COMPARISON AMONG PREDICTOR-CORRECTOR, SIMMETRICAL AND TVD UPWIND SCHEMES IN THE SOLUTION OF THE EULER EQUATIONS IN TWO-DIMENSIONS - RESULTS

Edisson Sávio de Góes Maciel, saviomaciel@pq.cnpq.br
CNPq Researcher – Rua Demócrito Cavalcanti, 152 – Afogados – Recife – Pernambuco – Brazil – 50750-080

Abstract. *The present work compares the MacCormack, the Harten, the Yee and Kutler and the Jameson and Mavriplis numerical methods, using a finite volume formulation and a structured spatial discretization, applied to the solution of the Euler equations in the two-dimensional space. All schemes are second order accurate in space. The MacCormack and the Jameson and Mavriplis schemes are also second order accurate in time and both use artificial dissipation operators to guarantee the convergence to the steady state solution. The steady state physical problems of the transonic flows along a convergent-divergent nozzle and around a NACA 0012 airfoil, the supersonic flow along a ramp and the "cold gas" hypersonic flow around a double ellipse configuration are studied. A spatially variable time step is implemented aiming to accelerate the convergence process. The results have demonstrated that the MacCormack scheme predicts the most critical solutions, although unphysical results were obtained in high Mach numbers. Shock pressure ratio, nozzle, and stagnation pressure, double ellipse, were best estimated by this scheme. This paper (RESULTS) presents the solutions of the numerical simulations performed by the schemes, comparisons between numerical and experimental or theoretical results and computational data.*

Keywords*: MacCormack algorithm, Harten algorithm, Yee and Kutler algorithm, Jameson and Mavriplis algorithm, Euler equations.*

1. INTRODUCTION

The development of aeronautical and aerospace projects require hours of wind tunnel essays. It is necessary to minimize such wind tunnel procedures due to the growing cost of electrical energy. In Brazil, there is the problem of this country has not yet wind tunnels of great capacity, able to generate supersonic flows or even high subsonic flows. So, Computational Fluid Dynamics, CFD, techniques have now great highlight in the aeronautical industry scenario. Analogous to wind tunnel essays, the numerical methods determine physical properties in discrete points of the spatial domain. Hence, the aerodynamic coefficients of lift, drag and momentum can be calculated.

The present work compares the MacCormack (1969), the Harten (1983), the Yee and Kutler (1985) and the Jameson and Mavriplis (1986) numerical methods, using a finite volume formulation and a structured spatial discretization, applied to the solution of the Euler equations in the two-dimensional space. All schemes are second order accurate in space. The MacCormack (1969) and the Jameson and Mavriplis (1986) schemes are also second order accurate in time and both use artificial dissipation operators to guarantee the convergence to the steady state solution. The Harten (1983) and the Yee and Kutler (1985) schemes are first order accurate in time. The steady state physical problems of the transonic flows along a convergent-divergent nozzle and around a NACA 0012 airfoil, the supersonic flow along a ramp and the "cold gas" hypersonic flow around a double ellipse configuration are studied. A spatially variable time step is implemented aiming to accelerate the convergence process. The results have demonstrated that the MacCormack scheme predicts the most critical solutions, although unphysical results were obtained in high Mach numbers. The pressure ratio at the shock, nozzle problem, and the stagnation pressure, double ellipse problem, were best estimated by the MacCormack (1969) scheme.

This paper (RESULTS), second part of this study, presents the solutions of the numerical simulations performed by the schemes, comparisons between numerical and experimental or theoretical results and computational data. To a brief description of the motivation, as well comments about the numerical methods implemented in this work, the reader is encouraged to read part I of this study (THEORY) and Maciel (2006).

2. RESULTS

Tests were accomplished in a CELERON-1.2GHz and 128 Mbytes of RAM memory microcomputer. Converged results occurred to 4 orders of reduction in the value of the maximum residue. The maximum residue is defined as the maximum value obtained from the discretized conservation equations. A value of 1.4 was used to γ. A value of 0.0° was used to the entrance (nozzle problem) or to the attack (others problems) angle.

2.1. Convergent-divergent nozzle physical problem

To this problem, an algebraic mesh with 61x71 points was used, which corresponds to 4,200 rectangular volumes and of 4,331 nodes, on a finite volume context. It was used an exponential stretching of 10% in both ξ and η directions.

Figures 1 to 4 show the pressure field generated by the MacCormack (196), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes, respectively. The pressure field generated by the Harten (1983) scheme is the most severe in relation to the others schemes.

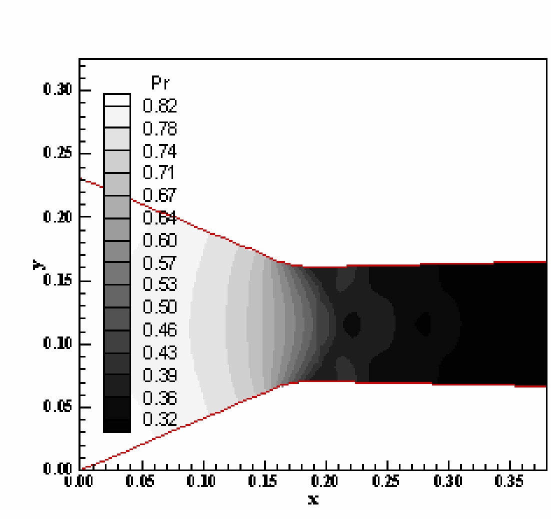

Figure 1. Pressure field (M/69).

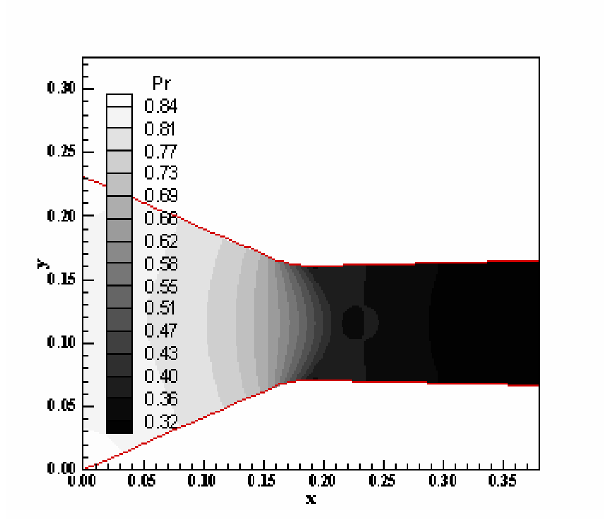

Figure 2. Pressure field (H/83).

