THERMAL STRESSES IN AN AL+SiC COMPOSITE DUE TO THE FABRICATION PROCESS

Carlos A. de J. Miranda, cmiranda@ipen.br
Rosani M. P. Libardi, rmpenha@ipen.br
Nuclear and Energy Research Institute, IPEN-CNEN/SP
Av. Lineu Prestes, 2242 - São Paulo, SP, Brazil

Zoroastro de M. Boari, zoroastr@uol.com.br
UNIABC – Universidade do Grande ABC
Av. Industrial, 3330 - Santo André, SP, Brazil

Abstract. In a previous work, one of the authors (Boari) developed an analytical methodology to predict the most probable thermal stress in a metallic matrix composite reinforced with particles, due to the fabrication process, when the temperature varies from 600 ^{o}C to 20 ^{o}C. The Eshelby method and dislocation mechanisms as well as the Maxwell-Boltzmann distribution model were employed in the formulation. All the analytical work was based on material linear elastic behavior due to the assumed basic hypothesis. To verify the analytical results, several numerical analyses were performed based on the finite element method using non-uniform random distributions of particles. About 30 analyses were performed with several volumetric ratios values and to stick with the analytical hypothesis particles with round geometry were considered. To obtain the most probable stress level in the material from the numerical results, firstly, each stress distribution, in terms of isostress curves at ΔT=-580 ^{o}C, was analyzed with an image analyzer. In the next step, a statistical procedure was applied to obtain that stress level. The analytical and numerical results compared very well. As the plastic deformation can alter significantly the stress field in the material, in this work, several numerical analyses were preformed considering the non-linear behavior in the aluminum matrix. To allow a direct comparison the same models (particle form, size and distribution) used previously were adopted in these new analyses using finer meshes. A very simple and fast algorithm, based on the Matlab image processing toolbox was developed to analyze the new results. The comparison of these results with the previous ones shows the strong influence of the elastic-plastic behavior of the aluminum matrix on the composite thermal stress distribution due to its manufacturing process. In a future work it is planned to make a new set of numerical simulations with quadrilateral particles to verify the influence of the particles geometry using a very tight volumetric ratio range closer to the value in an actual SiC composite.

Keywords: *Composite, Metallic matrix, Particles, Thermal stresses*

1. INTRODUCTION

The metallic matrix composites, MMC, specially those ones reinforced with particles, have important advantages when used as a structural material such as their high strength and good conformability. However, their properties depend, among others, on the volumetric ratio, the particles size and distribution, besides the matrix microstructure itself. While the mechanical properties of composites with long fibers are well known, and are based on the fiber resistance theory, for the composite made with particles it is very difficult to obtain a mathematical model relating its strength to the particles distribution, in particular the Aluminum matrix reinforced with SiC particles.

In a previous work, Boari (2003) developed an analytical methodology to predict the most probable thermal stress in this material due to the fabrication process, with the temperature varying from 600 ^{o}C to 20 ^{o}C. The Eshelby method and dislocation mechanisms as well as the Maxwell-Boltzmann distribution model were employed in the formulation. All the analytical work was based on linear elastic behavior of the material due to the assumed basic hypothesis.

As the plastic deformation can alter significantly the stress field in the material, in this work, the most probable thermal stress in the same MMC composite used previously, Aluminum matrix reinforced with SiC particles, was obtained considering the non-linear behavior in the aluminum matrix. To allow a direct comparison, the same models (particle form, size and distribution) used in the previous numerical analyses were adopted in these new analyses using the ANSYS program (version 11.0) with finer meshes. Also, a very simple and fast algorithm, based on the Matlab image processing toolbox was developed to analyze the new results.

2. THE LINEAR BEHAVIOR - THEORETICAL BACKGROUND

The Boari´s analytical methodology (2003) to predict the most probable thermal stress in an MMC composite will be briefly presented. The model is adapted from the Maxwell- Boltzmann statistics (Beiser, 1963). The main scope of the distribution law is to study how the particles, which form a system, are distributed in their phase space. When one found the probabilities of all possible distributions in the system nature it is possible to obtain that particular most probable distribution. And one can say the system has the tendency to behave according that distribution on position and momentum. This combination is the ‘phase space’.

