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INVERSE PROBLEM OF SIMULTANEOUS IDENTIFICATION OF THERMOPHYSICAL PROPERTIES AND BOUNDARY HEAT FLUX

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Abstract. *In this paper we apply a Bayesian approach for the simultaneous identification of volumetric heat capacity, thermal conductivity and boundary heat flux, in a one-dimensional nonlinear heat conduction problem. It is assumed that the measurement noise and the unknown parameters are normally distributed and independent, so that the solution of the inverse problem is obtained via the minimization of the Maximum a Posteriori Objective Function. Results are presented for an experiment involving the heating of a sample material with an oxyacetylene torch.*

Keywords: *Inverse problem, Bayesian estimation, statistical inversion approach, oxyacetylene torch*

1. INTRODUCTION

The Brazilian Space Agency is dedicating efforts to build a satellite that will reenter the atmosphere after staying a couple of days in orbit. The Laboratory of Heat Transmission and Technology of COPPE is involved in the design of the heat shield for such satellite, which can be constituted of ablating and non-ablating materials. An important requirement for the thermal design of heat shields of vehicles re-entering the atmosphere, which are subjected to extremely high heat loads, is to have prior accurate information regarding the thermal properties of the materials utilized. Rey Silva and Orlande (2002) presented the solution of the inverse problem of parameter estimation used for the identification of the thermal properties of ablating materials. The D-optimum approach was used, together with the analysis of the sensitivity coefficients, for the design of the experiment for the estimation of thermal conductivity, volumetric heat capacity and heat of ablation of materials with negligible thermal decomposition. A combination of the Levenberg-Marquardt method (Beck and Arnold, 1977, Ozisik and Orlande, 2000) and of the sequential parameter estimation technique (Beck and Arnold, 1977) was used as the estimation procedure. Simulated temperature measurements taken inside the specimen were used for the inverse analysis in order to examine the accuracy of the proposed approach. Other works can be found in the literature related to the identification of the thermophysical properties of ablating materials (see, for example, Kanevce et al, 1999).

With respect to the heat loads that the surface of vehicles re-entering the atmosphere are subjected to, a usual approach for the design of the heat shield is to use simplified models (DeJarnette et al, 1987). On the other hand, the use of inverse analysis techniques can provide the important information on how accurate such simplified models are. Oliveira and Orlande (2002) examined the use of simulated temperature measurements taken below the surface for the solution of such inverse problem, by using the conjugate gradient method with adjoint problem (Alifanov, 1994, Ozisik and Orlande, 2000). Accurate results were obtained for functions containing sharp corners and discontinuities, but only for small magnitudes of the peak-flux, when the portion of material removed during ablation was small. In (Oliveira and Orlande, 2004) we examined the use of surface position measurements, in addition to temperature measurements, for the estimation of the boundary heat flux at the surface of ablating materials. Surface position measurements are highly sensitive to variations on the unknown heat flux during ablation, when the temperature measurements become less sensitive due to the energy consumption at the ablating surface. The use of simulated surface position measurements containing random errors resulted on unstable inverse problem solutions when the discrepancy principle was used to stop the iterative procedure of the conjugate gradient method. However, in these cases the *a priori* filtering of the measured data provided accurate and stable solutions for the inverse problem.

More recently, (Mota et al, 2004) used inverse analysis techniques of function estimation in order to identify the heat flux at the surface of a reference material with known thermophysical properties. The experimental setup consisted of an oxyacetylene torch that was used to heat the specimen with unknown thermophysical properties. Such experimental setup was based on that described in the ASTM standard E-285 (1996). A high-quality graphite sample was used as the reference material for the identification of the imposed heat flux. The type of graphite utilized in this work was the same used in the aerospace industry, like in rocket nozzles. The thermophysical properties of such reference material were available from its manufacture for a wide temperature range, but the thermal conductivity, specific heat and thermal diffusivity of the sample, were also measured in a Flash-method apparatus based on the ASTM standard 1461 (2001). For the solution of the inverse problem, the temperature dependence of the thermophysical properties was taken into account in the form of functions selected to fit the experimental results, while temperature measurements taken below the surface were used for the identification of the imposed heat flux. In order to solve the inverse problem, a function estimation approach based on the conjugate gradient method with adjoint problem (Mota et al, 2004) was used.