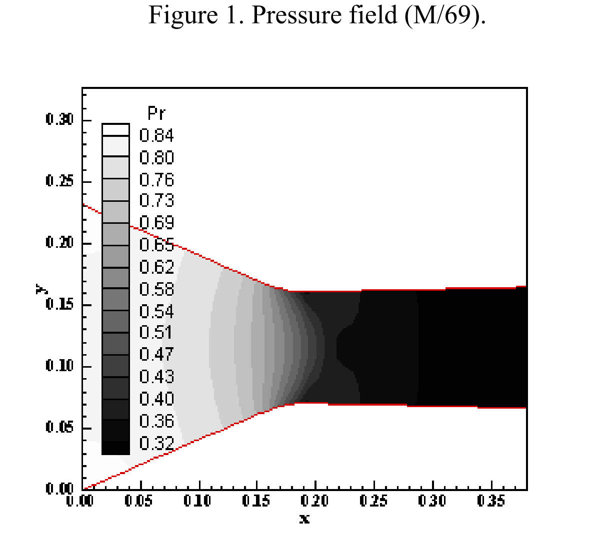

Figure 3. Pressure field (YK/85).

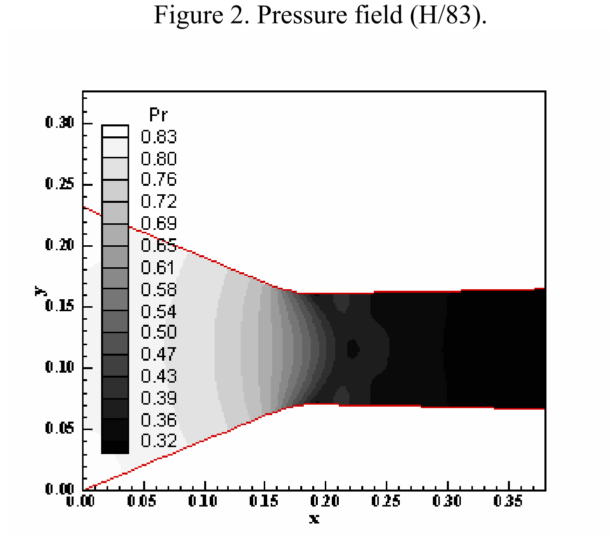

Figure 4. Pressure field (JM/86).

Figures 5 to 8 exhibit the Mach number contours obtained by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes, respectively. The Mach number field generated by the Yee and Kutler (1985) scheme is the most intense in comparison with the others schemes.

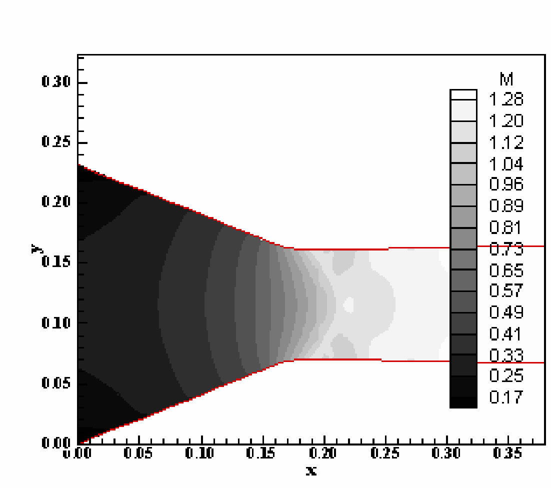

Figure 5. Mach number field (M/69).

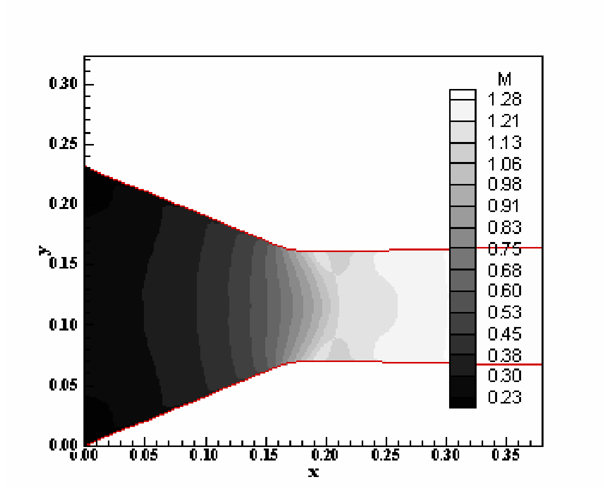

Figure 6. Mach number field (H/83).

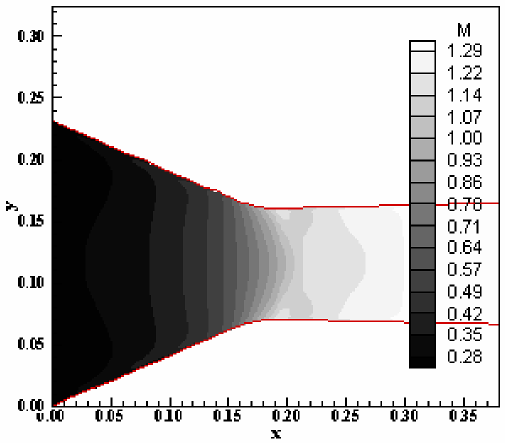

Figure 7. Mach number field (YK/85).

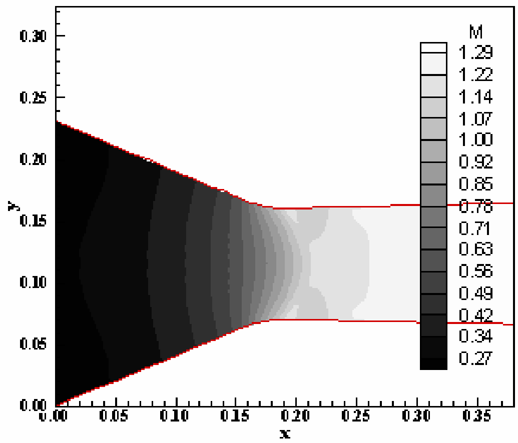

Figure 8. Mach number field (JM/86).

