

## THERMAL CONTACT RESISTANCE ON HEAT TRANSFER - REVIEW

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**Abstract.** *An interesting problem in thermal science is the study of heat transfer across a contact between two solids. To do authors defined commonly the concept of thermal resistance at the interface. In this paper we begin in the first with a comparison between electric and thermal diffusion phenomena in solids. The goal throughout this comparison is to understand if the concept of thermal resistance is available or not. Secondly, an apparatus is set to measure thermal resistance. It consists of two metallic blocks closed together, one is heated with a thermal resistance and the other is cooled with water. Temperatures measurements in both blocks are used to estimate the temperatures and the heat fluxes at the interface, with the use of an inverse method. Finally the results are presented to illustrate the conclusions.*

**Keywords:** *thermal contact resistance, heat transfer*

### 1. INTRODUCTION

Since 19<sup>th</sup> century mathematicians and physicians are interested by developing a Similitude between electromagnetism phenomena and fluid mechanics one (Jech, 2001; Domps, 2003; Rousseaux and Guyon, 2002). The reason is to understand the physical sense of the potentials on witch the Maxwell's equations are based and to understand many physical points that are evident in one area and not in the other. In this section a brief introduction to electromagnetism is given in order to review the concept of the electric resistance and to understand the limit of using the thermal one. In literature many authors use the concept of thermal resistance in transient regime and many others look to the sensible heat as an electric capacity one. These considerations are found in many works, manly that using notion of thermal impedance (Degiovanni, 1988; Antczak *et al.*, 2003). So the question is that we are sure that in regime permanent the concept of electric and thermal resistance is correct, but why it should be correct in transient regime for electric phenomena and no for heat diffusion one?

### 2. ELECTRIC AND THEMAL RESISTANCE

Details of contact between two solids are given in fig. 1. In practice many additive materials can be added to full the asperities in order to make better contact. The impact of pressure and thermo-elasticity on the contact zone between two solids is widely studied in literature (Degiovanni, 1992; Mikik, 1974; Negus *et al.*, 1989; Foucher *et al.*, 1975; Greenwood, 1991; Lambert and Fletcher, 1993). The analysis in this paper is focused on the heat transfer through a heat flux tubes covering a contact point or surface. Two microscopic points in contact together should have the same temperature when the assumption of local thermodynamic equilibrium is considered. In reality the heat conduction transfer in the contact zone is not 1D one, because of the irregular shape of the dendrites.

We beguine by considering the well-known Maxwell's equations:

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0} \quad (1)$$

$$\frac{1}{\mu_0 \mu_r} \vec{\nabla} \times \vec{B} - \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t} = \vec{J} \quad (2)$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0 \varepsilon_r} \quad (3)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (4)$$

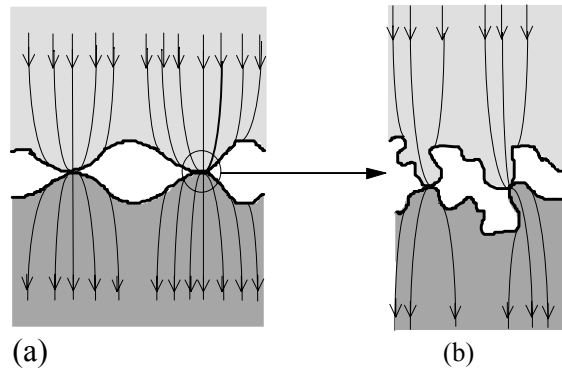


Figure 1: (a) macroscopic Details of contact and (b) microscopic details ones

The electric and the magnetic field are determined since the distribution of charge and current are given. The equation of charge conservation can be easily derived from the above system:

$$\frac{\partial \rho}{\partial t} + \text{div} \vec{J} = 0 \quad (5)$$

The general form of the electric and magnetic field respecting Maxwell equations consists of the use of a potential scalar and vector  $(V, \vec{A})$  as follows:

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (6)$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} V \quad (7)$$

These potentials are not unique but they are additive ones:

$$\vec{A} = \vec{A}_0 + \vec{\nabla} \varphi \quad (8)$$

$$V = V_0 - \frac{\partial \varphi}{\partial t} \quad (9)$$

In addition they are chosen with respect of the constraint called 'Lorentz gauge':

