AN INVERSE PROBLEM FOR THE THERMAL PROPERTIES OF POROUS MEDIA

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Abstract. *This paper is concerned with the determination of thermal diffusivity and conductivity. The experimental apparatus is a rigid cylindrical tank filled with a granular medium that is heated with an electric resistance placed in the center of the cylinder. Temperatures at different radiuses are measured and used for the estimation of the thermal proprieties. A model of transient heat conduction transfer including temperature measurements is set and we have used the concept of the sensibility coefficients to demonstrate that it can not be possible to determine simultaneously the diffusivity, conductivity and heat capacity when only temperatures measurements are used, in this case only the diffusivity will be done. Eventually we have developed an inverse scheme to determine the profile of temperature between one sensor and the resistance, then the heat flux of the resistance heater is used to determine the thermal conductivity. Finally the sensibility of the estimated conductivity to the radius and resistance position is investigated..*

Keywords*: problem inverse, media porous, thermal diffusivity, thermal conductivity, thermal capacity****.***

1. INTRODUCTION

Many physical and chemical processes in soil are temperature-dependent. An important one is the coupled heat and moisture transfer in soil that concerns many fields like meteorology, agronomy, geophysics and so on. The knowledge of the soil thermal properties is required to better predict these processes. There are two approaches, the first is a microscopic one that studies local processes and it seems to be so complicated, for example, in the case of a wet soil they have a so different scales such as masse and heat transfer by capillary flow and convective flow, radiation, heat diffusion between grain of rather complex structures, diffusion in the grains, phase change of water, adsorption phenomena (etc). The second one is a macroscopic approach treating the medium as homogenous system with apparent proprieties. These approaches are commonly the methods of choice for working engineers and researchers, and the questions of their accuracy are still open. In this study we will focus our interest to the second formulation in order to estimate the apparent thermal proprieties of dray soil.

In thermal science, many methods for the estimation of conductivity and diffusivity are developed, and each one is more or less appropriate to the variety of matter and the heat and mass transfer phenomena that can occur in the considered medium. The heat pulse technique one is frequently used based on analytic or numeric solution (Da Silva, Laurent and Baillis, 1998), thermal impedance technique using alternative fluxes or temperature (Defer, Antczak and Duthoit, 2001) are largely used. An apparatus used for measuring thermal diffusivity of granular matter is shown in Fig. 1. The granular medium is placed within the inner chamber of a cylinder and heated with an electric resistance. Three temperatures are measured in different positions r_1, r_2 and r_3 and they are used for the identification of the apparent thermal diffusivity. Note that there is no need to impose any form of the heat flux dissipated by the resistance because in the model the heat flux is not explained. But if the conductivity will be estimated, it will be shown that there is to possibilities to do. The first consists of using the temperature measurements at a permanent regime and then the constant known heat flux is used to estimate only one value of the conductivity. The second consists of using the transient regime and the instantaneous heat flux so that the conductivity is estimated at each instant.

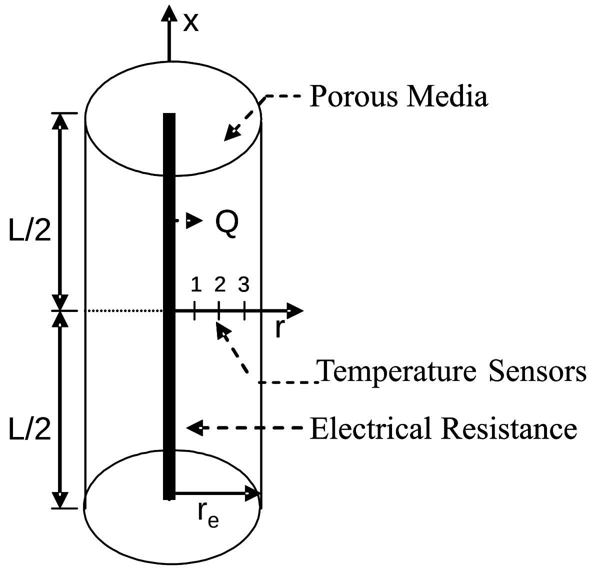

Figure 1. Apparatus for measurement of the thermal diffusivity and conductivity

2. ESTIMATION OF THE THERMAL DIFFUSIVITY

The direct model governing the heat conduction transfer accruing in the region between the sensors 1 and 2 can be in form:

