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INFLUENCE OF THE LOAD MODELLING AND HUMAN HEEL ON THE DYNAMICAL RESPONSE OF FOOTBRIDGES

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Abstract. *Considering the demands imposed by daring architectural projects pedestrian footbridges have been designed and constructed with light and large span structures. This fact have generated very slender structural footbridges and consequently changed the serviceability and ultimate limit states associated to their design. A direct consequence of this design trend is a considerable increase of structural vibration associated problems. In the particular case of pedestrian footbridges this phenomenon precisely occurs when the structural fundamental frequency is equal or near the existing loading frequencies, or even equal to some of its multiples. The present investigation is carried out based on a more realistic loading model developed in order to incorporate the dynamical effects induced by people walking when the dynamical response of pedestrian footbridges is investigated. In this particular loading model the movement of legs that cause an ascent and descending of the effective mass of the human body in each passing was considered and the position of the dynamical loading is changed according with the individual position and the generated time function, corresponding to the excitation induced by people walking, has a space and time description. The investigated structural model was based on several footbridges, with main spans varying from 10m to 35m. The structural system, used for pedestrian crossing, is composed of a composite (steel/concrete) solution made of an "I" steel beam section and a reinforced concrete deck. The proposed computational model, developed for the composite slab dynamic analysis, adopted the usual mesh refinement techniques present in finite element method simulations. A parametric study was developed based on the use of different loading models. The footbridges dynamical response was investigated and the peak accelerations are obtained and compared with results supplied by design standard objectifying human comfort evaluations.*

Keywords*: vibrations, structural dynamics, human comfort, composite footbridges, computational modelling.*

1. INTRODUCTION

Footbridges have been constructed with daring structures that consider the experience and knowledge of structural engineers by using newly developed materials and technologies boosted by the ever-growing investigations on this field. As the main objective of the structural engineer is related with the design of slight structures whose conception requires a substantial amount of theoretical knowledge associated to structural design and construction processes.

This fact have generated very slender structural footbridges and consequently changed the serviceability and ultimate limit states associated to their design (Bachmann and Ammann, 1987), (Ellis, 2000), (Figueiredo, 2005), (Lima, 2007), (Murray *et al*, 1997), (Pimentel *et al*, 2001), (Varela, 2004).

A direct consequence of this design trend is a considerable increase of structural vibration (Bachmann and Ammann, 1987), (Ellis, 2000), (Figueiredo, 2005), (Lima, 2007), (Murray *et al*, 1997), (Pimentel *et al*, 2001), (Varela, 2004). In the particular case of footbridges this phenomenon precisely occurs when the structural fundamental frequency is equal or near of the existing loading frequencies, or even similar to some of its multiples. Another important aspect that still deserves further investigation is related to the modelling of the harmonic dynamical loads induced by pedestrians walking on the footbridges.

The present investigation is carried out based on a more realistic loading model developed in order to incorporate the dynamical effects induced by people walking when the dynamical response of footbridges is investigated. In this particular loading model, the motion of legs that cause an ascent and descending of the effective mass of the human body in each passing was considered. The position of the dynamical loading is also changed according to the individual position and the generated time function, corresponding to the excitation induced by people walking, having a space and time description.

The investigated structural model was based on several footbridges, with main spans varying from 10m to 35m. The structural system, used for pedestrian crossing, is composed of a composite (steel/concrete) solution made of an "I" steel profile and a reinforced concrete slab. The proposed computational model, developed for the composite slab dynamic analysis, adopted the usual mesh refinement techniques present in finite element method simulations (ANSYS, 1998).

In this paper, the developed analysis methodology is described and discussed. Based on an extensive parametric study, the footbridges dynamic response in terms of peak accelerations was obtained and compared to the limiting values proposed by several authors and design codes (Murray *et al*., 1997), (ISO 2631-2, 1989) in order to provide a more realistic evaluation for vibration due to walking in this type of structure.

In specific design situations, the obtained results have shown that the code recommendations could supply values against the safety based on the adoption of excessively simplified load models (Murray *et al*., 1997), (Figueiredo, 2005), (Lima, 2007). Hence, it was detected that this type of structure can reach high vibration levels, compromising the footbridge user's comfort and especially its safety.

2. MODELLING OF THE DYNAMIC ACTIONS INDUCED BY PEOPLE WALKNG

In this paper a loading model were developed in order to incorporate the dynamical effects induced by people walking when the dynamical response of pedestrian footbridges is investigated. The mathematical model behind this strategy was proposed by Varela (2004) as well as a numerical approach to evaluate the floor structure reaction, as illustrated in Figure 1.

It must be emphasized that the geometry of the human body walking is an organized motion of legs that cause an ascent and descending of the effective mass of the body in each passing. The accelerations of the human body mass are associated with floor reactions, and it is approximately periodic in the frequency of the step. The two feet produce this type of loading, as function of the static parcel associated with the weight of the individual and three or four harmonic components of the loading. These harmonic appear due to interaction between the increasing load represented by a foot and the simultaneous unloading of the other foot.