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \cdot \frac{\partial V}{\partial t} = 0 \quad (10)$$

Now, to make a similitude between heat conduction and electric phenomena consider the Fourier law:

$$\vec{q} = k \cdot \vec{\nabla} T \quad (11)$$

It means that the heat conduction flux (vector) derives from the scalar potential temperature T. Thus, from the Maxwell equations, the only way that we have found to make the similitude between electromagnetism and heat conduction if it exists is:

$$\vec{q} \leftrightarrow \vec{E} \quad (12)$$

$$T \leftrightarrow V \quad (13)$$

When the energy balance is set, the heat conduction equation is obtained:

$$\vec{\nabla} \cdot \vec{q} = k \Delta T = c \frac{\partial T}{\partial t} \quad (14)$$

While the divergence of the electric field hasn't the same form:

$$\vec{\nabla} \cdot \vec{E} = - \left( \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} + \Delta V \right) = \frac{\rho}{\epsilon_0 \epsilon_r} \quad (15)$$

When the two last equations are compared, the conclusion is that in general cases the above similitude is still not correct. Eventually, when electro-magnetic waves are considered at the condition called quasi-stationary approximation regime (QSAR):

$$e = \frac{l}{c \Gamma} \ll 1 \quad (16)$$

Where  $l$  is the mean space dimension of the studied electric device,  $c$  is the velocity of the electromagnetic wave in the studied medium  $\sim 3.10^8$  m/s and  $\Gamma$  is the mean time of variation of the electric sources  $(\rho, \vec{j})$ . For periodic signal,  $\Gamma$  is taken equal to the period and one can guess the largest domain of frequency at which the (QSAR) is valid (for ordinary electric apparatuses  $l \sim 1$  m and  $f \sim 1$  GHz). By defining the retard time of response, the last condition (QSAR) can be explained as:

$$\tau_p = \frac{l}{c} \ll \Gamma \quad (17)$$

In transient regime, the Lorentz gauge leads to:

$$\frac{|\vec{\nabla} \cdot \vec{A}|}{\left| \frac{1}{c^2} \frac{\partial V}{\partial t} \right|} \approx \frac{\tilde{A}/l}{\tilde{V}/c^2 \Gamma} = 1 \Rightarrow \frac{\tilde{A}}{\tilde{V}} \approx \frac{c^2 \Gamma}{l} \quad (18)$$

One can compare the two terms constituting the electric field under the QSAR

$$\frac{\left| \frac{\partial \vec{A}}{\partial t} \right|}{|\vec{\nabla} V|} \approx \frac{\tilde{A}/\Gamma}{\tilde{V}/l} = \frac{\tilde{A}}{\tilde{V}} \frac{l}{\Gamma} \approx \left( \frac{l}{c \Gamma} \right)^2 \ll 1 \quad (19)$$

So that in QSAR we have:

$$\vec{E} = -\vec{\nabla} V \quad (20)$$

$$\vec{\nabla} \cdot \vec{E} = -\Delta V = \frac{\rho}{\epsilon_0} \quad (21)$$

This equation is still not similar to the heat diffusion one. But in the case of an electric conductor, the local Ohm law is:

$$\vec{E} = \gamma \cdot \vec{J} \quad (22)$$

According to the Drude model, this law is available under condition  $\tau_c \ll \Gamma$ .  $\tau_c$  is the relaxation time (time of collision) given by:

$$\tau_c = \frac{m \gamma}{n q^2} \quad (23)$$

For example for copper  $\tau_c \sim 10^{-14} \text{s}$

When Ohm low, Maxwell-Gauss equation and the charge conservation one are used, we obtain:

$$\rho = \rho_0 \exp\left(-\frac{t}{\tau_d}\right) \quad (24)$$

$$\tau_d = \gamma \epsilon_0 \rho_m = 0 \quad (25)$$

For a good conductor copper we have:  $\tau_d = 1.6 \cdot 10^{-19} \text{s} \ll \tau_c$  and for a bad conductor like the water of the sea:  $\gamma = 4.35 \text{ s/m}$ ,  $\epsilon_r = 81$  we found:  $\tau_d = 1.7 \cdot 10^{-10} \text{s}$ .  $\rho$  decrease exponentially and it can be taken zero with a good approximation when  $\tau_d \ll \Gamma$ . This result means that the volumetric charge is null  $\rho_m = 0$  and only charges at the surface can be considered in the transient regime. As a result, at the QSAR we have in electromagnetism transient regime in a Ohmic conductor:

$$\vec{E} = -\vec{\nabla}V = \gamma \vec{j} \quad (26)$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (27)$$