$$\frac{\partial T}{\partial t} = \alpha \Delta T, \quad T \equiv T(r,t) \tag{1}$$

with boundary conditions

$$T(r_1,t) = F_1(t) \tag{2}$$

$$T(r_2,t) = F_2(t) \tag{3}$$

and, initial condition is

$$T(r,0) = F_0(r) \tag{4}$$

where α is the thermal diffusivity:

$$\alpha = \frac{\lambda}{\rho c} \tag{5}$$

The third measured temperature $T(r_3,t) = F_3(t)$ will be used to estimate the diffusivity. It is important to remark that this model don't include the heat flux ϕ dissipated by the resistance.

The maximum likelihood estimate of the parameter is obtained by minimizing the quantity:

$$\chi^2 = \sum_{n=1}^{M} \left(F_3(t_n) - T(t_n)\right)^2 \tag{6}$$

We have used the Gauss method of minimization. Eventually to remove instability and reduce oscillations we add the Levenberg-Marquardt parameter noted μ that is less then unity. The iterative formula to compute the diffusivity is as follows:

$$\alpha_{k+1} = \alpha_k + \sum_1^M \frac{S_n}{\sum_1^M S_n^{\ 2} + \mu^k} \left(F_3\left(t_n\right) - T\left(t_n\right) \right) \tag{7}$$

The sensitivity coefficient S is:

$$S = \frac{\partial T\left(r_3, t\right)}{\partial \alpha} \tag{8}$$

Derivative of the system of equations (1), (2), (3) and (4) to α vanish:

$$\frac{\partial S}{\partial t} - \alpha \Delta S = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{9}$$

$$S\left(r_1, t\right) = 0 \tag{10}$$

$$S\left(r_2, t\right) = 0 \tag{11}$$

$$S\left(r, 0\right) = 0 \tag{12}$$

It can be easily verified that the diffusivity can not be estimated when only measurements at permanent regime are taken, the reason that S will be equal to zero all the time. A crucial remark to note is that $\alpha=\infty$ is a stationary point of convergence of the suite α_k. Eventually, system of equations (1), (2), (3) and (4), has a natural constraint on the range of temperature because T must be between $F(r_1,t)$ and $F(r_2,t)$ independently of the value of the diffusivity α_k. That means that $\frac{\partial T}{\partial t}$ is bounded too. Now, If we set at any step of the iterative procedure $\alpha_k=\infty$, so we have $\frac{1}{\alpha_k}\frac{\partial T}{\partial t} = 0$ and obviously the system of equations (9), (10), (11) and (12), leads to $S_n(\alpha_k)=0$. Finally we have $\alpha_{k+1}=\alpha_k=\infty$. In conclusion, a good estimate of the initial value of the diffusivity is required to avoid divergence.

Assuming that the temperature measurements errors are additive, having a zero mean, independent with a normal distribution and having a constant standard deviations σ_T . The standard deviation of the estimated diffusivity $\hat{\alpha}$ is obtained approximately using the statistical analysis given by Beck and Arnold (1977):

$$\sigma_\alpha^2 = \frac{\sigma^2}{\sum_1^M S_n^2} \tag{13}$$

The confidence interval at the 99% confidence level for the estimated diffusivity can be obtained as:

$$\text{Probability}\left(\hat{\alpha} - 2.57.\sigma_\alpha \le \alpha \le \hat{\alpha} - 2.57.\sigma_\alpha \right) = 99\% \tag{14}$$

As an example, simulated experimental temperature measurements are made under the conditions given by table (1):

Table 1. Experimental conditions.

