workpiece can present complex nonlinear behavior associated to extreme conditions, as chatter, which can severely affect the quality of the machined workpiece. Modeling can be very useful for establish optimal operational conditions.

Several authors proposed simple models to study tool dynamics during machining using one (Inspegerger et al, 2003; Patel, Mann e Young, 2008) or two (Inspegerger et al, 2008; Kim, Lee and Cho, 2007; Mann et al, 2008) degrees of freedom mass-spring-dashpot mechanical system models.

For one degree of freedom, the equation that represents the behavior of the tool tip displacement  $x$  and its relation to the force applied at this position is:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad (1)$$

Where  $m$  is the mass,  $k$  is the stiffness and  $c$  is the damping coefficient of the tool system. The external force  $F(t)$  is promoted by the workpiece to the tool during the machining process and  $t$  is the time. Although the simplicity, these models can capture important complex behaviors of the tool and furnish important information about the tool dynamics.

The characterization of the main parameters can be achieved through simple tests as from a condition of free vibration, where a tool free end initial displacement,  $x_0$ , is prescribed at  $t = 0$ . It is well know that for this condition the equation of motion for the tool free end with  $F(t)=0$  can be expressed as (Altintas, 2000):

$$x(t) = x_0 e^{-\xi \omega_n t} \cos \omega_d t \quad (2)$$

Where  $\omega_n = (K/m)^{1/2}$  is the undamped natural frequency,  $\omega_d = \omega_n [(1-\xi)^2]^{1/2}$  is the damped natural frequency,  $\xi = c/[2(Km)^{1/2}]$  is the damping ratio and  $x_0$  is static displacement.

Parameters  $k$  and  $\xi$  can be obtained from a simple free vibration experimental data using the equations above and the logarithmic decrement technique. The data signal peaks  $x_1, x_2 \dots x_n$ , observed at time instants  $t_1, t_2 \dots t_n$ , can be used to predict the stiffness and the damping ratio:

$$\xi = \frac{1}{n \cdot 2\pi} \ln \frac{x_1}{x_n} \quad (3)$$

The determination of accurate values for these parameters is essential to permit a precise assessment of the tool dynamics. This can be achieved through analytic, numerical or experimental approaches. Although analytic approaches generally results in lower costs and are less time consuming, the available models are very simple, and large errors can be obtained. Numerical models offer more precise predicted values, as more complex geometries and phenomena can be incorporated in the analysis. Finally, experimental approach can lead to very precise parameters evaluation, but at the expense of higher costs and larger time consuming. Although damping parameter evaluation requires an experimental analysis, mass and stiffness parameters can be obtained through analytic and numerical methods. Therefore numerical analysis offers an interesting approach to study the dynamical parameters of these simple models.

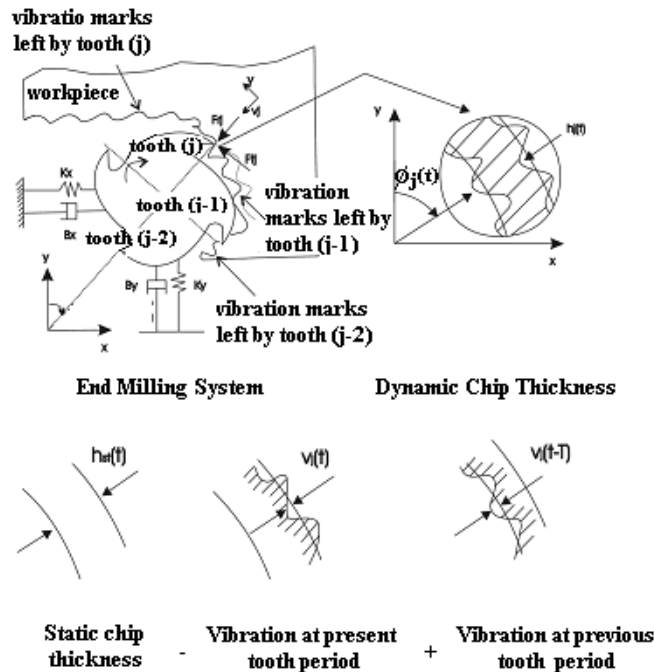
### 2.2 Regenerative Chatter Vibrations

Chatter vibrations are a result of auto-generative instability on chip formation. One of the natural frequencies of the machine-tool or the tool system can be achieved but not only on this critical situation. That is why it is needed a complex analysis that considered the time delay inherent to the milling process, related to the spindle speed and the number of teeth, as a potential source of instability. One tooth cuts more or less material and the next one will have more or less material to cut, consequently the forces will vary and the tool displacement will answer to the system variation closing the loop. Figure 1 shows the geometry that causes the chatter on end milling.