One major limitation of the technique used in (Mota et al, 2004) is that the thermophysical properties of the sample were assumed to be deterministic and known parameters. On the other hand, the use of the *statistical inversion approach* advanced in (Kaipio and Somersalo, 2004) permit that not only the mean values of the thermophysical properties, but also the uncertainties associated with them, be taken into account for the solution of the inverse problem. The statistical inversion approach is based on the following principles (Kaipio and Somersalo, 2004): 1. All variables included in the model are modeled as random variables; 2. The randomness describes the degree of information concerning their realizations; 3. The degree of information concerning these values is coded in the probability distributions; and 4. The solution of the inverse problem is the *posterior probability distribution*. Therefore, this approach relies fundamentally on the principles of the Bayesian statistics to obtain the solution of inverse problems (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004, Beck and Arnold, 1977, Winkler, 2003).

We present below the application of the statistical inversion approach to the solution of the inverse problem dealing with the estimation of the thermophysical properties and a boundary heat flux, in a one-dimensional heat conduction problem. The *a priori* information available for the parameters is assumed to be in the form of normal distributions with known means and known covariance matrices. A similar hypothesis is assumed valid for the measurement errors, so that the solution of the inverse problem can be obtained in terms of an optimization problem involving the Maximum a Posteriori Objective Function (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004, Beck and Arnold, 1977). Results are presented for an experiment dealing with the heating of a sample material with an oxyacetylene torch (Mota et al, 2004), as described below.

2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem considered in this work involves the heating of a slab of thickness L, with an imposed heat flux $q(t)$ at the boundary $x=L$, as illustrated in figure 1. The slab is initially at the uniform temperature T_{ini}, while the non-heated boundary temperature is $T_0(t)$. The slab is supposed to be made of a homogeneous, isotropic and thermally stable material, but its thermophysical properties may vary with temperature.

The mathematical formulation of such physical problem is given by:

$$C(T)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] \quad \text{in} \quad 0 < x < L,\ t > 0 \tag{1.a}$$

$$T = T_0(t) \quad \text{at} \quad x = 0, \quad t > 0 \tag{1.b}$$

$$k(T)\frac{\partial T}{\partial x} = q(t) \quad \text{at} \quad x = L, \quad t > 0 \tag{1.c}$$

$$T = T_{ini} \quad \text{for} \quad t = 0, \text{ in } 0 < x < L \tag{1.d}$$

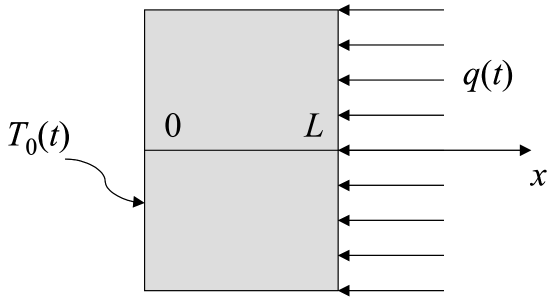

Figure 1. Physical problem

For the mathematical formulation of the physical problem given by Eqs. (1.a-d), we can devise a *direct problem* for which the thermophysical properties, initial and boundary conditions are known. The objective of the direct problem is to determine the transient temperature field $T(x,t)$ within the slab. However, we can also devise an *inverse problem* in which the imposed boundary heat flux $q(t)$, the volumetric heat capacity $C(T)$ and the thermal conductivity $k(T)$ are unknown, but transient temperature measurements taken at S selected positions within the slab are available.