ϕ (w/m^2)	α ($m^2.s^{-1}$)	λ ($w.m^{-1}.K^{-1}$)	r_0 (mm)	r_1 (mm)	r_2 (mm)	r_3 (mm)
10^3	10^{-7}	0.2	5	15	30	40

where, r_0 is the resistance radius. The exact temperatures at r_1, r_2 and r_3 are given in Fig. 2. The Fig. 3 presents these temperatures with addition of a noise term $a\upsilon$ the four one present the filtered noise temperatures. To respect the 99%

confidence interval, the modulus $|a|$ must be less than 2.57, in this case we have taken a=0.2. The noise υ is randomly generated using the GASDEV-RAN1 subroutine (Press, et al., 1992)

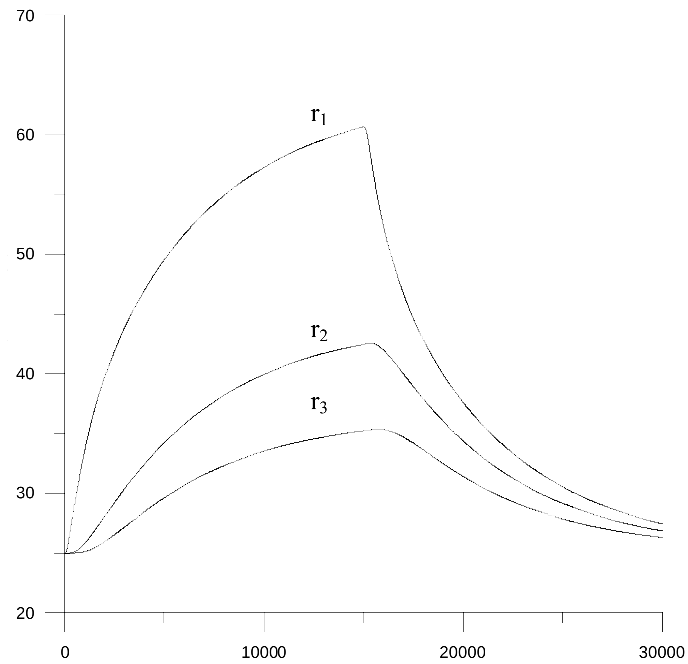

Figure 2. Exact temperatures histories at r_1, r_2 and r_3

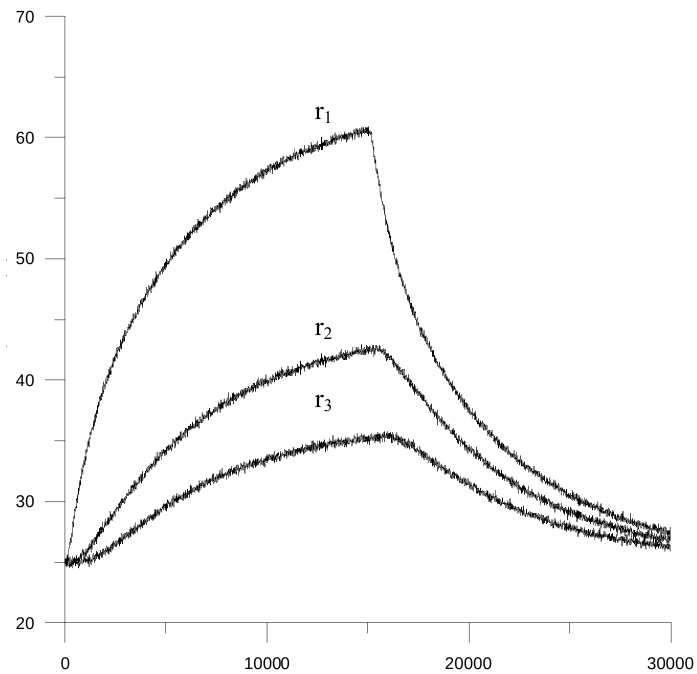

Figure 3. Noise temperatures

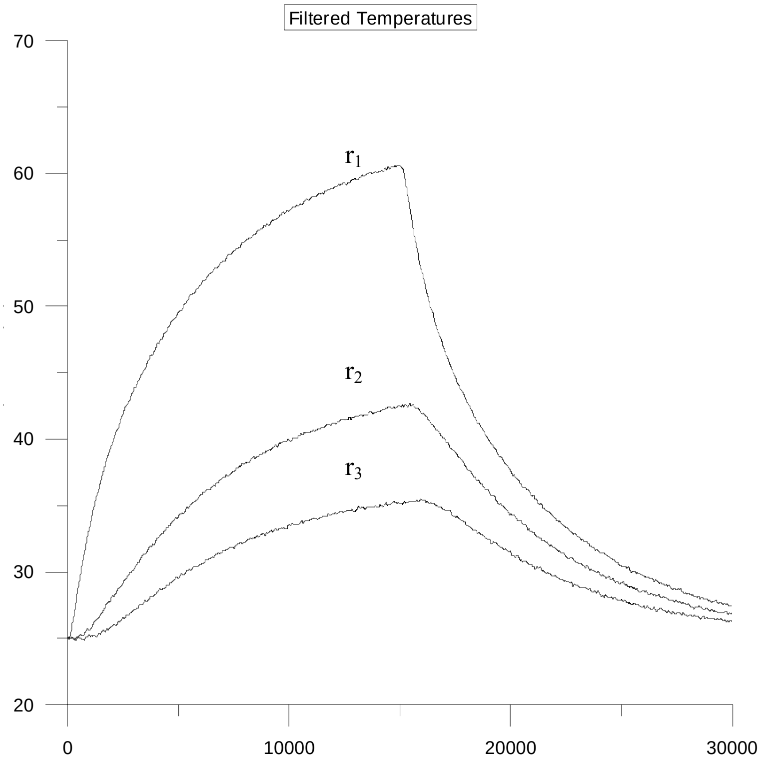

Figure 4. Noise and filtered temperatures

The standard deviation of temperature is taken σ_T=0.01°C. The value of the estimated diffusivity and its standard deviation with exact, noise and the filtered data are given in Tab. 2:

Table 2: Estimated diffusivity

	Exact data	Noise data	Noise and filtered data
α	10^{-7}	$1.006.10^{-7}$	$1.007.10^{-7}$
σ_α	$0.1408.10^{-10}$	$0.1416.10^{-10}$	$0.1418.10^{-10}$

3. ESTIMATION OF THE THERMAL CONDUCTIVITY

The value of the apparent thermal proprieties are largely influenced bay many heat and mass transfer processes, Eventually by the one due to the capillary flow, the interstitial gas molecular effects and so on. These phenomena are temperature dependant, obviously it is recommended to determine the effective diffusivity and conductivity under the same conditions and eventually through the same experience. Note that the last experience