The main parameter that excites the chatter is the depth of cut combined to the spindle speed values. That is the reason it is important to draw a stability graph with defined regions those two parameters that causes stable or unstable situations.

To understand the generation of the phenomenology it is important to understand that there are two sources of chatter: the superficial wave generated on the piece, when the external force is an input of the system and it modulates the system modifying displacements and, as a consequence, the force; and the second source it is the big change on

forces associated with the entry of cutting edge on the workpiece that create different impact forces on each teeth because of run out.



**Figure 1.** 2 DOF Dynamics of chatter on End Milling (Altintas, 2000)

Altintas (2000) presented on his book a procedure for predicting the occurrence of chatter based on block diagram theory (Fig. 2). The transfer function matrix  $\Phi(s)$  identified at the cutter-workpiece contact zone and it is a square matrix with direct transfer functions related to each direction (x and y) of the tool displacement that defined the modified chip thickness  $h(s)$ . It is an analytic method that is obtained from a Fourier series expansion of force coefficients based on the Eigen values ( $\Lambda$ ) and the chatter frequency ( $\omega_c$ ).

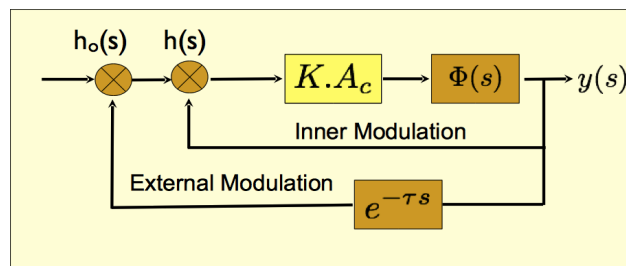
The procedure proposed by Altintas establishes the following steps:

- I. Identify the chatter frequency from transfer functions around a dominant mode
- II. Solve the Eigen value equation
- III. Calculate the critical depth of cut ( $a_{lim}$ )
- IV. Calculate the spindle speed for each stability lobe ( $N=0, 1, 2, \text{ecc}$ ) and construct the graphic

The final expression for chatter-free axial depth of cut is found as (Altintas, 2000):

$$a_{lim} = \frac{2\pi\Lambda_R}{NK_t} \left( 1 + \left( \frac{\sin \omega_c \tau}{1 - \cos \omega_c \tau} \right)^2 \right) \quad (4)$$

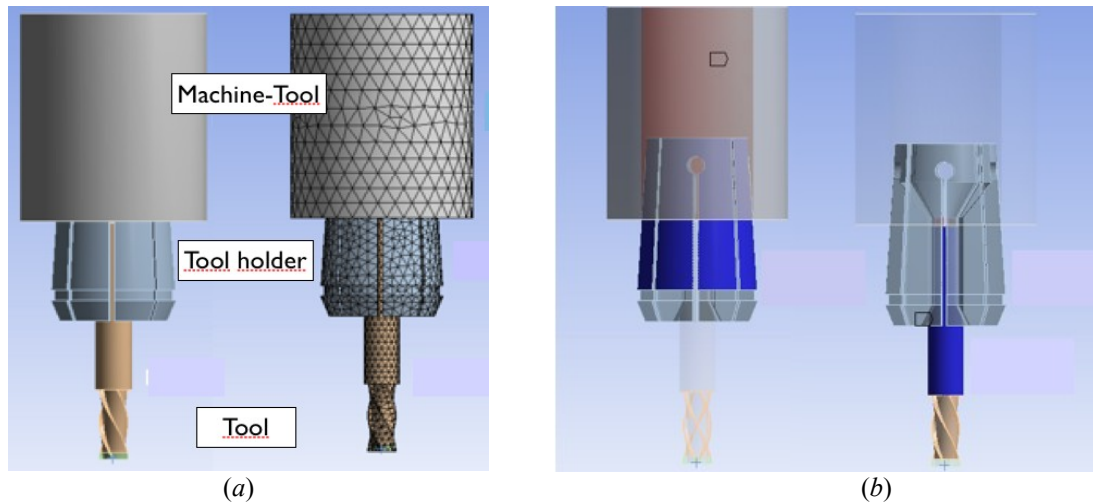
Where  $\Lambda_R$  is the real part of the eigen value and  $K_t$  is the cutting constant for specific cutting. The parameter  $\tau$  is the tooth passing period (s), the delay for the external modulation (Fig. 2).