The Maxwell-Boltzmann distribution law has two terms: one associated with the particle distribution and the other associated with kinetic energy. In the developed model the second term was changed by the elastic potential energy, which means, by the elastic stress. With this change it is possible to search for the most probable combination of particle position and elastic stress in the material. This redefines the 'phase space' concept for the scope of the work.

Considering this adaptation on the Maxwell-Boltzmann distribution law and the Eshelby´s method, based on the internal stress equilibrium, as presented elsewhere (Clyne & Withers, 1993; Taya & Arsenault, 1989; Withers et al., 1989; Taya et al., 1990; Arsenault & Taya, 1987, among others), we get the Eq. (1) which represents the stress distribution in the composite material with a random distribution of particles, *K* is given by Eq. (2).

$$n(\sigma) = 4\pi N \left(\frac{3}{2C_M K\pi} \right)^{3/2} \sigma^2 \, e^{-\frac{3\sigma^2}{K2C_M}} \quad (1)$$

$$K = f\, C_M \{(S-I)\{(C_M - C_I)[S - f\,(S-I)] - C_M\}^{-1} C_I \;(\alpha_I - \alpha_M)\Delta T\}^2 \quad (2)$$

The stress distribution is given by $n(\sigma)$, N is the number of particles, f is the volumetric ratio, S is the Eshelby´s tensor, I is the identity matrix, α_M and α_I are thermal expansion coefficients tensors respectively for the matrix and the inclusions (particles), and ΔT is the temperature range experienced by the material. The components of the Aluminum matrix (C_M) and SiC particles (C_I) elastic tensors are explicitly given in Eq. (3).

$$\begin{aligned}
C_{Mii} &= E_M\,(1 - \nu_M) / (1 - 2\nu_M)\,(1 + \nu_M) \\
C_{Mij} &= E_M\,\nu_M / (1 - 2\,\nu_M)\,(1 + \nu_M) \\
C_{M44} &= E_M / 2\,(1 + \nu_M) \\
C_{Iii} &= E_I\,(1 - \nu_I) / (1 - 2\nu_I)\,(1 + \nu_I) \\
C_{Iij} &= E_I\,\nu_I / (1 - 2\,\nu_I)\,(1 + \nu_I) \\
C_{I44} &= E_I / 2\,(1 + \nu_I)
\end{aligned} \quad (3)$$

E_M and E_I are the matrix and particles elastic (Young's) modulus and ν_M and ν_I are the matrix and particles Poisson's ratio.

The most probable stress in the material, σ_P, is associated with the maximum in Eq. (1) and is given by Eq. (4).

$$\sigma_p^2 = \frac{2f\,C_M^2\{(S-I)\{(C_M - C_I)[S - f\,(S-I)] - C_M\}^{-1} C_I\;(\alpha_I - \alpha_M)\Delta T\}^2}{3} \quad (4)$$

The Boari's original work (2003) presents this formulation in a very detailed way including equation deduction to get the average stress and the quadratic average stress. Also, it has some application examples and a comparison of his analytical results with numerical ones for a given MMC composite (Al+SiC).

2.1. Verification by Numerical Analyses

To verify the analytical results, about 30 numerical analyses were performed based on the finite element method using non-uniform random distributions of particles, each one with a different volumetric ratio and, to stick with the analytical hypothesis, particles with round geometry were considered. The analytical procedure was applied to each volumetric ratio to obtain the analytical stress distribution and the most probable stress value. To obtain the most probable stress level in the material from the numerical results, firstly, each stress distribution in terms of isostress curves at $\Delta T = -580\ ^oC$ was analyzed with an image analyzer. In the next step, a statistical procedure was applied to obtain that stress level. The analytical and numerical results compared very well. (The theoretical background on the finite element method as well as on the thermal stress in a given structure due to the difference in the thermal expansion coefficient and/or in the displacement restrictions will not be presented here.)