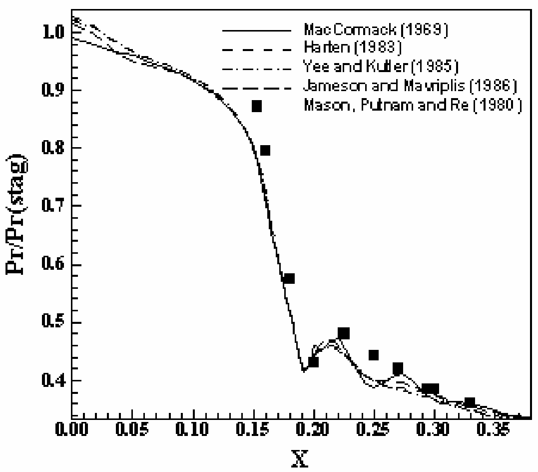

Figure 9. Wall pressure distributions.

Figure 9 shows the pressure distributions along the nozzle lower wall generated by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes. These distributions are compared with the experimental results of Mason, Putnam and Re (1980). It is possible to note that the pressure distribution generated by the MacCormack (1969) scheme is the closest with the experimental results. Moreover, the shock at the throat captured by the MacCormack (1969) scheme is the most severe in relation to the others schemes. The pressure ratio at the shock obtained by the MacCormack (1969) scheme has a value of 0.48, the Harten (1983) scheme presents a value to this pressure ratio equals to 0.47, the Yee and Kutler (1985) scheme presents a value of 0.46 and the Jameson and Mavriplis (1986) scheme detects a value of 0.46. So, the MacCormack (1969) scheme is the most critical algorithm in this problem.

2.2. Airfoil physical problem

To the physical problem of the airfoil configuration, an algebraic mesh of 49x100 points was used, which is constituted of 4,752 rectangular volumes and 4,900 nodes.

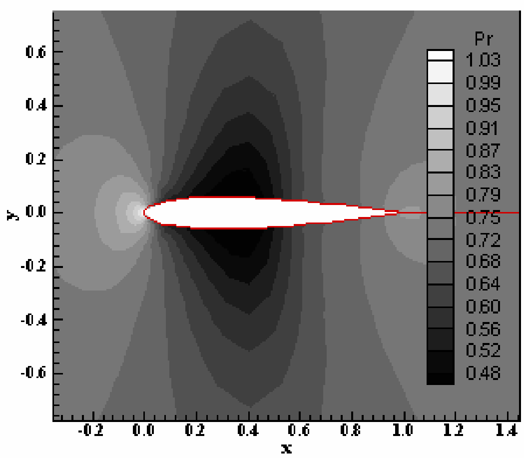

Figure 10. Pressure field (M/69).

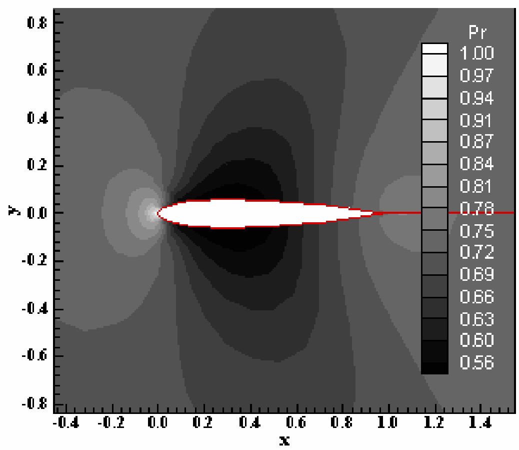

Figure 11. Pressure field (H/83).

An exponential stretching of 5% in the η direction was implemented. The far field was located at 10.0 times the airfoil chord in relation to the airfoil leading edge. The freestream Mach number adopted in this simulation was 0.8, characterizing a transonic flow regime.

Figures 10 to 13 show the pressure field generated by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes, respectively. The pressure field generated by the Yee and Kutler (1985) scheme is the most severe in comparison with the others schemes.

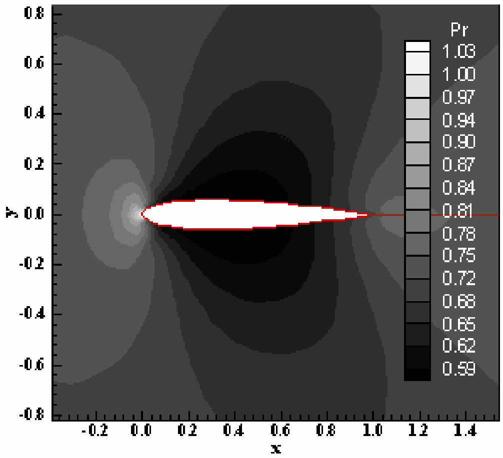

Figure 12. Pressure field (YK/85).

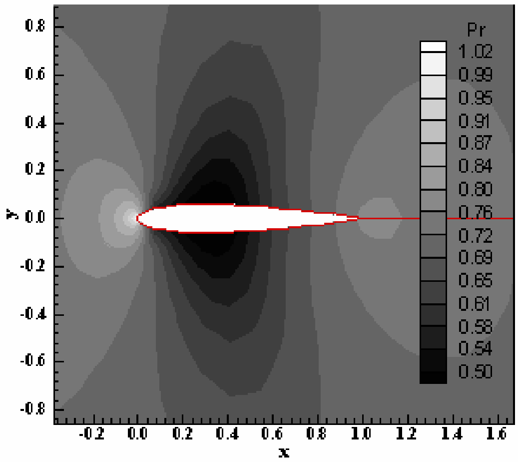

Figure 13. Pressure field (JM/86).

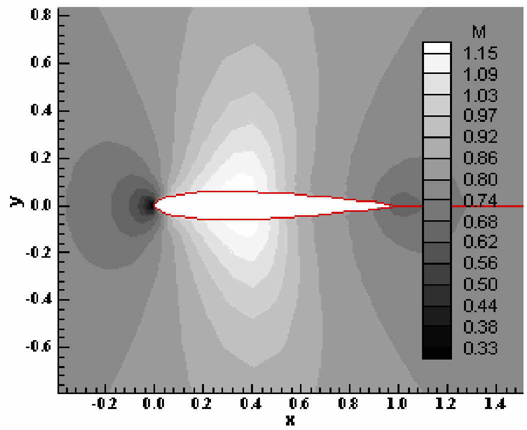

Figure 14. Mach number field (M/69).