involves the heat conduction equation and temperatures measurements as boundaries and initial conditions. With such model we can only estimate the thermal diffusivity. To proof this, one can make the derivative of the model (I) with respect to λ, ρc and α to give the sensitivity equations of the coefficients S_λ, $S_{\rho c}$ and S_α. The result is:

$$\alpha\left[\frac{\partial S_\alpha}{\partial t}-\alpha\Delta S_\alpha\right]=\lambda\left[\frac{\partial S_\lambda}{\partial t}-\alpha\Delta S_\lambda\right]=-\rho c\left[\frac{\partial S_{\rho c}}{\partial t}-\alpha\Delta S_{\rho c}\right]=\frac{\partial T}{\partial t} \quad (15)$$

The boundaries and initial conditions are zero for all the coefficients. So these coefficients are linearly dependent:

$$\alpha S_\alpha = \lambda S_\lambda = -\rho c S_{\rho c} \quad (16)$$

So they can not be simultaneously estimated. To estimate the conductivity we use the knowing value of the heat flux dissipated by the resistance as a boundary condition. Generally authors use only two temperature measurements in permanent regime, and the conservation of the one dimensional heat flux to determine one value of the conductivity (Gurgel et al, 1996, Gurgel et al, 2001). In this work we suggest the use of the transient regime to obtain large number of values. First we need to determine the temperature profile closely the electric resistance using an inverse scheme. The derivation of this space marching scheme consists of the use a variable transformation **y=lnr**, so the heat conduction equation leads to:

$$\frac{\partial T}{\partial t} = \alpha e^{-2y} \frac{\partial^2 T}{\partial y^2} \tag{17}$$