Considering a flux tube in a conductor, the concept of a const electric resistance is valid in transient regime under conditions  $\tau_p \ll \Gamma$ ,  $\tau_d \ll \Gamma$  and  $\tau_c \ll \Gamma$ . One can calculate the circulation of the electric vector a long the flux tube and found the expression of the electric resistance as follows:

$$R = \gamma \cdot \frac{\int \vec{j} \cdot d\vec{l}}{\int \vec{j} \cdot d\vec{S}} \quad (28)$$

Since we have  $\vec{\nabla} \cdot \vec{J} = 0$  in the (QSAR) and when a flux tube is considered in a pure conductive medium, the inlet current I is equal to the outlet one in transient regime. This current can be related to the difference of potential accounting an electric resistance (more than the conductive phenomena we must account the inductive and capacitive ones). But in thermal conduction and in transient heat transfer regime we can not deal with only one heat flux in the inlet side and the outlet one, this because of the sensible heat dissipation inside the flux tube:

$$q_1 - q_2 = \int_v \rho c \frac{\partial T}{\partial t} \quad (29)$$

Only in permanent regime we have:

$$\vec{q} = -k \cdot \vec{\nabla}T \quad (30)$$

$$\vec{\nabla} \cdot \vec{q} = 0 \quad (31)$$

Through the heat flux tube we have  $q_1 = q_2 = q$  and one can define such us the concept of the thermal resistance at a flux tube  $\Delta T = R_c \cdot q$ . In the QSAR we have:

$$\left| \frac{\epsilon_0 \frac{\partial \vec{E}}{\partial t}}{\sigma \vec{E}} \right| \approx \frac{\epsilon_0}{\sigma \Gamma} = \frac{\tau_0}{\Gamma} \ll 1 \quad (32)$$

In this assumption one can demonstrate that the electric field respects the diffusion equation:

$$\Delta \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} + \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} \approx \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} \quad (33)$$

Thus in 1D problem, one can set the analogies between T and E or T and B. But it rests only mathematical comparison because the electric resistance is still defined by using the electric potential V and the electric current j, as it seen in the above similitude.

### 3. APPARATUS AND HEAT CONDUCTION MODEL:

The apparatus used in practice is presented in fig. 3. It consists of making two cylindrical slaps one closer to the other, see fig. 2. The first is heated at one surface with electrical resistance. The power delivered by the resistance is sinusoidal:

$$P_e = \frac{V_m^2}{2R_e} (1 + \cos(2\omega t)) \quad (34)$$

Vacuum is assured by a pump and the pressure between the two samples too. Thermocouples at different position along them are implanted to measure transient temperature needed for the resistance estimation.

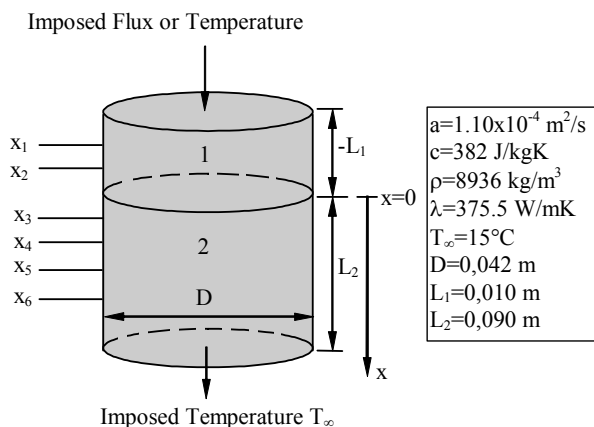


Figure 2: Samples used to thermal resistance determination

The mathematical model of the heat conduction transfer using the variable  $\theta_i(x, t) = T_i(x, t) - T_\infty$  is:

$$\frac{\partial^2 \theta_i}{\partial x^2} = \frac{1}{a} \frac{\partial \theta_i}{\partial t}, \quad i = 1, 2 \quad (35)$$