3. NON- LINEAR BEHAVIOR – NUMERICAL ANALYSES

From the numerical analyses in (Boari, 2003), it was noticed stresses over the Aluminum yielding limit. So, the obtained the most probable stress values in the material, analytical or numerical, are not realistic one as the material plastic behavior, in special the metal matrix one, can alter significantly the material stress field.

Considering the linear analysis limitation in this particular situation new numerical analyses were performed considering the Aluminum non-linear behavior.

To allow a direct comparison with the previous work the same particle round geometry was used. The ANSYS version 11.0 (2008) uniformly distributed random number generator was used, along with some other program resources, to get randomness in the particles position and size. The number of particles varies from case to case. The previous analyses data base were recovered and new meshes were generated with a doubled node density in each direction.

3.1. Adopted Material Properties

The material properties used in the simulations are presented in Tab. 1. The stress-strain curve adopted for the Aluminum matrix is presented in Fig. 1.

Table 1: Adopted Material Properties.

Property		Aluminum	SiC	SI unit
Young's modulus	E	73 10^9	450 10^9	Pa (N/m^2)
Density	ρ	2800	3200	kg/m^3
Thermal expansion coefficient	α	23,6 10^{-6}	4,0 10^{-6}	^{o}C^{-1}
Poisson's ratio	μ	0,33	0,17	-----
Transversal modulus	G	27,4 10^9	192,0 10^9	Pa (N/m^2)

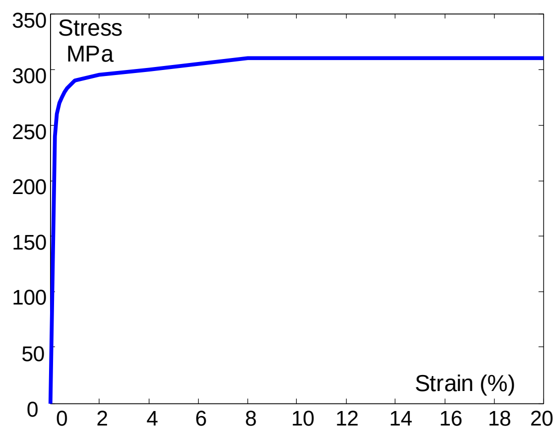


Figure 1. Adopted Stress-Strain Curve for the Aluminium Matrix.

3.2. Load and Boundary Conditions

The round particles random distribution is depicted in Fig. 2 for just two cases out of the 30 analyzed cases.

The thermal loading is originated from the composite manufacturing process: initially the material is at 600 ºC and after an extrusion process it is slowly cooled to the room temperature, 20 ºC. The stresses arise due to the differences in the thermal expansion coefficients of the Aluminum matrix and the SiC particles. Due to the Aluminum matrix non-linear behavior, this thermal loading is applied in 10 load steps and in each one the program is allowed to iterate until convergence is achieved. So, at the end of a given analysis the obtained stresses are associated with the ΔT = -580 ºC.

As boundary conditions, the nodes on the Y axis (coordinate X = 0) had their X displacement restrained while those on the X axis (coordinate Y = 0) had their Y displacement restrained.

With these hypotheses the model describes the most central region in a hypothetical bar formed by the composite extrusion.

3.3. Basic Results – Isostress Curves

Typical results, in terms of the isostress curves, are shown in Fig. 2 were each curve has its specific color to allow the image processing step and the 9 curves values are equally spaced. In the Fig. 2.a the stresses range from 20.9 MPa to 333 MPa and in the Fig. 2.b the stresses range from 18.9 MPa to 318 MPa.