To obtain a stable scheme for the calculation we introduce a bias for the temporal term used firstly by Sassi and Raynaud (1998) such as:

$$\left.\frac{\partial T}{\partial t}\right|_i = \frac{1}{2}\left(\left.\frac{\partial T}{\partial t}\right|_{i-1} + \left.\frac{\partial T}{\partial t}\right|_{i+1} \right) \tag{18}$$

In add, when using an explicit scheme to the spatial term we obtain:

$$T_{i-1}^n - \frac{2}{1+a_i} \cdot T_i^n + T_{i+1}^n = \frac{a_i}{1+a_i} \cdot \left(T_{i-1}^{n+1} + T_{i+1}^{n+1} \right)$$

The a_i terms are given by:

$$a_i = \frac{\Delta y^2}{2\alpha \Delta t} e^{2y_i} \tag{19}$$

It can be easily demonstrated that the scheme is stable. The details are discussed by Sassi and Raynaud (1998). Finally the conductivity is estimated as follows:

$$\lambda = \frac{\Phi}{2\pi r_0 l \left(-e^{-y} \cdot \left.\frac{\partial T}{\partial y}\right|_{y_0} + \frac{\Delta y}{2\alpha} \cdot \left(1 + \frac{\Delta y}{4}\right) . e^{y} . \left.\frac{\partial T}{\partial t}\right|_{y_0} \right)} \tag{20}$$

As it is seen the estimation of the conductivity needs only an inverse model to extrapolate the field of the temperature. Only the value of the diffusivity and temperature measurements are needed to make calculation and this inverse problem can be regarded as an inverse heat conduction problem to determine heat flux at a boundary. This problem is widely discussed in literature by Sassi and Raynaud (1998) and the inverse procedure is efficient under the following constraint:

$$\frac{\alpha \Delta t}{(r_1 - r_0)^2} > 0.02 \tag{21}$$

From literature the averaged value of the diffusivity is $\alpha \approx 10^{-7}$. Many values of the time step and the distance between the resistance and the sensor that respect this condition are given in the table 3:

Table 3: Condition to success the inverse heat conduction problem

r_1-r_0	15mm	10mm	7mm	5mm	3mm
Δt (s)	45	20	9.8	5	1.8
Time of 1000 measures (h)	12.5	5.5	2.7	1.4	0.5

The space marching molecule is derived using a const step for the variable y. The distribution of the real nodes between r_1 and r_2 has an exponential behavior. It is easy to prove that the space nodes are given by:

$$r_n = r_1 e^{(n-1)\Delta y} \tag{22}$$

Figure presents the distribution of the space grid using a 21 nodes.