The vector containing the measured temperatures is written as:

$$\mathbf{Y}^T = \left(\vec{Y}_1\ ,\ \vec{Y}_2\ , \ldots , \vec{Y}_I\right) \tag{2.a}$$

where $\vec{Y}_i$ contains the measured temperatures for each of the S sensors at time t_i, $i = 1, \ldots, I$, that is,

$$\vec{Y}_i = \left(Y_{i1}\ ,\ Y_{i2}\ ,\ldots ,Y_{iS}\right) \qquad \text{for } i=1,\ldots,I \tag{2.b}$$

Inverse problems are mathematically classified as *ill-posed*, whereas standard heat transfer problems are *well-posed* (Alifanov, 1994, Ozisik and Orlande, 2000, Kaipio and Somersalo, 2004, Tan et al, 2006). The solution of a well-posed problem must satisfy the conditions of existence, uniqueness and stability with respect to the input data. The existence of a solution for an inverse problem such as the one under picture in this work may be assured by physical reasoning. On the other hand, the uniqueness of the solution of inverse problems can be mathematically proved only for some special cases. Also, the inverse problem is very sensitive to random errors in the measured input data, thus requiring special techniques for its solution in order to satisfy the stability condition, as discussed next.

3. BAYESIAN TECHNIQUE FOR THE SOLUTION OF INVERSE PROBLEMS

A variety of techniques is nowadays available for the solution of inverse problems. However, one common approach relies on the minimization of an objective function that generally involves the squared differences between measured and estimated variables, like the least-squares norm, as well as some kind of regularization (stabilization) term. Despite the fact that the minimization of the least-squares norm is indiscriminately used, it only yields *maximum likelihood* estimates if the following statistical hypotheses are valid (Beck and Arnold, 1977, Kaipio and Somersalo, 2004): the errors in the measured variables are additive, uncorrelated, normally distributed, with zero mean and known constant standard-deviation; only the measured variables appearing in the objective function contain errors; and there is no prior information regarding the values and uncertainties of the unknown parameters.

Although very popular and useful in many situations, the minimization of the least-squares norm is a non-Bayesian estimator. A Bayesian estimator is basically concerned with the analysis of the *posterior probability density*, which is the conditional probability of the parameters given the measurements, while the likelihood is the conditional probability of the measurements given the parameters (Kaipio and Somersalo, 2004, Tan et al, 2006). In the Bayesian approach to statistics, an attempt is made to utilize all available information in order to reduce the amount of uncertainty present in an inferential or decision-making problem. As new information is obtained, it is combined with any previous information to form the basis for statistical procedures. The formal mechanism used to combine the new information with the previously available information is known as Bayes' theorem (Winkler, 2003). Therefore, the term *Bayesian* is often used to describe the so-called *statistical inversion approach*, which is based on the principles enumerated above in the Introduction of this paper.

Consider the vector of parameters appearing in the physical model formulation as

$$\mathbf{P}^T \equiv [P_1,P_2,\ldots,P_N] \tag{3}$$

where N is the number of parameters. Bayes' theorem can then be stated as (Beck and Arnold, 1977, Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004, Winkler, 2003):

$$\pi_{posterior}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi_{prior}(\mathbf{P})\pi(\mathbf{Y}|\mathbf{P})}{\pi(\mathbf{Y})} \tag{4}$$

where $\pi_{posterior}(\mathbf{P})$ is the posterior probability density, that is, the conditional probability of the parameters $\mathbf{P}$ given the measurements $\mathbf{Y}$; $\pi_{prior}(\mathbf{P})$ is the prior density, that is, the coded information about the parameters prior to the measurements; $\pi(\mathbf{Y}|\mathbf{P})$ is the likelihood function, which expresses the likelihood of different measurement outcomes $\mathbf{Y}$ with $\mathbf{P}$ given; and $\pi(\mathbf{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant.

4. MAXIMUM A POSTERIORI OBJECTIVE FUNCTION

If we assume the parameters and the measurement errors to be independent Gaussian random variables, with known means and covariance matrices, and that the measurement errors are additive, a closed form expression can be derived for the posterior probability density. In this case, the likelihood function can be expressed as (Kaipio and Somersalo, 2004, Beck and Arnold, 1977):

$$\pi(\mathbf{Y}|\mathbf{P}) = (2\pi)^{-M/2}\left|\mathbf{W}^{-1}\right|^{-1/2}\exp\left[-\frac{1}{2}(\mathbf{Y}-\mathbf{T})^T\mathbf{W}(\mathbf{Y}-\mathbf{T})\right] \tag{5}$$

where $M = SI$ is the number of measurements and **W** is the inverse of the covariance matrix of the measurement errors.