$$\theta_1(0, t) - \theta_2(0, t) = -R\lambda \frac{\partial \theta_1(0, t)}{\partial x} \quad (36)$$

$$\frac{\partial \theta_1(0, t)}{\partial x} = \frac{\partial \theta_2(0, t)}{\partial x} \quad (37)$$

$$(1 - F)\theta_1(-L_1, t) - F\lambda \frac{\partial \theta_1(-L_1, t)}{\partial x} = (1 - F)[T_0(1 + \cos \omega t)] + F[\varphi_0 + \varphi_0 \cos(\omega t)] \quad (38)$$

$$\theta_2(L_2, t) = 0 \quad (39)$$

$$\theta_1(x, 0) = 0 \quad (40)$$

$$\theta_2(x, 0) = 0 \quad (41)$$

The solution of  $\theta_i(x, t)$  is decomposed to three fields: permanent ( $T_p$ ), transient ( $T_t$ ) (related to the initial condition) and a sinusoidal ( $T_s$ ) as follows:

$$\theta_i(x, t) = T_{pi}(x) + T_{ti}(x, t) + T_{si}(x, t) \quad (42)$$

In an established regime the transient one is settled equal to zero  $T_t=0$  and the final field of temperature have the form:

$$\theta_i(x,t) = a_i + b_i \cdot x + M_i(x, R) \cdot \cos(\omega t - \varphi_i(x, R)) \quad (43)$$