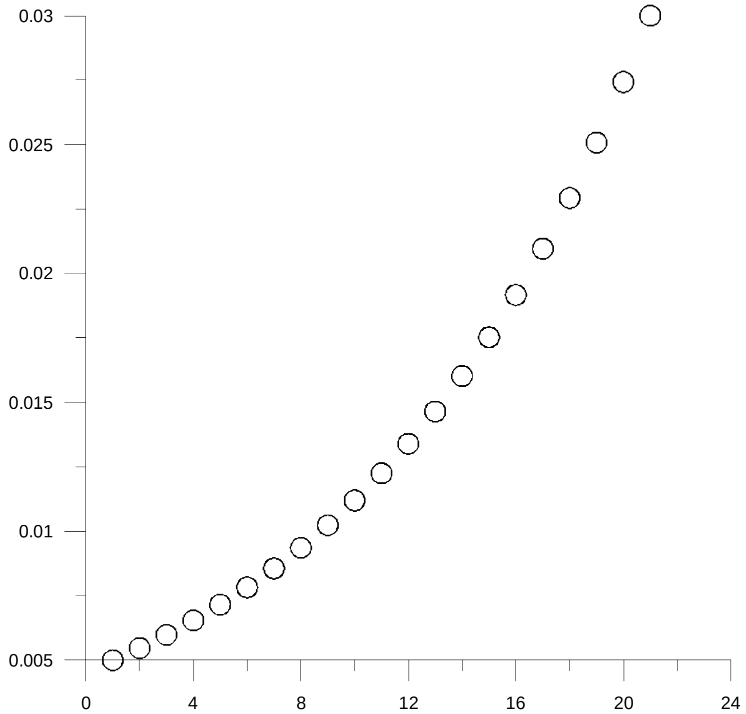

Figure 5. Space mesh with 21 nodes

The bias introduced in the back scheme is a function of the used space step. As it is seen the grid is not a homogenous one, so we have running the inverse procedure with different node number to determine the one that introduce the minimum of bias. It is found 21 nodes. Figs. 6, 7 and 8 presents the heat conduction flux estimated with different data.