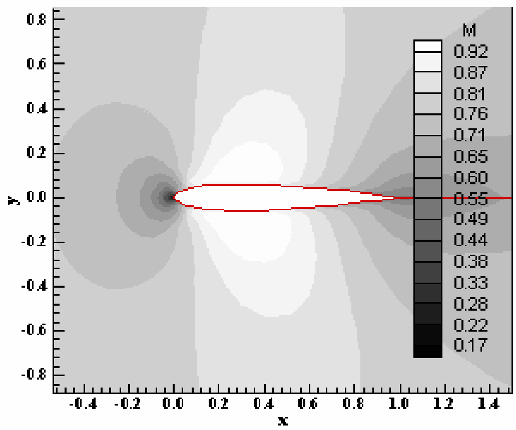

Figure 15. Mach number field (H/83).

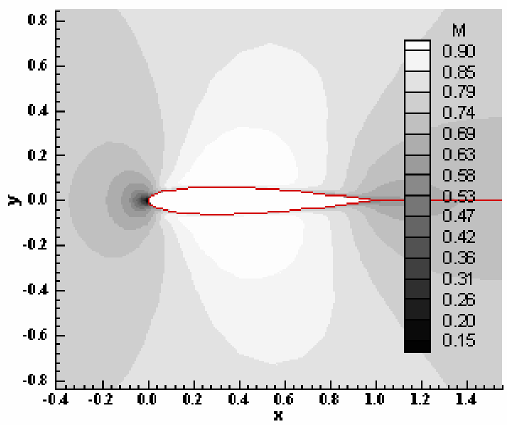

Figure 16. Mach number field (YK/85).

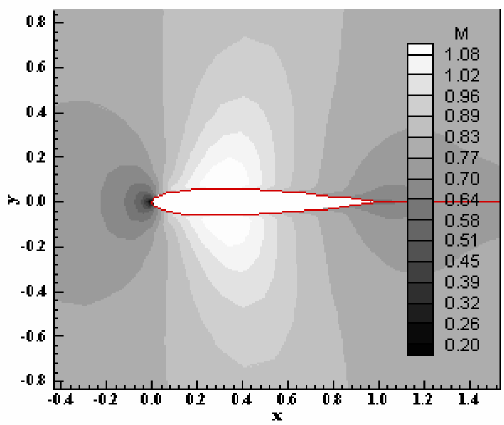

Figure 17. Mach number field (JM/86).

Figures 14 to 17 exhibit the Mach number field generated by the algorithms. The Mach number field generated by the MacCormack (1969) scheme is the most intense in relation to the others schemes. The TVD upwind schemes do not detects regions of supersonic flow, while the predictor-corrector and the symmetrical schemes present such regions.

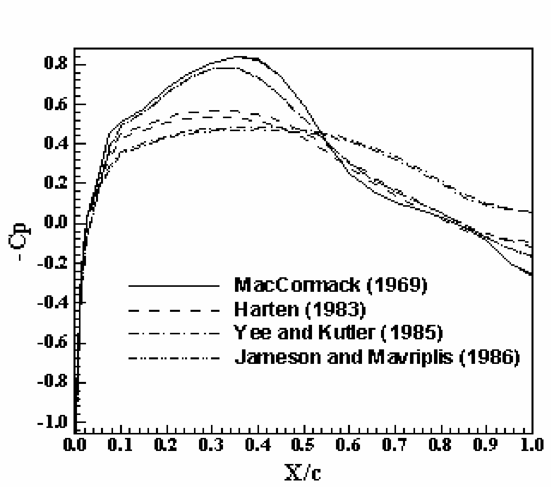

Figure 18. –Cp distributions.

Figure 18 shows the -Cp distributions around the airfoil geometry. The shock detected by the MacCormack (1969) scheme is the most severe. The -Cp curve generated by the MacCormack (1969) scheme presents a minimum -Cp value of 0.84 at x = 0.37m, which corresponds to the shock at the airfoil. The Harten (1983) scheme presents a minimum –Cp value of 0.56, at x = 0.32m, while the Yee and Kutler (1985) scheme presents such minimum with a value of 0.48, at x = 0.42m. The Jameson and Mavriplis (1986) scheme presents a minimum –Cp value of 0.78 at x = 0.32m.

Table 1 exhibits the aerodynamic coefficients of lift and drag. As the simulation considered a zero value to the attack angle and due to the configuration symmetry in relation to the flow direction, zero value to the lift aerodynamic coefficient is the expected result. The Jameson and Mavriplis (1986) scheme presents the most accurate c_L in relation to the others schemes.

Table 1. Aerodynamic coefficients of lift and drag to the airfoil problem.

Algorithm:	c_L:	c_D:
MacCormack (1969)	$7.451x10^{-5}$	$2.329x10^{-4}$
Harten (1983)	$-2,514x10^{-2}$	$-5,954x10^{-5}$
Yee and Kutler (1985)	$-1,183x10^{-2}$	$-1,377x10^{-4}$
Jameson and Mavriplis (1986)	$3.565x10^{-9}$	$2.791x10^{-10}$

2.3. Ramp physical problem

To this physical problem, an algebraic mesh of 61x100 points or composed of 5,940 rectangular volumes and 6,100 nodes was used. The angle of inclination of the ramp is 20º. The freestream Mach number adopted for this simulation is 2.0, which characterizes a supersonic flow.

Figures 19 to 22 show the pressure field generated by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes, respectively. The pressure field generated by the MacCormack (1969) scheme is the most severe in relation to the others schemes.

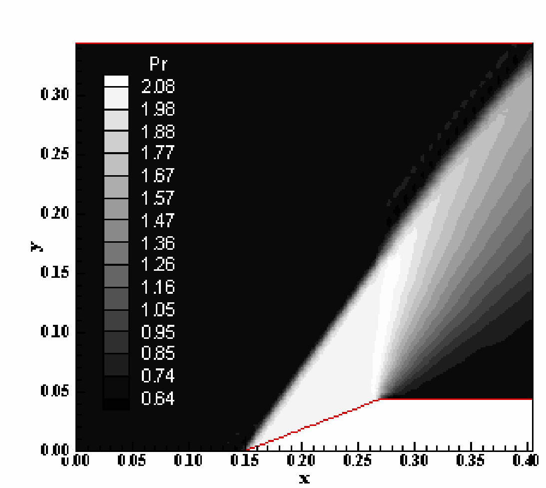

Figure 19. Pressure field (M/69).