For uncorrelated measurements:

$$\mathbf{W} = \begin{bmatrix} 1/\sigma_1^2 & & & 0 \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_M^2 \end{bmatrix} \tag{6}$$

where σ_i is the standard-deviation of the measurement Y_i, $i = 1, \ldots, M$.

Similarly, for the case involving a prior normal distribution for the parameters we can write:

$$\pi(\mathbf{P}) = (2\pi)^{-N/2} \left|\mathbf{V}\right|^{-1/2} \exp\left[-\frac{1}{2}(\mathbf{P}-\boldsymbol{\mu})^T \mathbf{V}^{-1}(\mathbf{P}-\boldsymbol{\mu})\right] \tag{7}$$

where $\boldsymbol{\mu}$ and **V** are the known mean and covariance matrix for **P**, respectively.

By substituting equations (5) and (7) into Bayes' theorem, except for the normalizing constant in the denominator we obtain:

$$\ln\left[\pi(\mathbf{P}\,|\,\mathbf{Y})\right] \propto -\frac{1}{2}\left[(M+N)\ln 2\pi + \ln|\mathbf{W}^{-1}| + \ln|\mathbf{V}| + S_{MAP}(\mathbf{P})\right] \tag{8}$$

where

$$S_{MAP}(\mathbf{P}) = \left[\mathbf{Y}-\mathbf{T}(\mathbf{P})\right]^T \mathbf{W}\left[\mathbf{Y}-\mathbf{T}(\mathbf{P})\right] + (\boldsymbol{\mu}-\mathbf{P})^T \mathbf{V}^{-1}(\boldsymbol{\mu}-\mathbf{P}) \tag{9}$$

Equation (8) reveals that the maximization of the posterior distribution function can obtained with the minimization of the objective function given by equation (9), denoted as the *maximum a posteriori objective function* (Kaipio and Somersalo, 2004, Beck and Arnold, 1977). Equation (9) clearly shows the contributions of the likelihood and of the prior distributions in the objective function, given by the first and second terms on the right-hand side, respectively.

For nonlinear estimation problems, where the sensitivity matrix is a function of the unknown parameters, the iterative procedure of the Gauss-Newton method for the minimization of the maximum a posteriori objective function is given by (Beck and Arnold, 1977):

$$\mathbf{P}^{k+1} = \mathbf{P}^k + [\mathbf{J}^T\mathbf{W}\mathbf{J} + \mathbf{V}^{-1}]^{-1}\{\mathbf{J}^T\mathbf{W}[\mathbf{Y}-\mathbf{T}(\mathbf{P}^k)] + \mathbf{V}^{-1}(\boldsymbol{\mu}-\mathbf{P}^k)\} \tag{10}$$

where the superscript k denotes the number of iterations and **J** is the sensitivity matrix defined by:

$$\mathbf{J}(\mathbf{P}) \equiv \left[\frac{\partial \mathbf{T}^T(\mathbf{P})}{\partial \mathbf{P}}\right]^T = \begin{bmatrix} \frac{\partial \vec{T}_1^T}{\partial P_1} & \frac{\partial \vec{T}_1^T}{\partial P_2} & \frac{\partial \vec{T}_1^T}{\partial P_3} & \cdots & \frac{\partial \vec{T}_1^T}{\partial P_N} \\ \frac{\partial \vec{T}_2^T}{\partial P_1} & \frac{\partial \vec{T}_2^T}{\partial P_2} & \frac{\partial \vec{T}_2^T}{\partial P_3} & \cdots & \frac{\partial \vec{T}_2^T}{\partial P_N} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial \vec{T}_I^T}{\partial P_1} & \frac{\partial \vec{T}_I^T}{\partial P_2} & \frac{\partial \vec{T}_I^T}{\partial P_3} & \cdots & \frac{\partial \vec{T}_I^T}{\partial P_N} \end{bmatrix} \tag{11}$$

We note in equation (10) that the inverse of the covariance matrix for the parameters, $\mathbf{V}^{-1}$, has a regularization effect mathematically analogous to that obtained with Tikhonov's regularization. While traditional regularization techniques produce smooth solutions by removing the ill-posed character of the inverse problem with empirical techniques, the statistical inversion approach permits that prior information be used for the sake of regularization of the inverse problem solution (Kaipio and Somersalo, 2004). On the other hand, the minimization of the maximum a posteriori objective function results on biased estimates (Beck and Arnold, 1977), which clearly shows the importance of selecting the appropriate prior distributions for the parameters. If the prior probability densities for the parameters cannot be assumed as normal distributions, the posterior probability distribution does not allow an analytical treatment such as presented above. In this case, Markov Chain Monte Carlo (MCMC) methods are used to draw samples of all possible parameters, so that inference on the posterior probability becomes inference on the samples (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004).