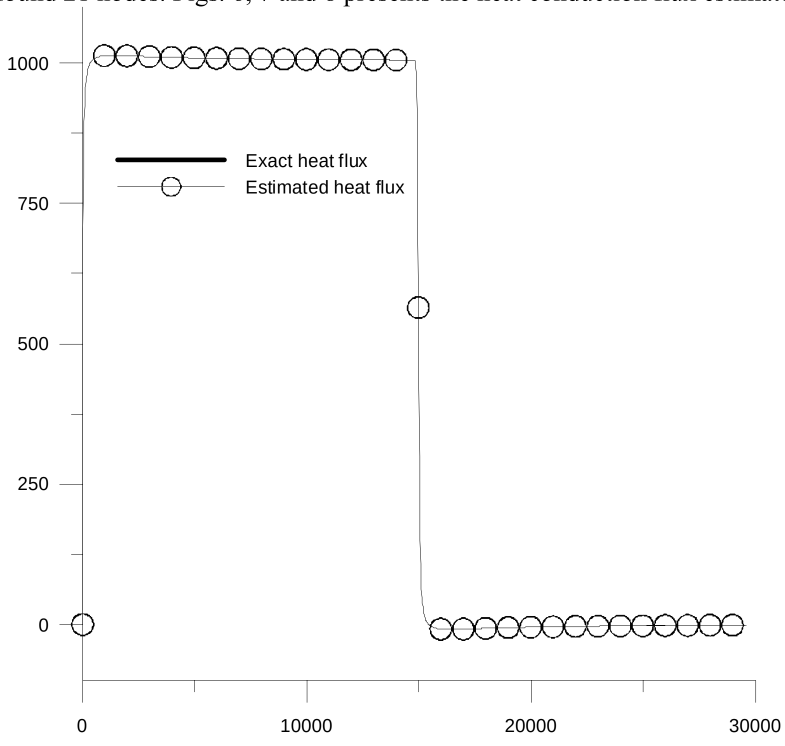

Figure 6. Exact and estimated heat flux using exact data

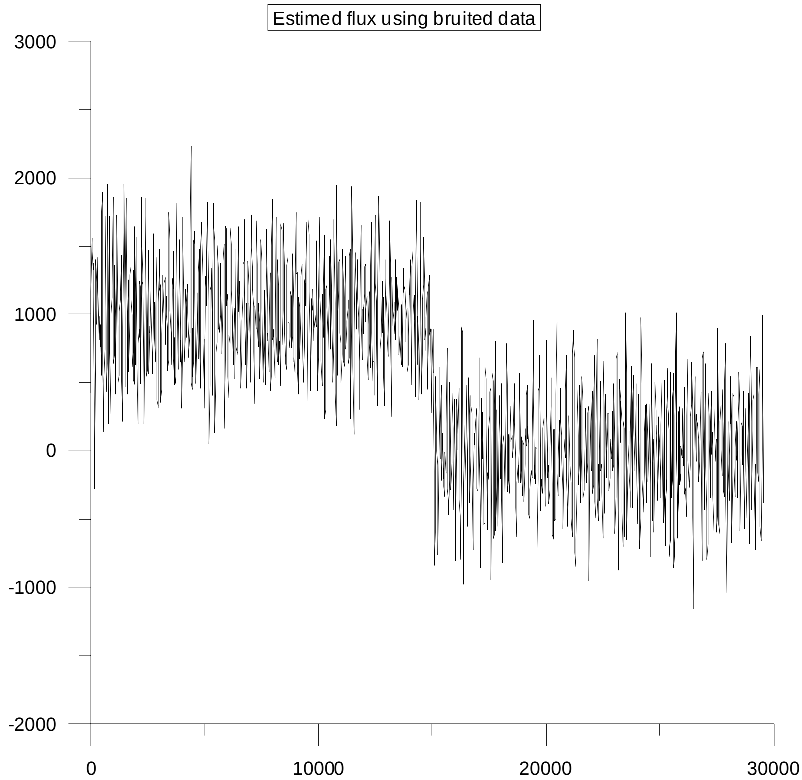

Figure 7. Estimated heat flux using noise data.

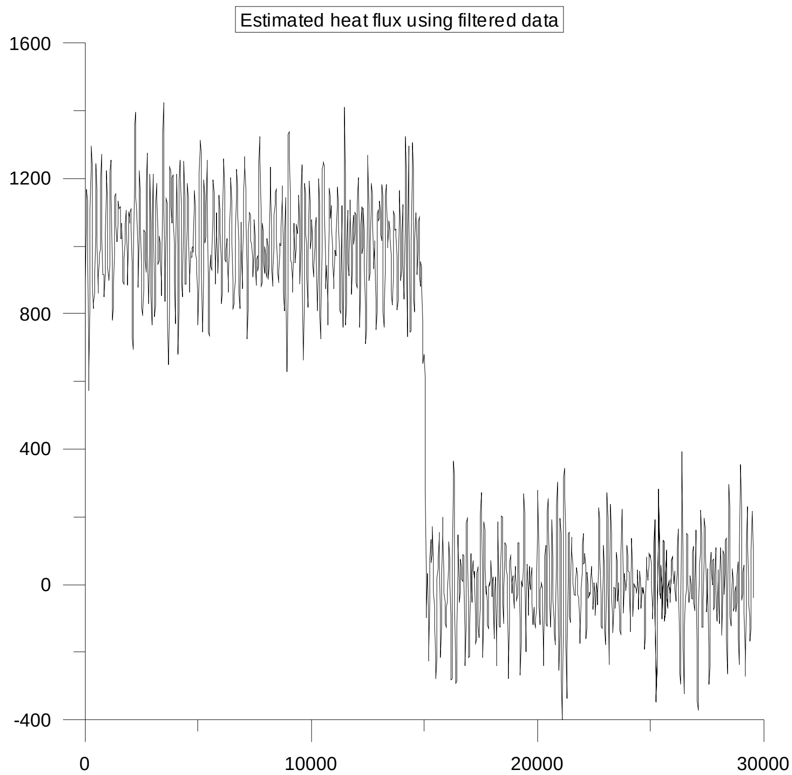

Figure 8: Estimated flux using filtered-noise data

Table 4. present the conductivity estimated using the simulated temperatures described above.

Table 4: The estimated conductivity

	Exact data	Noise data	Filtered Noise data
$\hat{\lambda}$	0.1974	0.2487	0.2015
σ_λ	0.0011	0.2060	0.0290

Furthermore, the sensitivity analysis to the resistance radius chows that a 5% precision in estimated conductivity requires in the maximum 5% errors of the resistance radius or position see figure (9). So for accuracy, it is required to use a resistance having a large radius. This a crucial point because if we suppose an error Δr=0.5mm on the position the diameter of the resistance must be at least d=20mm, so it will have a large heat capacity that implies errors for the knowledge heat flux generated by the resistance.