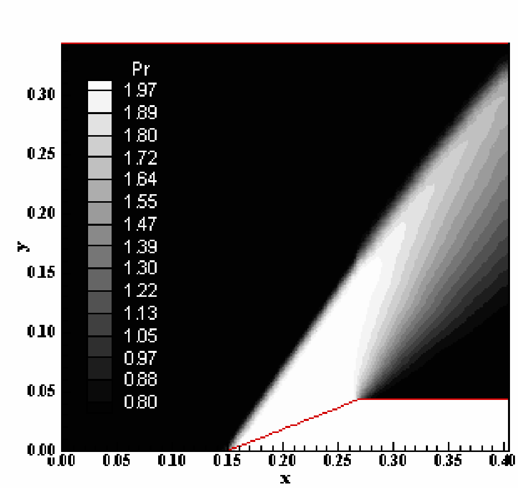

Figure 20. Pressure field (H/83).

Figures 23 to 26 exhibit the Mach number field obtained by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes, respectively. It is possible to not pre-shock oscillations in the MacCormack (1969) solution. The Mach number field reaches a peak greater than the freestream

Mach number, which is physically incorrect. The MacCormack (1969) scheme presents an error in this numerical solution. The Jameson and Mavriplis (1986) solution is the most intense in relation to the tested schemes, considering physically relevant solutions.

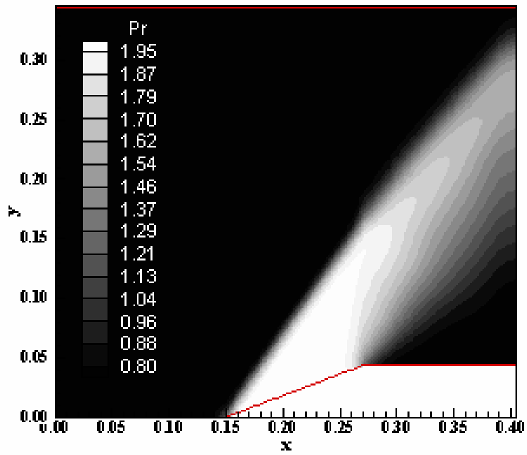

Figure 21. Pressure field (YK/85).

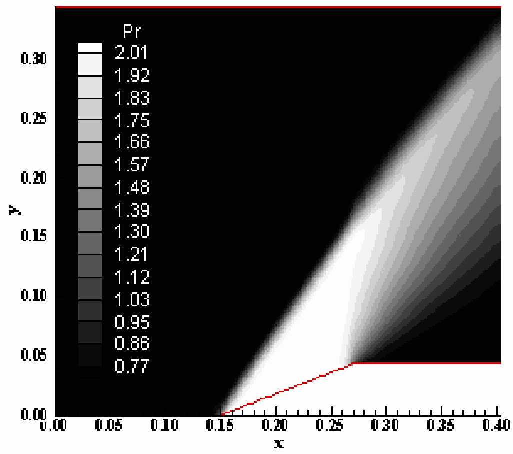

Figure 22. Pressure field (JM/86).

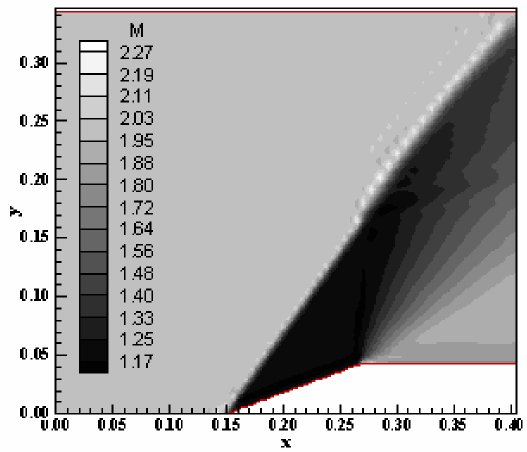

Figure 23. Mach number field (M/69).

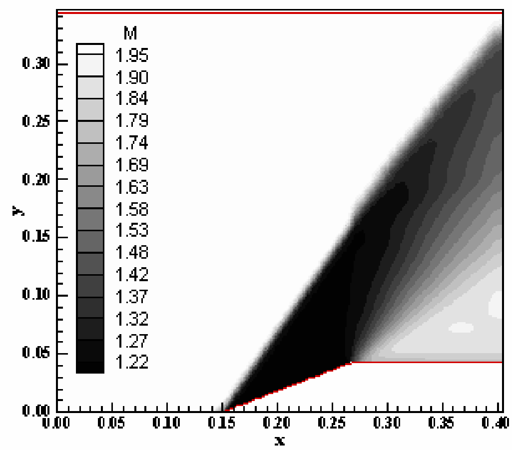

Figure 24. Mach number field (H/83).

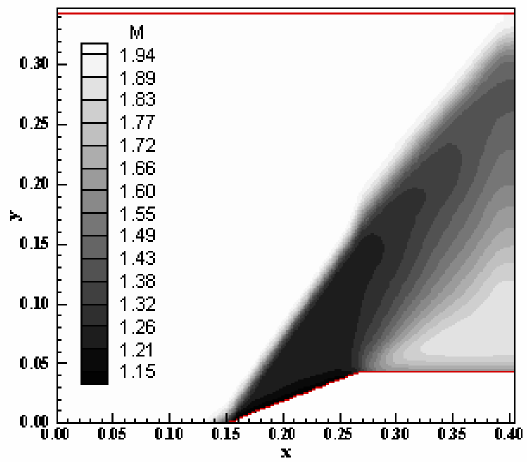

Figure 25. Mach number field (YK/85).

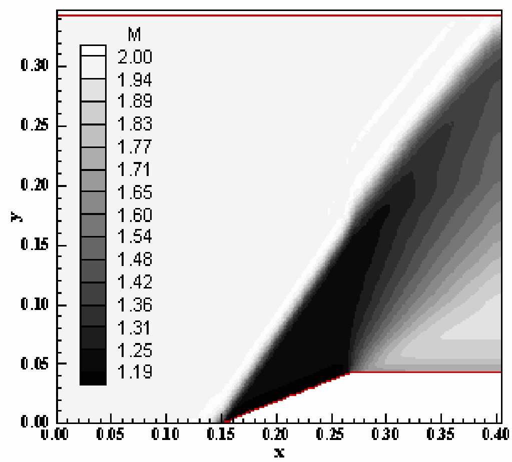

Figure 26. Mach number field (JM/86).