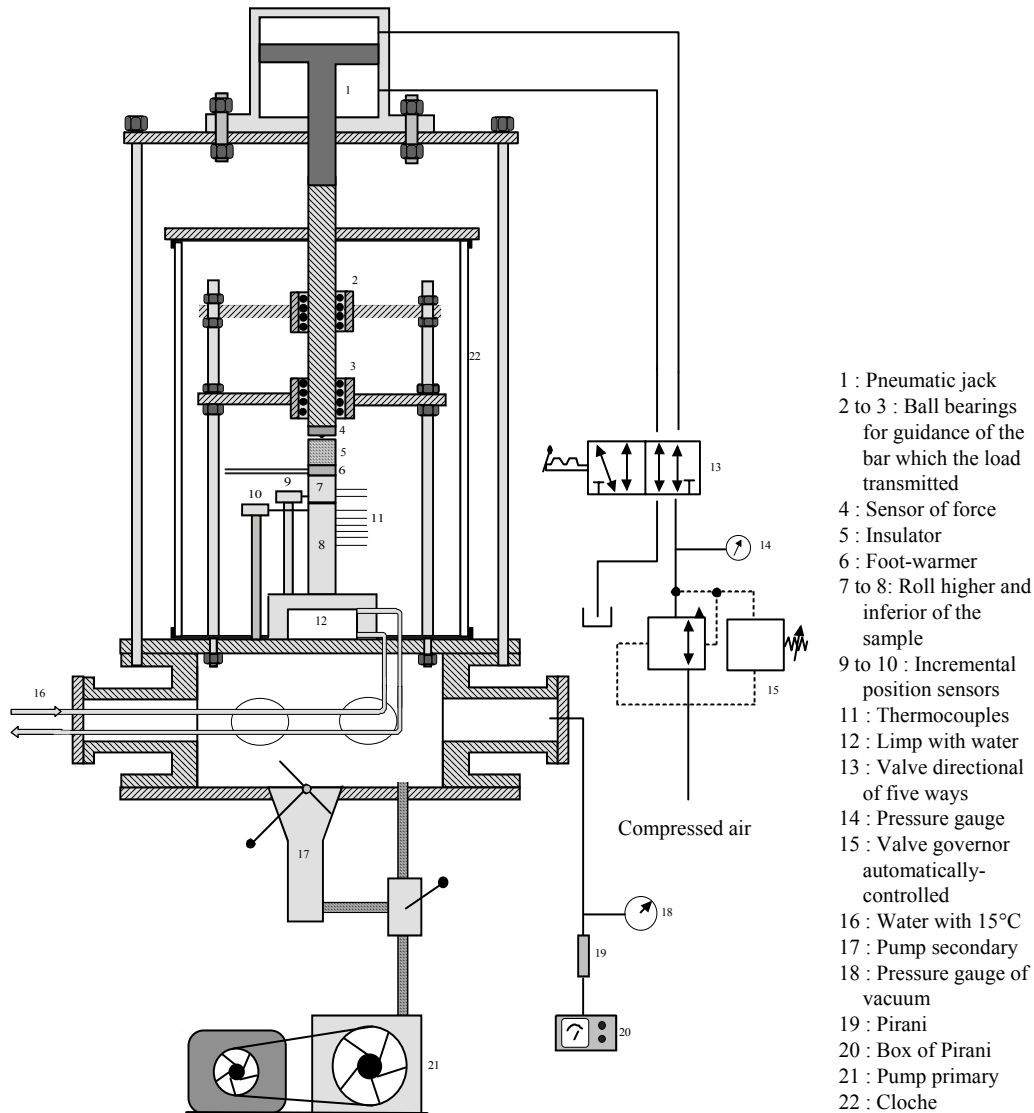


Figure 3: Apparatus to measure thermal resistance

Analytic solution is obtained and the expression of the amplitude  $M$  and the phase one  $\varphi$  are determined in function of the position  $x$  and the resistance  $R$ . The ratios of the amplitude and the phase of two different points of the same sample are found independent of the factor  $F$ . Thus we determine the thermal resistance with two ways,  $R_a$  using the ratio of amplitudes of two measured temperatures or  $R_\varphi$  using the ratio of there phases. The measurements are made with many samples of the same material and having different smoothed surfaces. Our intention is to conclude the common behavior of the determined resistance independently of the influence of the parameters related to the nature of the surface. The figs. (5, 6, 7) present the Thermal resistance of two samples under different pressure. It shows that the estimated resistance  $R_a$  and  $R_\varphi$  are different and frequency dependent. Note that the two samples have different smoothness and the behavior of the  $R_a$  and  $R_\varphi$  via the frequency variation is similar. Both of them are decreasing when the frequency is increased. The fig. 8 presents the evolution of  $R_\varphi$  in time, the frequency is varied and it is found that the thermal resistance has the same above behavior. Thus the thermal resistance is not only frequency dependent but its value depends of the transient temperature history in the samples. These remarks assure that they are not intrinsic values and the concept of thermal resistance is not correct in transient regime.

#### 4. CONCLUSION

The physical-mathematical similitude between electric conduction and heat one leads to that thermal resistance is only defined in permanent regime. While the electric one remains available in transient regime for a large domain of

frequency. The practical measurements confirm that we can not define an intrinsic resistance thermal value in transient regimes. One important point is that in transient regime, the heat conduction fluxes at the contact depends of the temperature fields in both solids and obviously of the boundaries conditions of them. So that they can not be set equals as we do in the thermal transfer models in general. Finally, these remarks done for the study of a contact between two solids, do them available for a contact between fluid-solid and the Newton's condition will be in ambiguity?