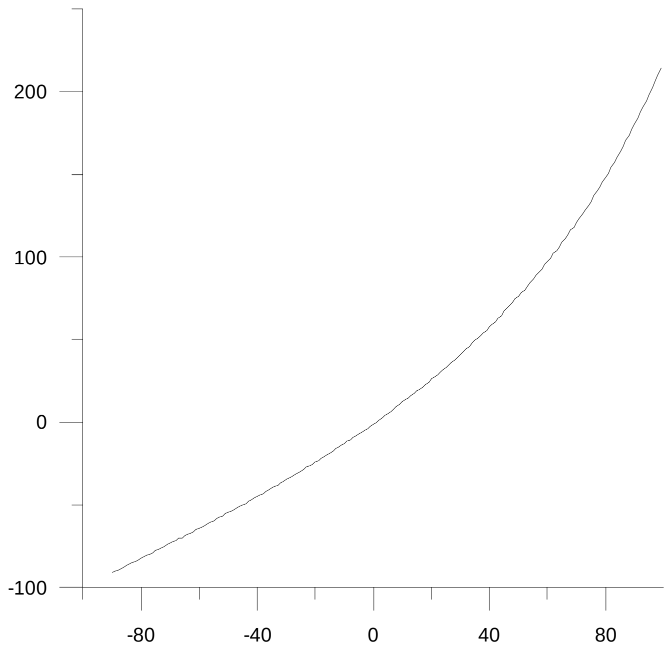

Figure 9. The conductivity relative error function of the radius one

4. CONCLUSION

This work develops an inverse procedure to determine simultaneously the thermal proprieties of soil. The analysis of the inverse models show that the estimation of the diffusivity using least-squares sense adding the Levenberg-Marquardt parameter is accurate and easily implemented. On another hand the estimation of the thermal conductivity is more delicate. Eventually, the space grid must be at first optimized to minimize the bias introduced by the space marching scheme in the cylindrical geometry. Secondly, the position of the electric resistance and the large of its radius influence considerably the accuracy of the estimated conductivity. Finally, it is interesting to note that the procedure is independent of the heat flux field that makes easy experimental procedure.

5. REFERENCES

Beck J. V. and Arnold K.J., 1977, Parameter Estimation in Engineering and Science, John Wiley & Sons, New York, P. 379.

D. Defer, E. Antczak and B. Duthoit, 2001, "Measurement of Low-Thermal Effusivity of Building Materials Using Thermal Impedance Method", Meas. Sci. Technol, Vol. 12**,** pp. 549–556

J. M. Gurgel and R. P. Kluppel, 1996, "Thermal Conductivity of Hydrated Silica-Gel", The Chemical Engineering Journal, v. 61, pp. 133-138..

J. M. Gurgel, L. S. A. Filho, PH. Grenier and F. Meunier, 2001, "Thermal diffusivity and Adsorption Kinetics of Sílica-Gel/Water", Kluwer Academic Publishers. Manufactured in The Netherlands, Adsorption, Vol. 7, pp. 211-219..

M. Sassi and M. Raynaud, 1998, "New Space Marching Method for Solving Inverse Boundary Problems", Numerical Heat Transfer, Part B-Fundamentals, EUA, v. 34, n. 1, p. 562-571, 1998.

Silva, Z. E.; Laurent, M.; Baillis, D. D., 1998, “Inverse Analysis of Transient Coupled Conduction-Radiation - Conductive and Radiative Properties and Measurements” AIAA/ASME Joint Thermophysics and Heat Transfer Conference, 7th, Albuquerque, NM; UNITED STATES; Paper AIAA, 98-2842, EUA.
William H. P., Brian P. F., Saul A. T., William T. V., Numerical Recipes (FORTRAN Version), Cambridge University Press, New York, P. 529 - P. 546.

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