Figure 27 exhibits the pressure distributions along the ramp obtained by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) scheme. They are compared with the exact solutions from oblique shock theory and the Prandtl-Meyer expansion wave theory. It is possible to note that the Yee and Kutler (1985) solution presents the smoothest behavior in comparison with the others schemes.

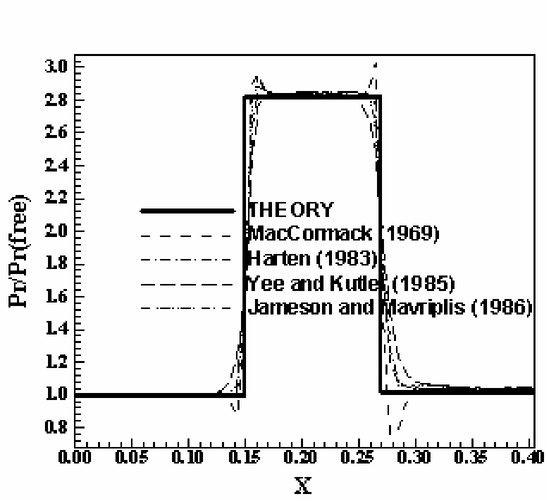

Figure 27. Pressure distributions.

It does not present oscillations at the shock region, what characterizes the best solution, in comparison with the theory, in relation to the others schemes. The Harten (1983) distribution presents a sharp defined shock, but with a small peak in relation to the theory. The Jameson and Mavriplis (1986) distribution also predicts a peak at the shock, but it behaves better than the MacCormack (1969) solution, which presents a reduction in the pressure before the shock, characterizing an unphysical solution. This behavior is due to numerical oscillations typical of second order schemes, recognized in the literature as "Gibbs oscillations". The MacCormack (1969) scheme also predicts a pressure peak at the ramp beginning. The expansion fan is well detected by all schemes as compared with the theory, with the smoothest behavior obtained by the Yee and Kutler (1985) scheme. The width of the constant pressure region after the shock (the plateau) at the ramp is well captured by all schemes, except by the Yee and Kutler (1985) scheme.

Other way to quantitatively verify if the solutions generated by each scheme are satisfactory consists in determining the shock angle of the oblique shock wave, β, measured in relation to the initial direction of the flow field. Anderson (1984) (pages 352 and 353) presents a diagram with values of the shock angle, β, to oblique shock waves. The value of this angle is determined as function of the freestream Mach number and of the deflection angle of the flow after the shock wave, ϕ. To $\phi = 20^{o}$ (ramp inclination angle) and to a freestream Mach number equals to 2.0, it is possible to obtain from this diagram a value to β equals to 53.0º. Using a transfer in Figures 19, 20, 21 and 22, it is possible to obtain the values of β to each scheme, as well the respective errors, shown in Tab. 2. The results highlight that the Harten (1983) scheme is the most accurate of the studied schemes in this problem.

Table 2. Shock angle and percentage errors to the ramp problem.

Algorithm:	β:	Error (%):
MacCormack (1969)	54.5	2.8
Harten (1983)	54.0	1.9
Yee and Kutler (1985)	54.5	2.8
Jameson and Mavriplis (1986)	54.5	2.8

2.4. Double ellipse physical problem

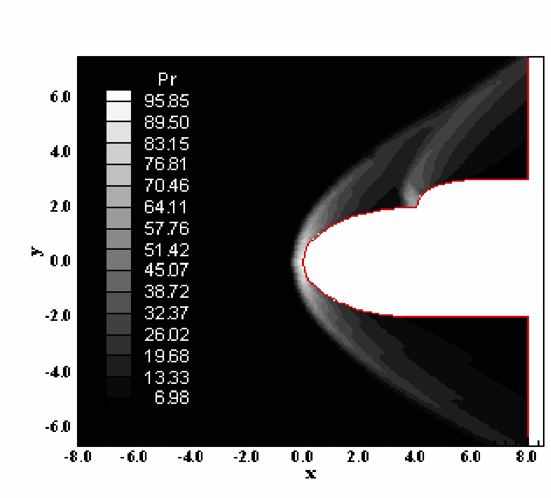

Figure 28. Pressure field (M/69).

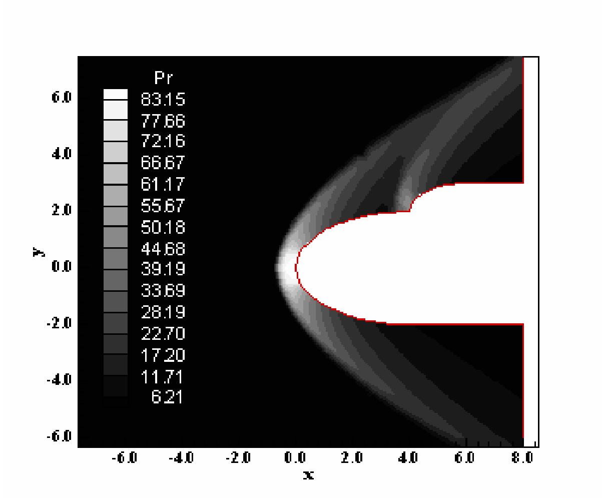

Figure 29. Density field (H/83).

To the physical problem of the double ellipse configuration, an algebraic mesh of 125x100 points was used, which is constituted of 12,276 rectangular volumes and 12,500 nodes. The freestream Mach number adopted in this simulation was 10.0, characterizing a hypersonic “cold gas” flow.

Figures 28 to 31 show the pressure field generated by the algorithms. The pressure field generated by the MacCormack (1969) scheme is again the most severe in comparison with the others schemes.

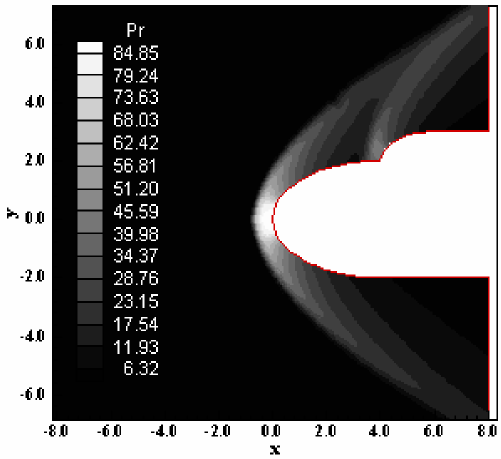

Figure 30. Pressure field (YK/85).