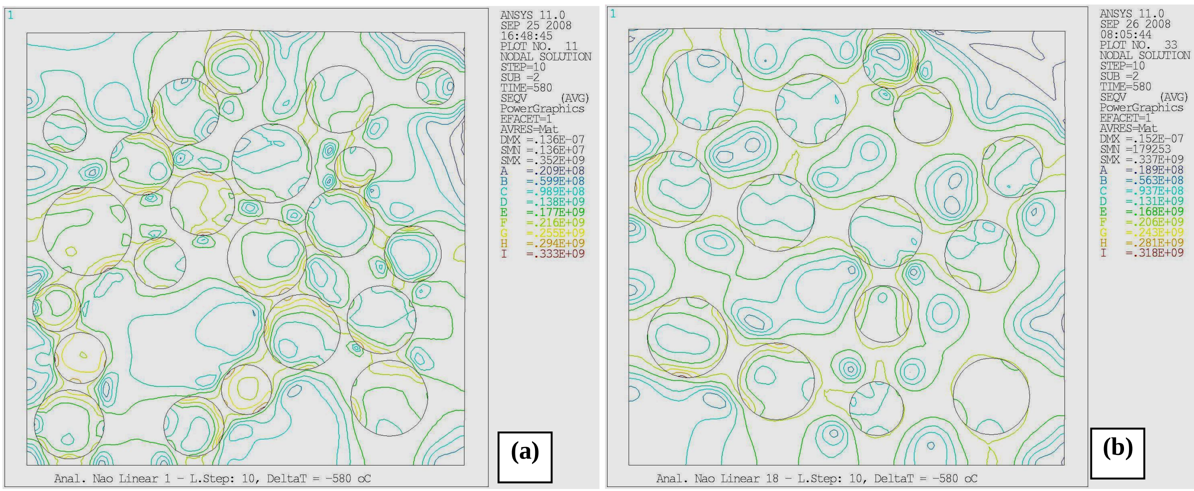


Figure 2. Example of stress distributions with round particles
(9 isostress values each - a given value/color can be repeated in the same figure).

4. IMAGE PROCESSING ALGORITHM

A simple and very fast algorithm, based on the Matlab (2008) image processing toolbox was developed to analyze the new (non-linear) results. The algorithm script is shown below, Fig. 3.a. The “cor1.mat” file should be created previously in the Matlab environment with the R(ed), G(reen) and B(lue) values associated with each one isostress curve/value (as defined in the ANSYS pos-processor when the figures were generated in .TIFF format).

After reading it onto the Matlab environment, the figure is cropped to eliminate unnecessary information generated by ANSYS like the legend, the color scale and the border line around it. Figures 2.a and 2.b were already cropped. The ‘x1’, ‘x2’, ‘y1’ and ‘y2’ crop coordinate/values are intended to be used with figures generated with a 600 pixels of resolution. If the figure is generated with a different resolution, 1200 pixels for example, these values should be adjusted proportionally.

The algorithm results are the percentages of each color present in a given figure. As the generated isostress curves are equally spaced, for the scope of this work, the percentage value given by the algorithm is being interpreted as the weight to be applied at each curve/stress value to obtain the averaged stress value in the material, for a given analysis. The 30 averaged values from each analysis can be also averaged to obtain the overall averaged stress in the composite.

4.1. Tests

The algorithm was tested against some geometric figures with several curves, each one with known perimeter and area (one of them is shown in Fig. 3.b). It compared very well with the values obtained by the geometry as shown in Tab. 2.

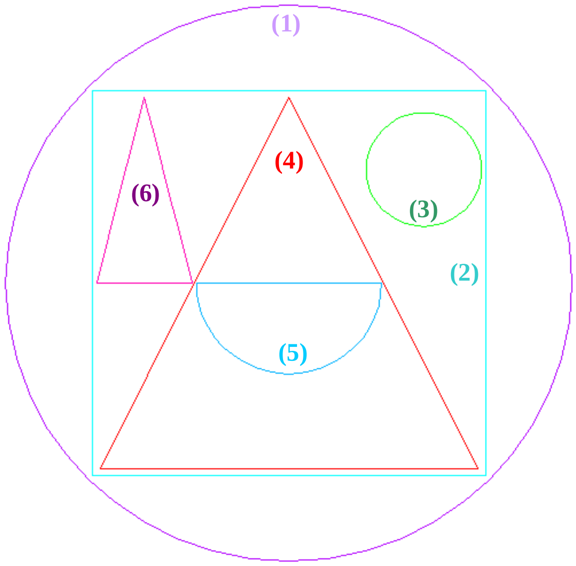
Table 2: Results (%) of the Matlab Script Testing.