**Figure 2.** Block Diagram on Chatter Dynamics

### 3. NUMERICAL MODEL

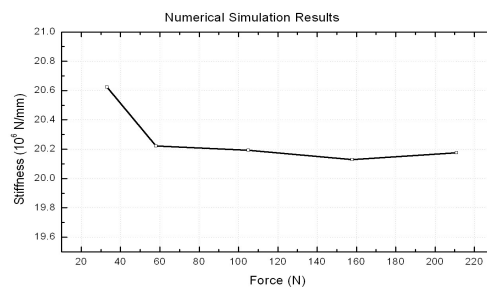
A nonlinear tridimensional model based on the finite element method is developed to study the global stiffness of the assembly composed by the end milling tool and the tool holder using the commercial software Ansys/Workbench (Ansys, 2010). The model considers the nonlinearities associated to the contact phenomenon between the end milling tool and the tool holder. As the tool fixation is obtained through the application of a radial pressure by the tool holder on the end milling tool, the magnitude of the pressure can affect the global stiffness of the assembly.



**Figure 3.** Tool Holder Assembly: (a) solid model and (b) finite element mesh. Numerical model.

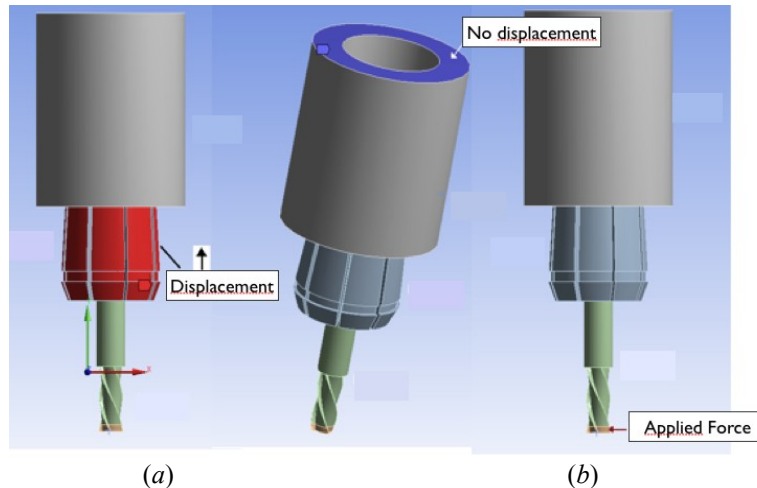
First, a solid tridimensional model is developed using the solid modeling software package Solidworks (2007) considering three geometry elements of the tool: end milling tool, mandrill and support. Figure 3 shows the three elements. With this solid model is possible to predict the total mass of the assembly, considering the density of the materials. Following, the solid model is imported to Ansys/Benchmark finite element package. Figure 3 shows the final mesh obtained after a convergence analysis.

To study the assembly stiffness, a prescribed load was applied at the tip of the end milling tool. The ratio between prescribed load and the end milling tool tip displacement can be associated to the assembly stiffness. The numerical analysis considers two steps: (1) fixture and (2) loading. The first step considers the fixture between the end milling tool and the mandrill, whereas the second step considers the application of a load at the end tool. The finite element model considers the contact between the three elements. Figure 4 shows the contact surfaces between the three elements.



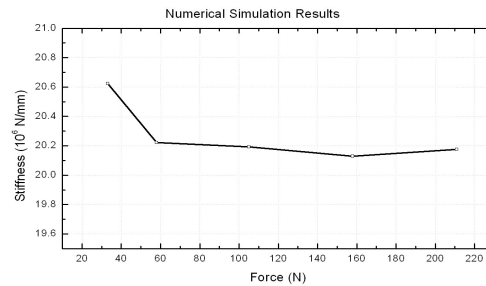
**Figure 4.** Contact surfaces between elements. Numerical model.