5. EXPERIMENTAL SETUP

The experimental setup used in this work is based on that specified in the ASTM standard E285-80 (1996). This standard specifies the experimental procedure for a qualitative comparison of ablating materials used as heat shields. The setup makes use of an oxyacetylene torch (Soldox-201) to heat a cylindrical specimen (20 mm of diameter and 20 mm of length). The torch type and the specimen diameter were selected so that the heat flux imposed on the specimen was practically uniform, in order to model the heat conduction through the specimen as one-dimensional, as described in section 2 above (Mota et al, 2004).

The temperature measurements were performed with thermocouples type K, with 30 gauge (0.25 mm diameter) wires. The thermocouples were calibrated against the readings of a standard PT-100 in a dry TECAL 650S bath. The maximum standard deviation of the temperature measurements is 1 ^{o}C, from ambient temperature to 600 ^{o}C. During the experiments, temperatures were recorded with an AGILENT 34970A data acquisition system with a frequency of 1 measurement per second. The specimen was instrumented with three thermocouples located at the depths of 2mm, 5mm and 10 mm below the heated surface and with another thermocouple at the non-heated surface, as illustrated in figure 2.

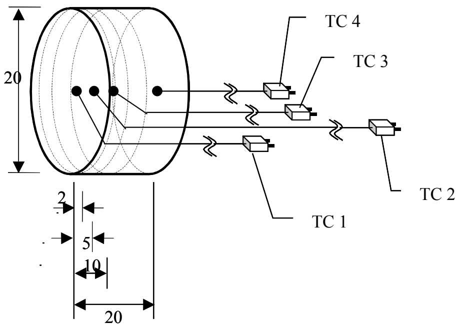

Figure 2. Sensors' locations

A high quality graphite was selected as the material for the specimen because of its thermal stability at high temperatures. Experimental data were available from the manufacturer of this material for its thermophysical properties at temperatures above 400 ^{o}C. From room temperature up to 200 ^{o}C, the thermal diffusivity and the specific heat of such material was determined with the Flash method, in the Laboratory of Heat Transmission and Technology of COPPE (ASTM, 2001). Figure 3 shows the thermal conductivity and thermal diffusivity for the sample material at different temperatures. This figure also shows functions used to fit the thermophysical properties (continuous lines), which were taken as exponentials in the form:

$$C(T) = A_1 + A_2 e^{-T/A_3} \tag{12.a}$$

$$k(T) = B_1 + B_2 e^{-T/B_3} \tag{12.b}$$

with the adjusted parameters given by table 1. Parameters A_1, A_2, A_3, B_1, B_2 and B_3 were supposed normally distributed with mean and standard-deviation values given by table 1.