Test curve #	(1)	(2)	(3)	(4)	(5)	(6)
Script	29,2	28,2	5,9	20,2	8,0	8,4
Geometry	30,6	26,5	6,1	20,6	8,0	8,2

A more sophisticated but slower algorithm was also developed; based on the curve smoothing. However, the tests had shown only a few percent of discrepancy between both algorithms, far less then the involved uncertainties as those associated with the material properties, for instance. So, the simpler one was applied to analyze each of the new numerical results in terms of the non-linear stress distribution at ΔT=-580 °C.

```
load cor1.mat; % load the color map
FilN=input('FigureFileName (tif): ','s');
a=imread(FilN); % fig. resol. = 600ppi
x1=30; x2=585; y1=25; y2=585;
%crop the figure to eliminate unnecessary
acrop=a(x1:x2, y1:y2, :);  % information
colmap=cor1;
aT= rgb2ind(acrop,colmap);
imshow(aT,colmap) % verify cropped fig
[i1 j1]=find(aT==1);curv1=length(i1);
[i2 j2]=find(aT==2);curv2=length(i2);
[i3 j3]=find(aT==3);curv3=length(i3);
[i4 j4]=find(aT==4);curv4=length(i4);
[i5 j5]=find(aT==5);curv5=length(i5);
[i6 j6]=find(aT==6);curv6=length(i6);
[i7 j7]=find(aT==7);curv7=length(i7);
[i8 j8]=find(aT==8);curv8=length(i8);
[i9 j9]=find(aT==9);curv9=length(i9);
aT(i4(1),j4(1));
curvs=[curv1;curv2;curv3;curv4; ...
curv5;curv6;curv7;curv8;curv9];
totalCurvs=sum(curvs);
percentCurvs=(curvs./totalCurvs)*100;
StressL=(1:9);
tenEprc= [StressL; percentCurvs'];
FigureFileName = FilN
b=fprintf(1, 'Stress Level %2g: %04.2f%% \n',
tenEprc);
```

(a) Matlab Script



(b) Geometric curves with known perimeter and area to test the Matlab script.

Figure 4. Matlab Script and figure to test the script.

5. RESULTS ANALYSIS AND DISCUSSION

Table 3 shows some typical results from the image processing for some runs/analyses like those ones presented in Fig. 2. For only one analysis the associated isostress figure was generated with three different resolutions (600, 1200 and 2400 pixels) to verify its influence on the results. Again, the difference among the results obtained from these figures were very small allowing the figures in all analyses be generated with a resolution of 1200 pixels.

Table 3: Typical Image Processing Results for Some Analyses.

Isostress	run #01		run #02		run #04		run #05	
curve #	MPa	%	MPa	%	MPa	%	MPa	%
#1	20.9	1.08	23.3	1.41	24.7	2.29	25.3	0.91
#2	59.9	3.85	62.6	5.62	64.9	11.07	63.4	6.91
#3	98.9	13.51	102.	15.89	105.	18.07	102.	17.79
#4	138.	25.70	141.	30.29	145.	28.29	140.	26.74
#5	177.	31.88	180.	31.86	185.	29.61	178.	29.42
#6	216.	15.77	220.	10.01	225.	6.53	216.	13.17
#7	255.	6.24	259.	3.88	265.	3.18	254.	3.90
#8	294.	1.71	298.	0.97	306.	0.83	292.	1.06
#9	333.	0.25	338.	0.07	346.	0.12	330.	0.10

Figure 5 shows the maximums and the minimums stress values for each analysis. From the figure it becomes clear the thermal stresses in the composite range from ~0 MPa until about ~330 to ~380 MPa. For each isostress curve there is a range of values that arises from the own randomness nature of the particles distributions (their size and distances to each other).

Figure 6 shows the averaged percentage for each isostress curve where the ‘error’ bar is the standard deviation (from 30 values at each ‘point’). It depicts how the stress distribution in the composite should look like when the Aluminum non-linear behavior is taken into account.