Some simplifications are adopted in the proposed model. First, the fixation disk used to compress the mandrill onto the support in order to furnish a fixture pressure on the end milling tool is not represented in the model. Instead, an upward prescribed displacement is applied to the lower mandrill surface in order to represent the effect of the fixation disk as it is tightened into the support threads. The prescribed displacement is associated to the thread pitch. For the presented simulations a 0.25 mm upward displacement was applied, corresponding to a 45° thread rotation. Also, null displacements and rotations are applied to the upper surface of the support. Therefore in this model, except for the three elements represented, the tool machine is considered as rigid. Figure 5a presents the imposed boundary conditions. Figure 5c shows the prescribed load applied at the end tool. To reduce concentration effects, the load is applied as a pressure at a small area at the end tool which load resultant is equal to the magnitude of the desired load.



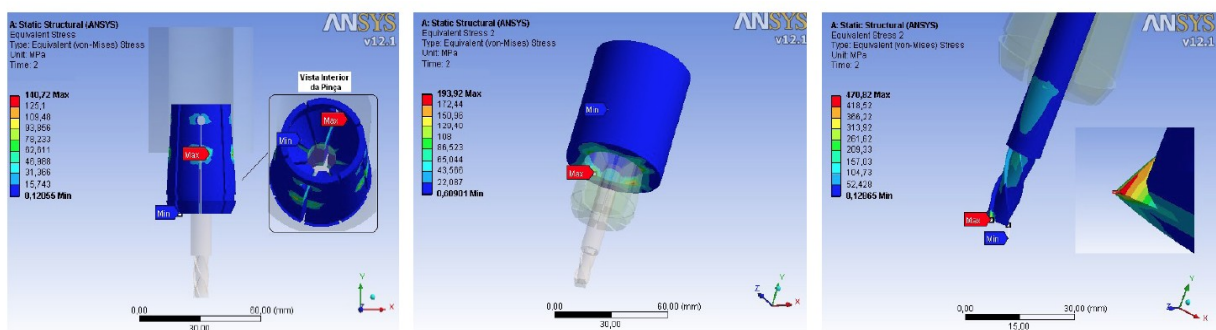
**Figure 5.** Numerical analysis: (a) Step 1 – Fixture and (b) Step 2 – Loading. Numerical model.

To study the effect of the applied load on the assembly stiffness, several loads are applied to the end milling tool end. Figure 6 presents the obtained assembly stiffness for several load magnitudes. A near constant value of 2.03 MN/m is obtained.



**Figure 6.** Assembly stiffness as a function of the applied load. Numerical Model.

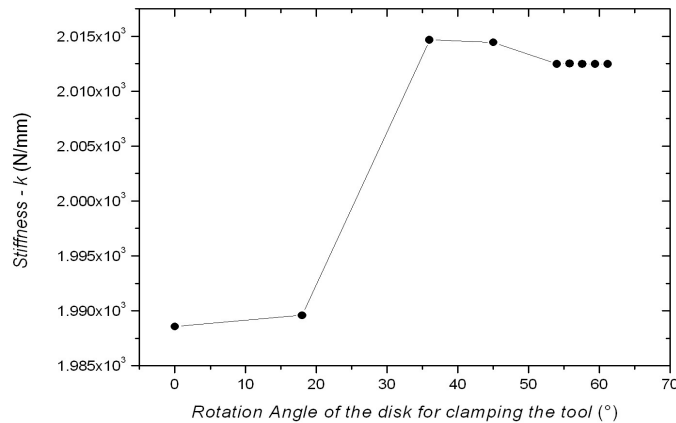
The proposed finite element model can be used to assess the critical regions of the three elements in terms of stress in order to study the assembly geometry optimization. Figure 7 shows von Mises equivalent stress distribution.



**Figure 7.** Assembly von Mises equivalent stress distribution. Numerical Model.

As the prescribed displacement is associated to the thread pitch and the rotation angle of the disk that clamps the tool, it was simulated different situations of clamping angles to analyze the stiffness variation. The same force was used on the tool tip, with a value of 128 N, and the displacement was imposed by the rotation angle of the clamping disk up to 0,32 mm on vertical direction that correspond to 61,2 degrees.

The results of these situations can be seen on Fig. 8. It is noted that on the range of the experiments, around 45 degrees, the stiffness does not depend anymore on the disk displacement.