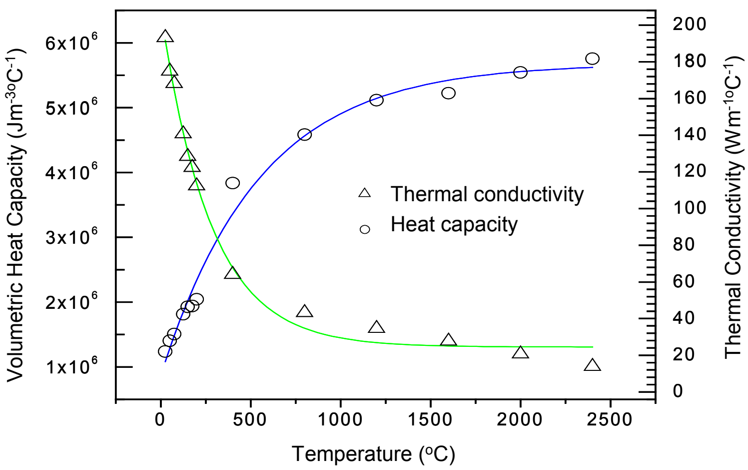

Figure 3. Thermophysical properties

Table 1. Parameters of the functions fitted for the thermal properties

Parameter	Mean	Standard-deviation
A_1 (Jm^{-3} ºC^{-1})	5,681,006	63,378
A_2 (Jm^{-3} ºC^{-1})	−4,813,057	66,750
A_3 (ºC)	547.00	71.42
B_1 (Wm^{-1} ºC^{-1})	24.52	1.08
B_2 (Wm^{-1} ºC^{-1})	183.05	2.02
B_3 (ºC)	277.00	7.85

During assembly, the specimen was inserted into its holder, which is made of a refractory brick. The torch and the holder were assembled on a structure that permits an accurate centralization of the torch with respect to the specimen axis. Such structure also permits the control of the distance between the torch and the specimen. The experiments were conducted with oxygen and acetylene static pressures of 1.2 bar and 1.1 bar, respectively, and flow rates of 20 scfh for oxygen and for acetylene. The torch was located at a specified distance from the specimen and then lighted. During the period that the torch flame was adjusted, the direct heating of the specimen was avoided by a shield made of a refractory brick and a stainless-steel plate. The steel plate was used as the radiation shield between the brick and the specimen. The exact instant when the shield was removed, and the torch heated the specimen directly, was carefully recorded. Figure 4 depicts one of the experiments performed.

Figure 4. Experiment

6. RESULTS AND DISCUSSIONS

We now present the results obtained for the simultaneous estimation of the heat flux imposed by the oxyacetylene torch, as well as of the volumetric heat capacity and the thermal conductivity of the graphite specimen. For the estimation of such quantities, the heat flux was parameterized in the form

$$q(t) = \sum_{i=1}^{I} q_i \, \delta(t - t_i) \tag{13}$$

where I is the number of transient measurements and $\delta(t - t_i)$ is the Dirac delta function. The volumetric heat capacity and the thermal conductivity were parameterized in the form given by equations (12.a,b), respectively. Therefore, the vector of unknown parameters is given by

$$\mathbf{P}^T = [q_1, q_2, \ldots, q_I, A_1, A_2, A_3, B_1, B_2, B_3] \tag{14}$$

For the solution of the inverse problem, the prior information for the parameters was assumed to be in the form of normal distributions. In addition, the parameters were supposed uncorrelated. Mean and standard-deviation values for the heat flux parameters, $q_1, q_2, \ldots, q_I$, were taken from previous estimations obtained by the conjugate gradient method with adjoint problem (Mota et al, 2004). Meanwhile, the values shown in table 1 were used for the mean and standard-deviation values for the parameters in the functions describing the variation of the thermophysical properties with temperature, that is, A_1, A_2, A_3, B_1, B_2 and B_3.

Figure 5 presents the temperature measurements obtained with the sensor located 2 mm below the heated surface, for three different experiments and their repetitions. The different experiments involve distances of 200 mm, 150 mm and 100 mm between the torch and the specimen. This figure clearly shows a larger rate of temperature increase for smaller distances, caused by larger heat fluxes imposed by the torch. In addition, figure 5 shows a quite good repeatability of the experimental conditions; except for the experiment with the distance of 150 mm between the torch and the specimen, the temperatures obtained with the two experimental runs are practically identical within the graphic scale.