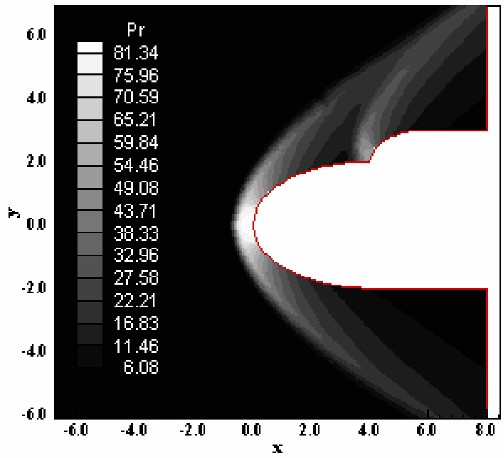

Figure 31. Pressure field (JM/86).

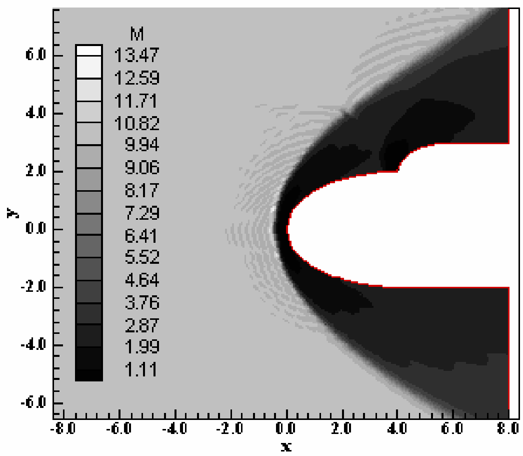

Figure 32. Mach number field (M/69).

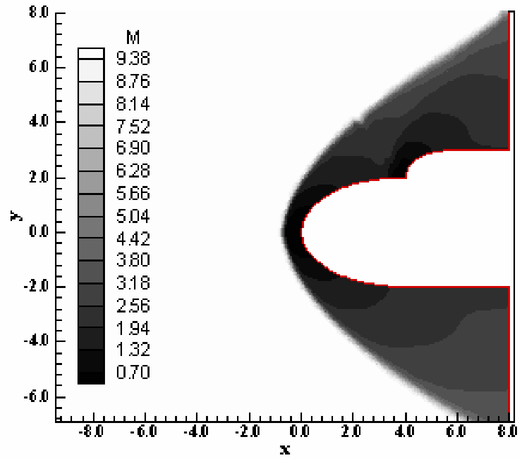

Figure 33. Mach number field (H/83).

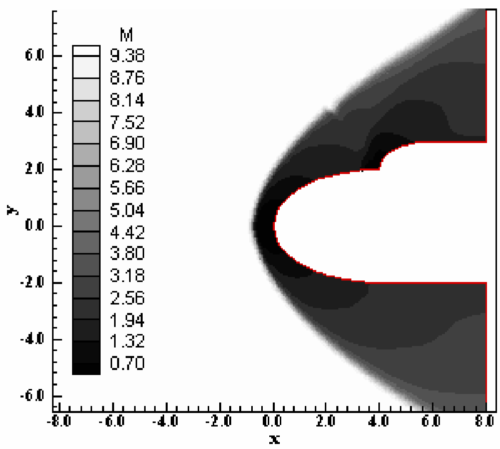

Figure 34. Mach number field (YK/85).

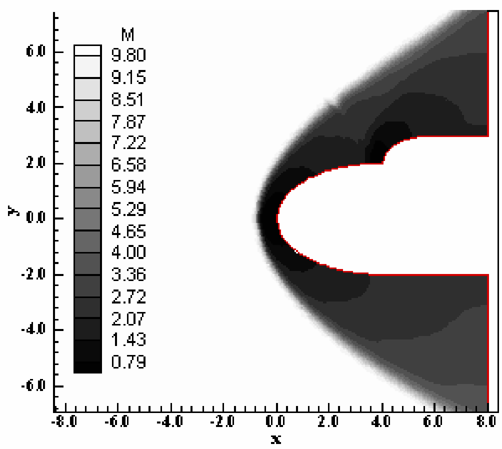

Figure 35. Mach number field (JM/86).

Figures 32 to 35 exhibit the Mach number field generated by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes, respectively. The Mach number field generated by the MacCormack (1969) scheme again presents pre-shock oscillations. It is important again to note that the Mach number peak obtained by the MacCormack (1969) scheme is bigger than the freestream value, which is physically impossible. It constitutes a limitation of the MacCormack (1969) scheme as treating supersonic and "cold gas" hypersonic flows. Considering relevant physical results, the Jameson and Mavriplis (1986) scheme presents the most intense Mach number field.

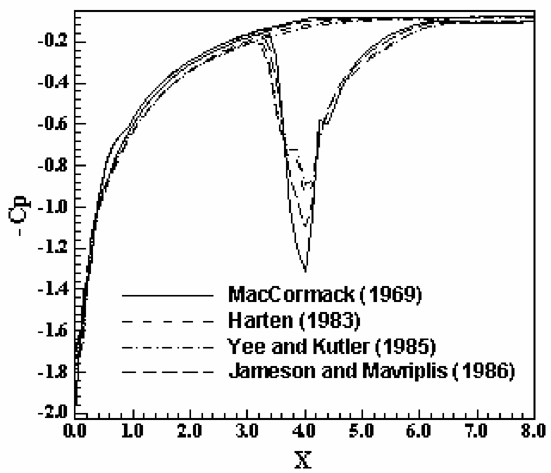

Figure 36. –Cp distributions.

Figure 36 shows the -Cp distributions around the double ellipse obtained by the MacCormack (1969), by the Harten (1983), by the Yee and Kutler (1985) and by the Jameson and Mavriplis (1986) schemes. The MacCormack (1969) scheme presents a Cp value of 2.04 at the first shock and a Cp value of 1.32 at the second shock. The Harten (1983) scheme presents a Cp value at the first shock of 1.74 and at the second shock of 0.92. The Yee and Kutler (1985) presents at the first shock a value of Cp of 1.78, while at the second shock its value is of 0.88. The Jameson and Mavriplis (1986) scheme presents a Cp value of 1.72 at the first shock and of 1.1 at the second shock. Hence, as the first shock is more intense than the second shock, the MacCormack (1969) scheme presents the most critical solution.