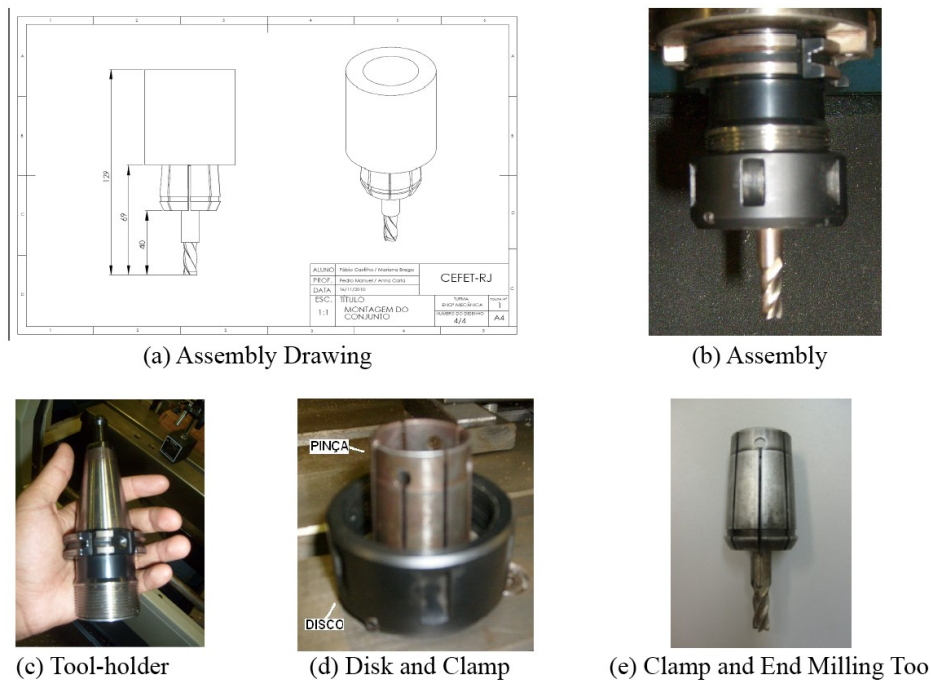


**Figure 8.** Stiffness obtained for different disk rotation angle to clamp the tool

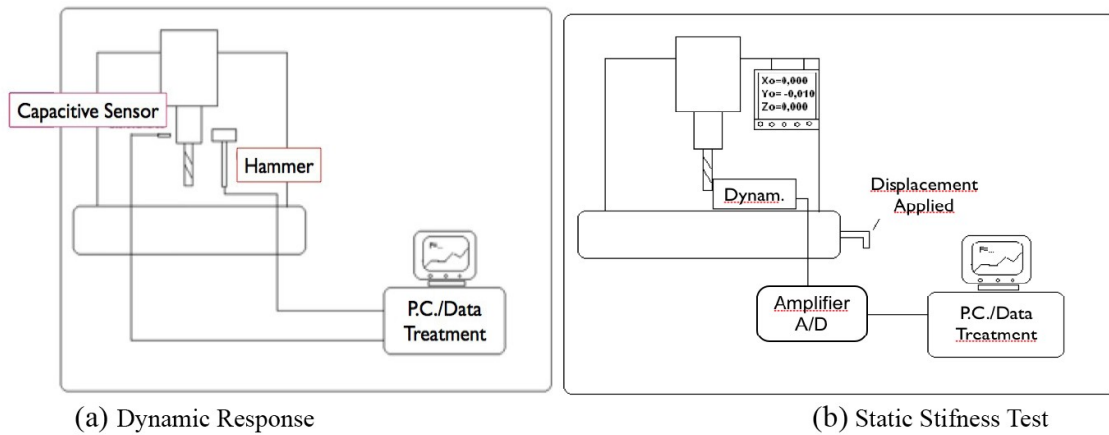
#### 4. EXPERIMENTAL PROCEDURE

Two different approaches were taken to experimentally measure stiffness and natural frequency to use as input data in chatter model and also to compare with numerical results. The first one is the conventional one, when the discussion is chatter prediction: impact hammer test. The second one is static procedure that relates applied force and displacement. For both experiments, it was used a capacitive sensor with 0.6 mm range.