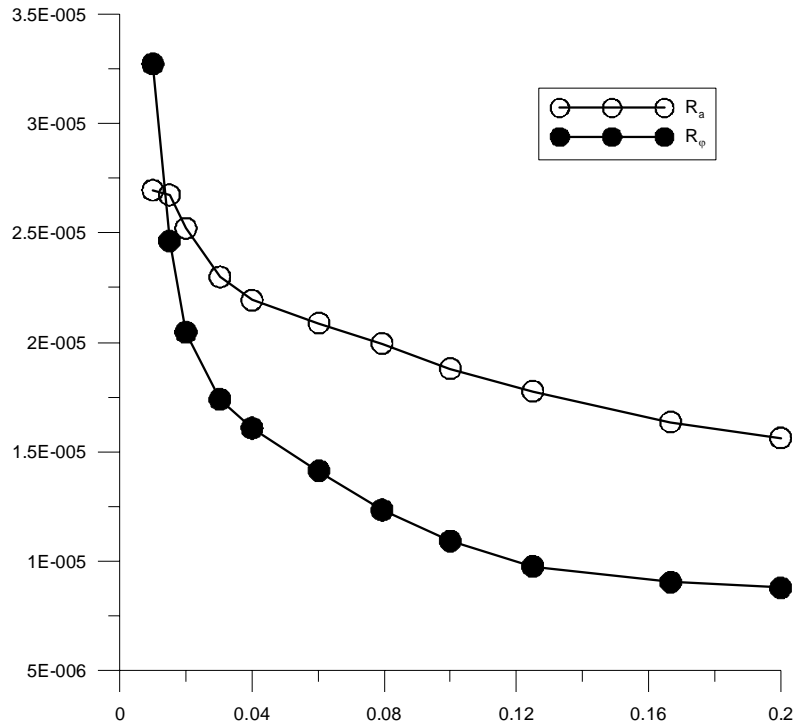


Figure 4: Thermal resistance (sample 1) , P=2.21 MPa

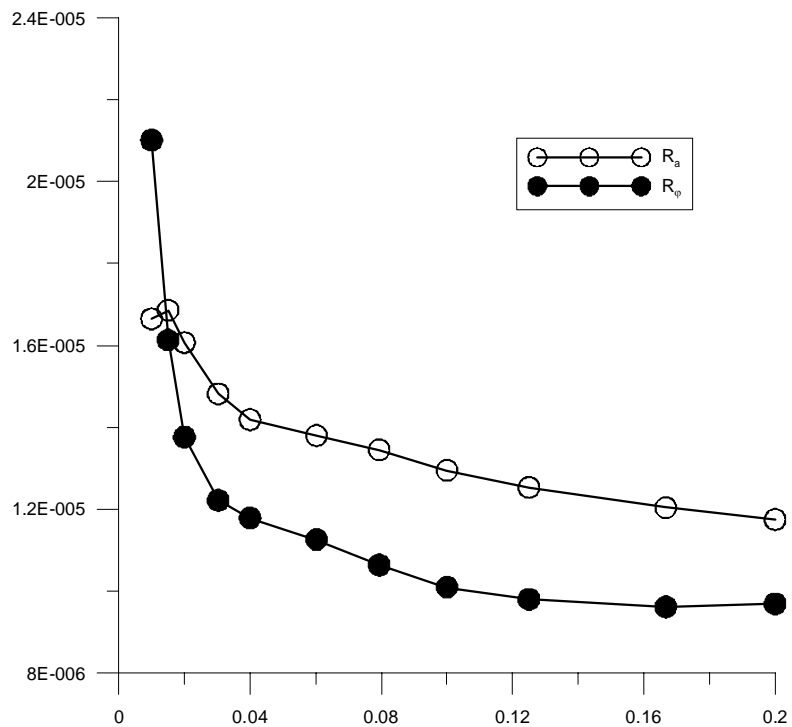


Figure 5: Thermal resistance (sample 1), P=5.79 MPa

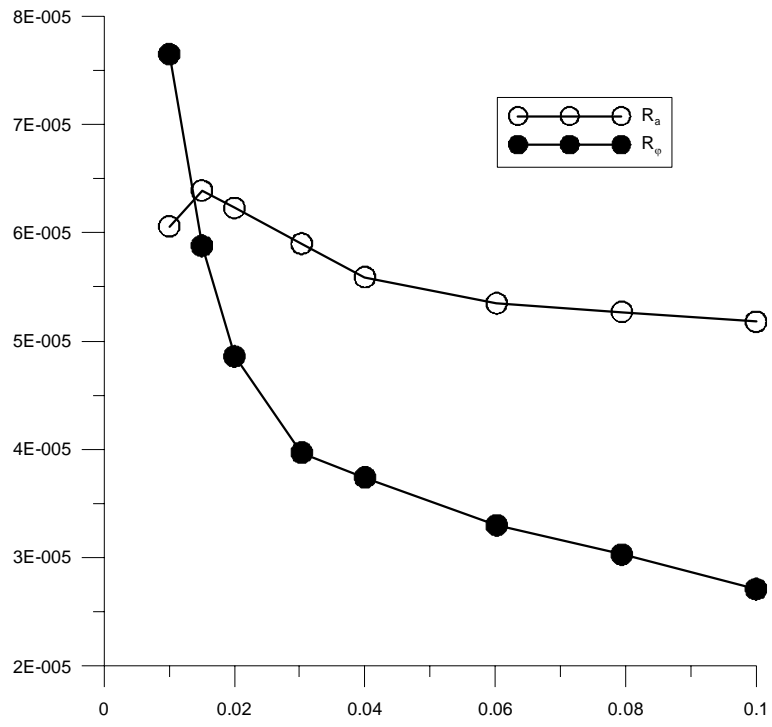


Figure 6: Thermal resistance (sample 2), P=2.12 MPa.