Another possibility to quantitative comparison of all schemes is the determination of the stagnation pressure ahead of the configuration. Anderson (1984) presents a table of normal shock wave properties in its B Appendix. This table permits the determination of some shock wave properties as function of the freestream Mach number. In front of the double ellipse configuration studied in this work, the shock wave presents a normal shock behavior, which permits the determination of the stagnation pressure, behind the shock wave, from the tables encountered in Anderson (1984). So it is possible to determine the ratio pr_0/pr_∞ from Anderson (1984), where pr_0 is the stagnation pressure in front of the configuration and pr_∞ is the freestream pressure (equals to $1/\gamma$ to this problem nondimensionalization).

Hence, to this problem, $M_\infty = 10.0$ corresponds to $pr_0/pr_\infty = 129.2$ and remembering that $pr_\infty = 0.714$, it is possible to conclude that $pr_0 = 92.25$. Values of the stagnation pressure and the respective percentage errors are described in Tab. 3. These results indicate that the MacCormack (1969) scheme yields the most accurate solution to the stagnation pressure among the studied algorithms in this work to this problem.

Table 3 – Stagnation pressure and percentage errors to the double ellipse problem.

Algorithm:	pr_0:	Error (%):
MacCormack (1969)	95.85	3.9
Harten (1983)	83.15	9.9
Yee and Kutler (1985)	84.85	8.0
Jameson and Mavriplis (1986)	81.34	11.8

2.5. Numerical data

Table 4 presents the numerical data of the simulations to all problems, involving the four schemes tested in this work. As can be seen, the Harten (1983) scheme (the most expensive) is approximately 495% more expensive than the Yee and Kutler (1985) scheme (the cheapest). In all problems, the Jameson and Mavriplis (1986) scheme always used a CFL number greater than the others schemes, resulting, hence, in its excellent convergence ratio. In the airfoil problem, the Jameson and Mavriplis (1986) scheme is approximately 1,058% faster than the MacCormack (1969) scheme.

Table 4 – Numerical data of the simulations.

	Nozzle		Airfoil		Ramp		Double Ellipse		
Algorithm:	CFL:	Iterations:	CFL:	Iterations:	CFL:	Iterations:	CFL:	Iterations:	Cost(1):
M (1969)	0.4	20,068	0.1	38,297	0.5	1,897	0.1	6,098	0.0000336
H (1983)	0.9	7,173	0.4	5,550	0.9	1,059	0.3	1,717	0.0000494
YK (1985)	0.8	8,239	0.3	5,930	0.3	3,348	0.4	1,457	0.0000083
JM (1986)	2.2	2,635	0.9	3,307	2.0	463	0.5	930	0.0000196

(1): measured in seconds/per volume/per iteration.

3. CONCLUSIONS

The present work is the second part of the study that aims a comparison among the predictor-corrector scheme of MacCormack (1969), the TVD flux difference splitting schemes of Harten (1983) and of Yee and Kutler (1985), and the symmetrical scheme of Jameson and Mavriplis (1986), all algorithms second order accurate in space, applied to aeronautical and aerospace problems in the two-dimensional space. The MacCormack (1969) scheme is integrated in time using a predictor/corrector method, the Harten (1983) and the Yee and Kutler (1985) schemes employ a time splitting method, and the Jameson and Mavriplis (1986) scheme uses a Runge-Kutta method of five stages. The Euler equations were solved using a finite volume formulation and a structured spatial discretization. A spatially variable time step was employed to accelerate the convergence process of the algorithms to the steady state solution. The physical problems of the transonic flows along a convergent-divergent nozzle and around a NACA 0012 airfoil, the supersonic flow along a ramp and the "cold gas" hypersonic flow around a double ellipse configuration were solved.

In the nozzle problem, the Harten (1983) scheme presented the most intense pressure field in relation to the others schemes. However, the MacCormack (1969) scheme predicted more accurately the pressure ratio of the shock, at the nozzle throat, as compared with the experimental results of Mason, Putnam and Re (1980). Hence, the MacCormack (1969) scheme predicted the most critical solution. In the airfoil problem, the Yee and Kutler (1985) scheme presented the most severe pressure field; Nevertheless, the MacCormack (1969) scheme detected the biggest value of Cp at the shock in comparison with the others schemes. Again, the MacCormack (1969) scheme predicted the most critical situation. The aerodynamic coefficient of lift was more accurately calculated by the Jameson and Mavriplis (1986) solution, which estimated a value closer to zero, the expected solution. In the ramp problem, the MacCormack (1969) scheme predicted the most severe pressure field. The pressure distribution along the ramp was best described by the Yee and Kutler (1985) scheme. All schemes detected a pressure peak at the ramp beginning, with exception of the Yee and Kutler (1985) scheme, which predicted the smoothest solution, as many in terms of shock resolution as in terms of expansion fan resolution. In comparison with the shock-expansion fan theory, the Yee and Kutler (1985) scheme presented the best solution. However, the Harten (1983) scheme predicted the best value of the shock angle of the oblique shock wave. In the double ellipse problem, the pressure field generated by the MacCormack (1969) scheme was the most critical in comparison with the others schemes. The biggest value of Cp at the first shock, the most severe, was estimated by the MacCormack (1969) scheme. Moreover, the stagnation pressure in front of the double ellipse configuration was best calculated by this scheme. The Harten (1983) scheme (the most expensive in this study) is approximately 495% more expensive than the Yee and Kutler (1985) scheme (the cheapest in this study).

It is possible to conclude that the MacCormack (1969) scheme presents the most severe, the most critical, pressure results in relation to the others schemes in all examples studied in this work. In terms of accuracy, MacCormack (1969) scheme was the most accurate in the nozzle and in the double ellipse problems, the Harten (1983) scheme was the most accurate in the ramp problem and the Jameson and Mavriplis (1986) scheme was the most accurate in the airfoil problem. A limitation of the MacCormack (1969) scheme is due to pre-shock oscillations and unphysical solutions that appear in supersonic and hypersonic "cold gas" flows. The implementation of an entropy condition could improve the results of the MacCormack (1969) scheme.

To initial project studies, the MacCormack (1969) scheme is recommended because of its simplicity in numerical implementation and more conservative results that can be obtained (more severe pressure fields). To more accurate results, the Harten (1983) and the Jameson and Mavriplis (1986) schemes are more recommended.

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5. RESPONSIBILITY NOTICE