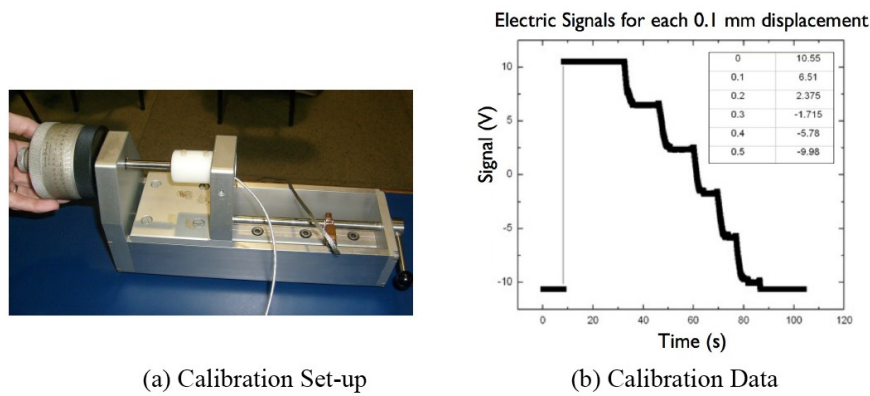
The calibration of the sensor was checked in order to verify if the acquisition system correspond to the calibration given by the manufacturer. To compare calibration it was used a micrometer system shown on Figure 9 and it was computed from the nearest point of displacement with a sequence of known discrete displacements.



**Figure 9.** Tool holder – End Milling Tool System (Castilho and Braga, 2010)



**Figure 10.** Experimental Set-up



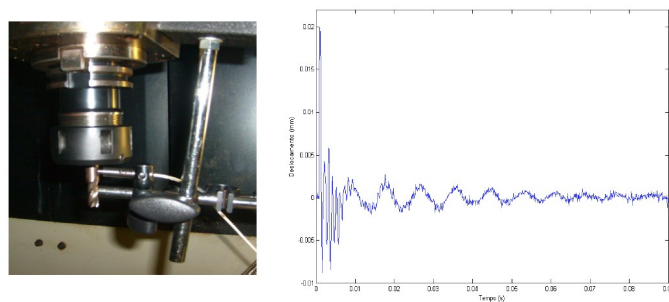
**Figure 11.** Calibration of the Capacitive Sensor

#### 4.1 Dynamic Response

The system dynamic response needs an experimental set-up, shown on Figure 10a, to be identified. The set presented on Fig. 9 is fixed to the machine tool and a hammer is used to introduce the impact force and its range of mechanical signals in the system. The capacitive sensor is used to acquire the displacement of the tool as a response to the impact. Natural frequency and damping coefficient are the parameters that are the aim of this experiment.

Before running experiments using the capacitive sensor, a calibration set-up is used to relate the electrical signal and displacement comparing with the calibration chart given by the supplier (Figure 11).

The direction of the force should be in the same direction the displacement sensor (Figure 12a). Nevertheless, the position on the vertical direction is not the same. The distance of measuring and impact is an important parameter and it is compatible to the numerical simulation.



(a) Capacitive Sensor on the measuring position

(b) Displacement Results

**Figure 12.** Impact Test

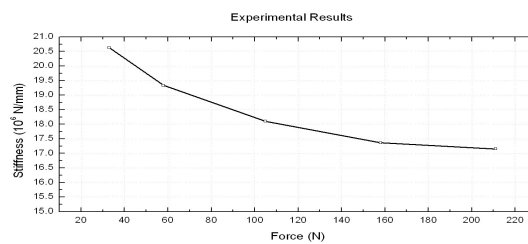
The results of the impact test with a steel head hammer shown on Fig.12b are used to calculate the logarithmic decrement. From this graphic it is possible to identify the damping frequency of 118.7 Hz, using the first and the fourth pics. The damping coefficient is then 0.027.

#### 4.2. Static Stiffness Experiment

A simplified experimental procedure was developed to review the parameters obtained from dynamic identification (Figure 10b). In this activity, sequences of several displacements were imposed to the workpiece in contact with the tool, and measure reaction force for each displacement. The holding system and the tool were the same used one dynamic response procedure. A dynamometer, an amplifier and an A/D converter was used to acquire the force data.

The dynamometer was fixed in the machine tool table. the results shown when there is a contact with the tool and the displacement. The displacements were taken from the display of the machine tool table in steps of 0.1mm.

Experimental data from static stiffness test is presented on Figure 13.