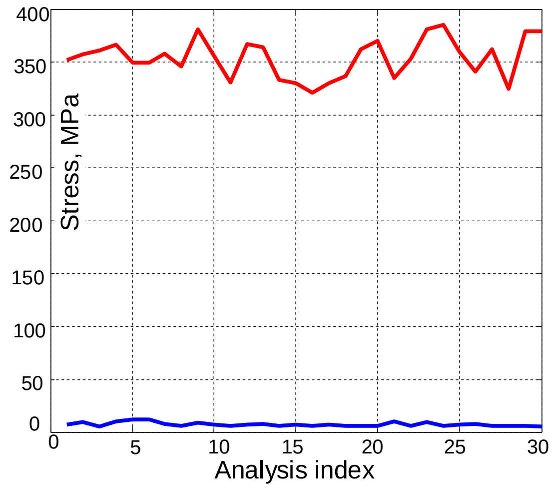


Figure 5. Min & Max stress values (MPa) curves for each analysis.

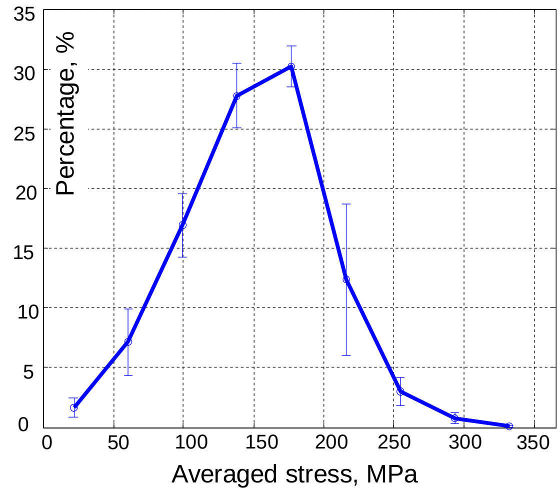


Figure 6. Averaged percentage for each isostress curve and respective error (Std. Dev).

Figure 7 shows the comparison among the present versus the previous results (Non-Linear X Linear behavior): averaged & mode stress as well as the volumetric ratio. From the statistics definition, the 'mode' stress is the value that appears most in the material (in a given analysis). The 'averaged stress' is the value obtained for a given analysis, weighing the isostress values with the percentages from the image analyzing step.

This Fig. 7 shows clearly the strong influence of the Aluminum matrix behavior in the results. This behavior has been influenced by misfit strain which is relaxed by the punching of dislocation loops at the microscopic level. The observation of plastic deformation in aluminum matrix indicates high work-hardening around the particles, increasing the dislocation density through the relaxation of thermal stress.

In the elastic regime the volumetric ratio has a greater influence on the stress values and when the Aluminum matrix plasticity behavior is taken into account this influence is attenuated. Considering the linear behavior, the trend in the thermal stresses is to increase as increases the volumetric ratio (in the analyzed volumetric ratio range of values) with the values oscillating in the range from ~400 MPa to ~580 MPa. The material can not afford those so high stress values as predicted by the Analytical results and the correspondent numerical analyses (both considering linear behavior). So, the actual average thermal stress in the material is much lower than the predicted by the linear analyses.

Using the same statistical procedure, as used previously by Boari (2003), and from the processed non-linear analyses results shown above, the average stress level in the Al+SiC composite, due to the manufacturing process, from 600 ºC to 20 ºC, was evaluated as 171,9 MPa with a standard deviation 12,8 MPa.