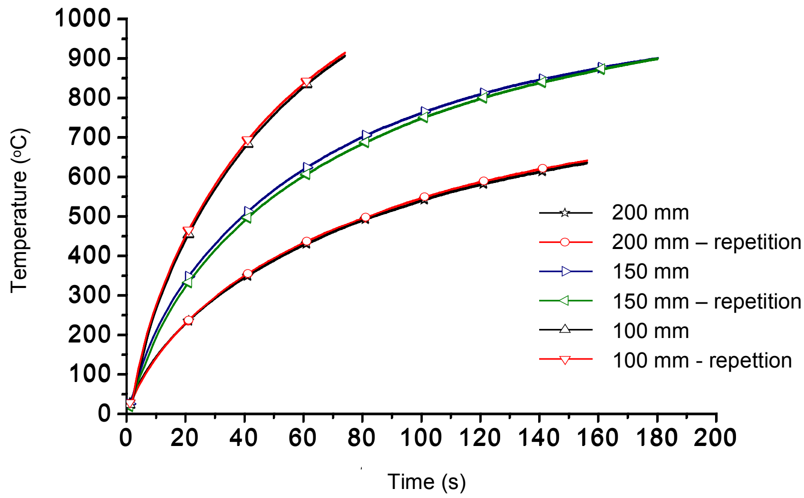

Figure 5. Temperatures measured by the sensor 2 mm below the heated surface on three experiments and their repetitions

The temperature measurements obtained with the sensors 2 mm and 5 mm below the heated surface were used for the inverse analysis, while the measurements of the sensor located 10 mm below the heated surface were used as boundary condition in equation (1.b).

Estimations obtained for the volumetric heat capacity, thermal conductivity and boundary heat flux, for a distance of 200 mm between the torch and the specimen, are presented in figures 6.a-c. The 99% confidence intervals for each of these quantities are also presented in figures 6.a-c. We note in figures 6.a,b that the experimental points used to obtain the prior information for the volumetric heat capacity and thermal conductivity (see figure 3) generally fell within the 99% confidence intervals for these properties. However, discrepancies between the prior information and the estimated properties can be attributed to thermal degradation of the graphite as it was heated. The estimated heat flux presented in figure 6.c was quite stable and shows a reduction as time increased, due to the larger heat losses by radiation as the specimen was heated. The residuals obtained in this case are shown in figure 7, for the thermocouples located 2mm (sensor 1) and 5 mm (sensor 2) below the heated surface. Except for times near the begin and end of the specimen heating, when the heat flux function underwent a step change, the residuals were of the order of 1 ^{o}C, that is, very small as compared to the magnitude of the measurements. The correlation observed in the residuals can be resultant from the effects of the prior information on the estimation procedure, which is apparent from the analysis of the maximum a posteriori objective function given by equation (9).

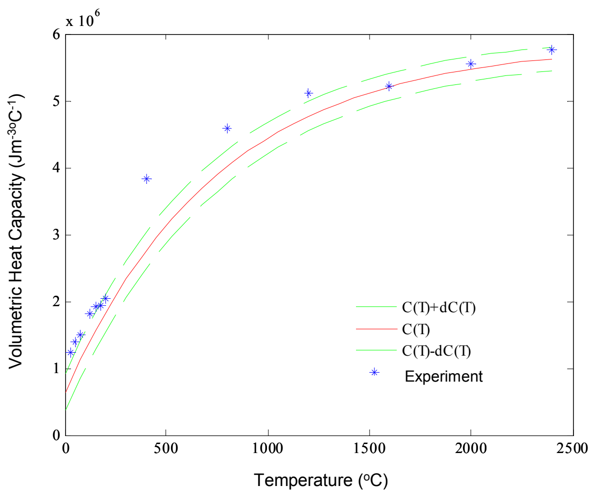

Figure 6.a. Estimated volumetric heat capacity for a distance of 200 mm between the torch and the specimen

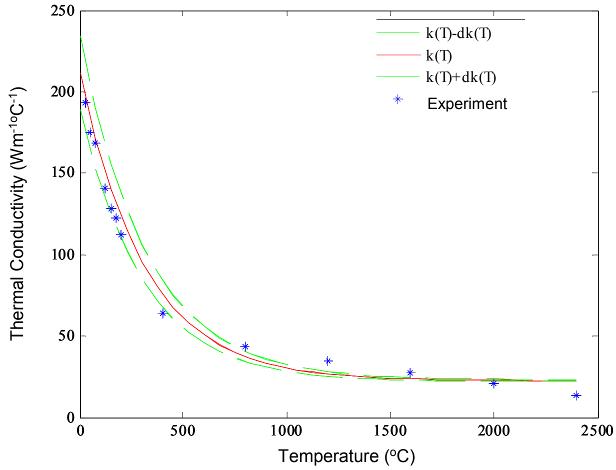

Figure 6.b. Estimated thermal conductivity for a distance of 200 mm between the torch and the specimen