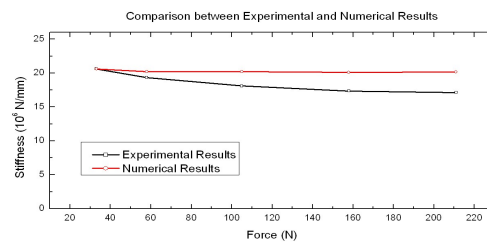
**Figure 13.** Stiffness as a function of the Applied Force

The results shown the non linearity of the system. The average value for the range experimented is calculated as  $1.85 \cdot 10^6$  N / m.

#### 5. COMPARISON BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS

Collecting data from Fig. 6 and 13, it can be claimed that the numerical results presented higher stiffness then the experimental data. It can be explained by the absence of more elements in the machine-tool in the numerical simulation. The machine-tool presents elements that can be modified with the machining time and wear of components. Experimental stiffness considers the mechanical and geometrical models of each solid.

Both curves are presented in Fig. 14 to show the difference between the two procedures. The maximum error between the curves is 15%.



**Figura 14.** Stiffness coefficient versus force applied to experimental and numerical models.

In order to have another reference, an analytical evaluation was analyzed. A simple analytic model based on Mechanics of Solids can be achieved considering the simplified geometry associated to a cantilever beam with two regions of different transversal area. Using the end milling tool parameters, a value of 2.16 MN/m is obtained, near 6% higher than the numerical model result. It can be obtained as function of Inertia of different transversal areas  $I_1$  and  $I_2$  (Craig, 2000):

$$K_a = \left( \frac{L^3}{E} \cdot \left( \frac{1}{24I_1} + \frac{7}{24I_2} \right) \right)^{-1} \quad (5)$$

#### 6. CHATTER SIMULATION

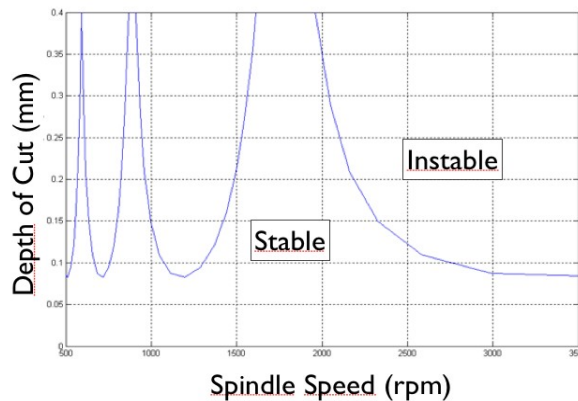
Altintas model predicts stable and unstable regions of space defined by axial depth of cut and spindle speed parameters as it was presented previously. It is based on the experimental impact test that identity the stiffness, damping



and its natural frequency. As shown, after successively expansions of Fourier series to calculate the force coefficients, it is possible to generate the lobus diagram.

Using the dynamic parameters simulated in this article and the Altintas model equations, the Figure 15 shows the first four lobes for stability on end milling in a full machining operation using the tool parameters described on the experiments. In full machining the radial depth of cut is defined by the diameter and the entry and exit angles on the working material are respectively 0 and 180 degrees.

The algorithm presents the dynamic behavior of the milling tool as two degrees of freedom system. The parameters in both directions (x and y) were considered with same modules are per the symmetric shape of the four flute milling tool. Matlab software was used for this prediction.



**Figure 15.** Lobes Diagram using Dynamic Parameters from Model

It can be seen that the lobus diagram defines the range of axial depth of cut and spindle speed under the continuous line where machining is safe from chatter.

## 7. CONCLUSION

The numerical methodology for tool dynamic parameters is used based on a nonlinear model based on the Finite Element Method (FEM). A comparison from the ones obtained through FEM and experimental data was done to evaluate if the procedure can be used.

The model allows an approach that is flexible and easy rebuilt for different cases of tool-holder and end milling tool positions and set-up. Different clampings can be simulated to find a better result of the set-up without executing experiments that can expend more time and resources.

Using the Altintas model for predicting chatter, the two sets of dynamic parameters (taken from experiments and from simulation) are computed separately and lobe diagrams for each set are constructed and compared. Results indicate that dynamic parameters obtained from FEM model can be used with literature models, as the Altintas model, to predict chatter in end milling.

The influence of the disk rotation, contact pressure and the distance between the tool tip and the clamping on the chatter dynamics can be studied with this procedure to propose to the operator changes on the machining parameters when there is a chatter occurrence during the process.

## 8. ACKNOWLEDGEMENTS

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