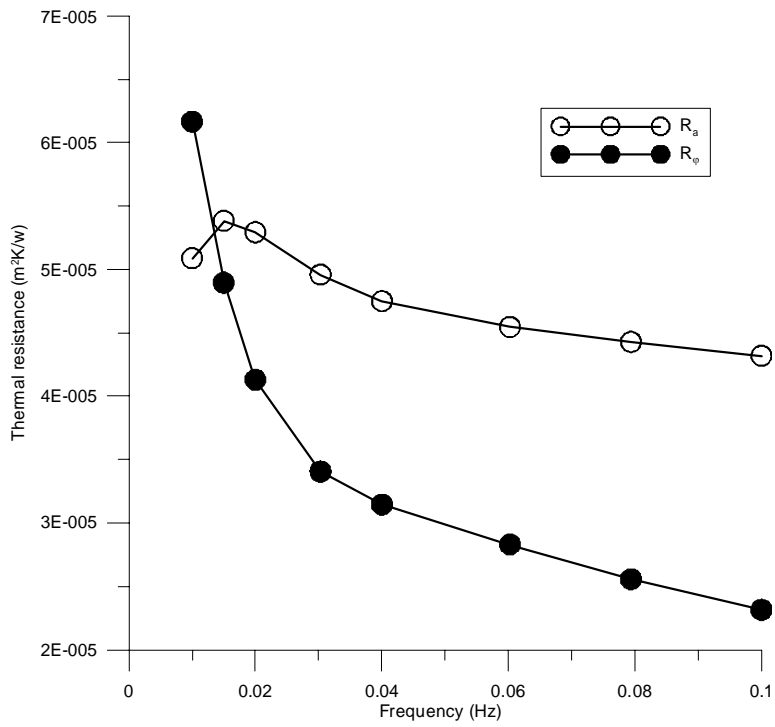


Figure 7: Thermal resistance (sample 2), P=5.80 MPa.



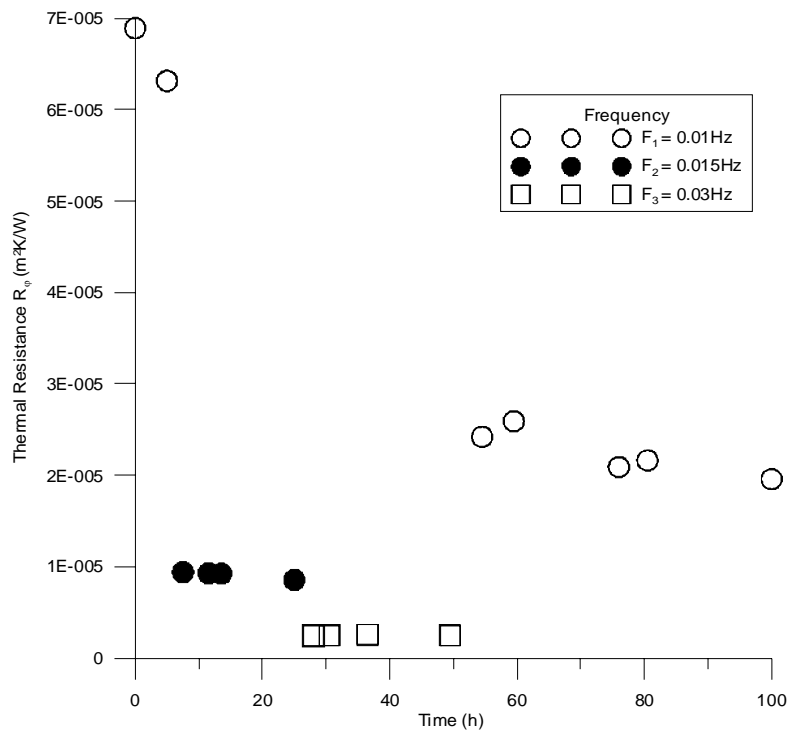


Figure 8: Thermal resistance (sample 3),  $P=2.12\text{ MPa}$

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