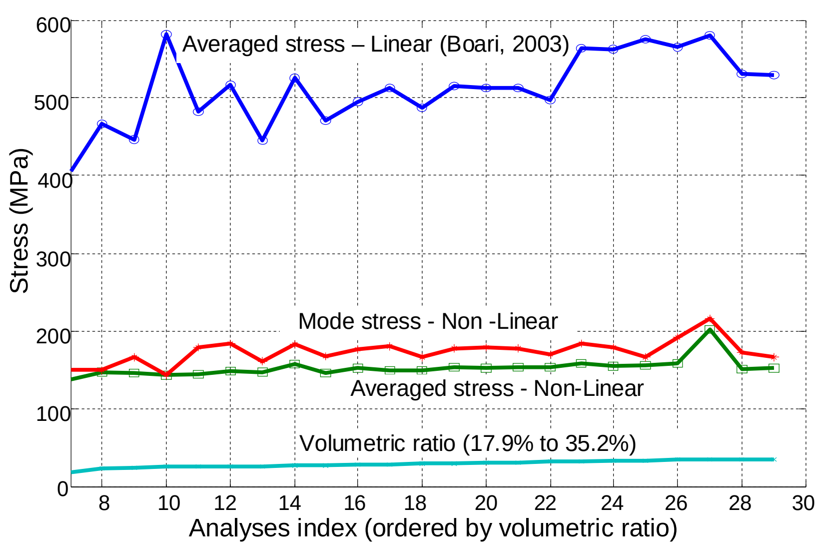


Figure 7. Comparison – Present (Non-Linear) X Previous (Linear) Numerical Results.

6. CONCLUSIONS

As well known, the plastic behavior alters the stress field in a given material, as is the case investigated by this work: the thermal stresses in an MMC (Al+SiC) composite due to its manufacturing process. The briefly presented analytical approach covers the material linear behavior supposing round particles. However, the predicted thermal stresses locally reach values over the Aluminum yielding level which is confirmed by numerical linear analyses. So, to predict a more realistic thermal stress value in this material it is necessary to perform numerical analyses considering the non-linear behavior in the aluminum matrix which was done in this work, Also, a simple and very efficient and fast algorithm, based on the Matlab image processing toolbox was developed to analyze the non linear results in terms of the isostress curves. The comparison of these new results with the previous ones shows the strong influence of the aluminum matrix elastic-plastic behavior on the composite thermal stress distribution due to its manufacturing process (from 600 ºC to 20 ºC). The average stress was evaluated as ~172 MPa with a standard deviation of ~13 MPa. Although this average value is still a high one it is from thermal loading and it should be considered as so.

One can argue about the particle geometry as well as the wide range of volumetric ratios. As mentioned before, these choices were adopted in this work to allow a direct comparison with the previous analytical & numerical work. In a future work, it is planned to make a new set of numerical simulations with quadrilateral particles to verify the influence of the particles geometry using a very tight volumetric ratio near the value in an actual SiC composite. The expectation is to find a less influence of the particles geometry due to the reasons already presented (the strong relaxation of thermal stress due to the increasing aluminum dislocation density).

7. REFERENCES

Boari, Z. M., 2003, Mathematical model – the influence of the particles distribution on the thermal stresses in a metallic matrix composite (in Portuguese – Modelo Matemático da Influência da Distribuição de Partículas de SiC nas Tensões Térmicas em Compósitos de Matriz Metálica), IPEN, São Paulo, Brazil – PhD Thesis.

Beiser, A., 1963, **Concepts of Modern Physics**. 2ª ed. Mc Graw-Hill Book Company, Inc., p. 235-246.

Clyne, T. W.; Withers, P. J., 1993, **An Introduction to Metal Matrix Composites.** Cambridge University Press. First ed.; p. 44-64.

Taya, M.; Arsenault R. J., 1989, **Metal Matrix Composites - Thermomechanical Behavior.** Pergamon Press. First ed.; p. 32-35.

Withers, P. J.; Stobbs, W. M.; Pedersen, O. B., 1989, **The Application of the Eshelby Method of Internal Stress Determination to Short Fiber Metal Matrix Composites**. Acta Metall. v. 37, n⁰. 11; p. 3061-3084.

Taya, M.; Lulay, K. E.; Lloyd, D. J., 1990, **Strengthening of a Particulate Metal Matrix Composite by Quenching.** University of Washington. Seattle-U.S.A. Acta Metall. v.39. n⁰.1; p. 73-87.

Arsenault, R. J.; Taya, M., 1987, **Thermal Residual Stress in Metal Matrix Composite**. Acta Metall. v.35, n⁰.3; p. 651-659.

MATLAB version 7.6 (R2008a), 2008, Mathworks Inc..

ANSYS v11.0, 2008, *ANSYS Structural Analysis Guide & Theory Reference Manual*.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.