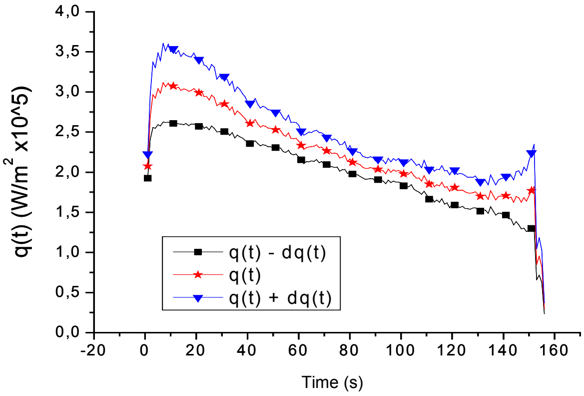

Figure 6.c. Estimated heat flux for a distance of 200 mm between the torch and the specimen

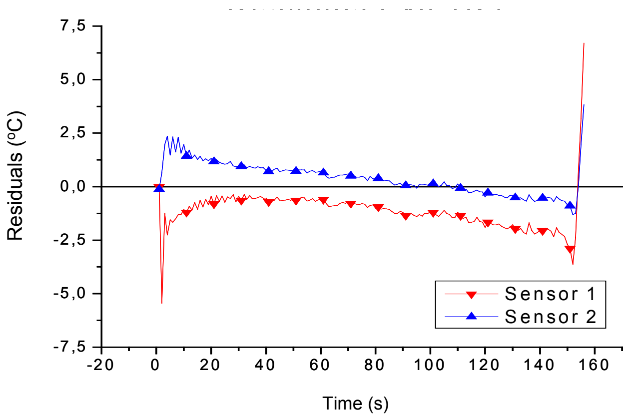

Figure 7. Residuals for a distance of 200 mm between the torch and the specimen

Figure 8 presents the heat flux estimated for a distance of 100 mm between the torch and the specimen. A comparison of figures 6.c and 8 reveals a large increase in the magnitude of the heat flux as such distance is reduced from 200 mm to 100 mm. The larger heat flux results on a larger rate of temperature increase, as shown by figure 5. The estimated volumetric heat capacity and thermal conductivity for this case are identical to those estimated with the distance of 200 mm between the torch and the sample (see figures 6.a,b). This result reveals the robustness of the statistical inversion methodology utilized in this paper. The residuals for the case involving the distance of 100 mm between the torch and the specimen are also of the order of 1 ^{o}C, except in the neighborhood of the begin and end of the heating.

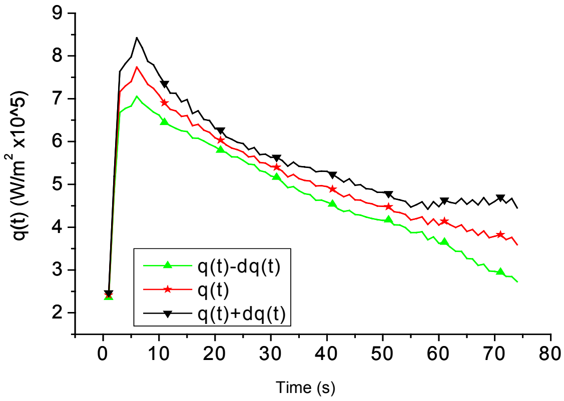

Figure 8. Estimated heat flux for a distance of 100 mm between the torch and the specimen

7. CONCLUSIONS

In this paper we solved the inverse problem of simultaneously estimating the applied heat flux and the thermal properties of a material, in a one-dimensional nonlinear heat conduction problem. The inverse problem was solved with the statistical inversion approach, based on the minimization of the maximum a posteriori objective function. Prior information for the unknown parameters was taken in the form of normal distributions.

Actual experimental data, obtained from the heating of a graphite cylindrical specimen with an oxyacetylene torch, were used in the inverse analysis. The present solution approach was stable, robust and resulted on estimates with very small residuals. Although the magnitude of the heat flux was very sensitive to the distance between the torch and the specimen, the estimated properties were not affected by such quantity.

8. ACKNOWLEDGEMENTS

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