In this particular model the position of the dynamical loading is changed according with the individual position, and the generated time function has a space and time description. In this modelling, the motion of legs that cause an ascent and descending of the effective mass of the human body in each passing was considered. However, it is necessary the study of several other parameters in this type of modelling like the step distance and step frequency (Bachmann and Ammann, 1987), as illustrated in Table 1.

Table 1. Characteristics of the human walking.

Activity	Velocity (m/s)	Step Distance (m)	Step Frequency (Hz)
Slow Walking	1.1	0.6	1.7
Normal Walking	1.5	0.75	2.0
Fast Walking	2.2	1.0	2.3

The pedestrian motion on the footbridge was modelled based on the Equation (1) to (4) and four harmonics were used to generate the dynamical forces, see Table 2. Like in the previous model, the third harmonic with step frequency of 1.79Hz, as shown in Table 2, was the resonant harmonic of the walking load (3 x 1.79Hz = 5.37Hz), see Figure 2. In this situation, the finite element mesh has to be very refined and the contact time of application of the dynamical load with the structure depends of the step distance and step frequency, see Table 2.

According to Varela (2004), the proposed mathematical function, Equations (1) to (4), used to represent the dynamical actions produced by people walking on floor slabs is not a Fourier series simply because the equation also incorporates in its formulation the effect of the heel impact.

This load model considers a space and temporal variation of the dynamic action over the structure that is evaluated considering four harmonics, see Table 2. Additionally, also incorporates the transient effect due to the human heel impact. The present investigation used a heel impact factor equal to 1.12 ($f_{mi} = 1.12$) (Varela, 2004). However, it must be emphasized that this value can vary substantially from person-to-person.

Figure 1 presents the transient peak representative of the human heel impact over the floor (Varela, 2004). An increase of the dynamic load due to the human heel impact could be observed when compared to an analysis that did not consider this effect. In sequence, Figure 2 illustrates the dynamical load function for an individual walking at 5.37Hz (3 x 1.79Hz = 5.37Hz), based on Equations (1) to (4) and Tables 1 and 2.

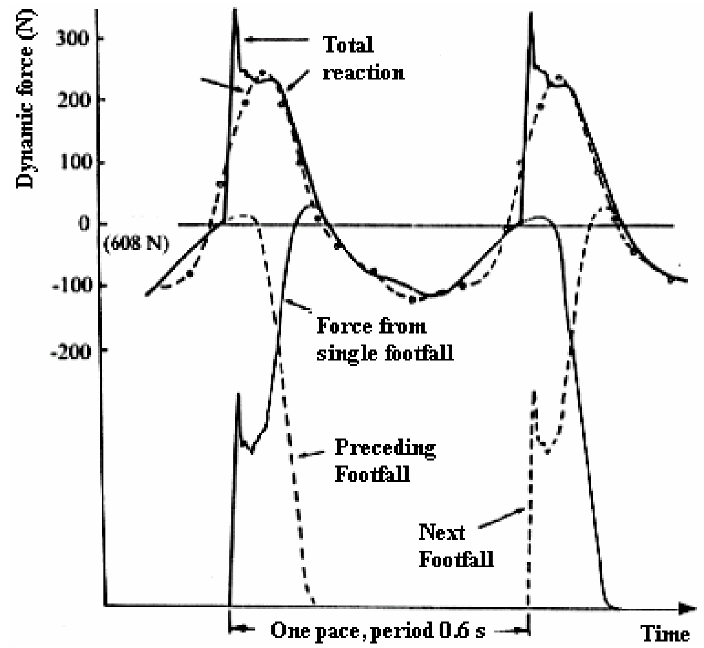


Figure 1. Footfall force and reaction on the floor structure (Varela, 2004).

$$F(t)=\begin{cases} \left(\dfrac{f_{mi}\,F_m-P}{0{,}04\,T_p}\right)t+P & \text{if } 0\ \le t<\ 0{,}04\,T_p \\[2ex] f_{mi}\,F_m\left[\dfrac{C_1\left(t-0{,}04\,T_p\right)}{0{,}02\,T_p}+1\right] & \text{if } 0{,}04\,T_p\ \le t<\ 0{,}06\,T_p \\[2ex] F_m & \text{if } 0{,}06\,T_p\ \le t<\ 0{,}15\,T_p \\[2ex] P+\sum\limits_{i=1}^{nh}P\,\alpha\,\mathrm{sen}\left[2\,\pi\,i\,f_c\left(t+0{,}1\,T_p\right)+\phi_i\right] & \text{if } 0{,}15\,T_p\ \le t<\ 0{,}90\,T_p \\[2ex] 10\left(P-C_2\right)\left(\dfrac{t}{T_p}-1\right)+P & \text{if } 0{,}90\,T_p\ \le t<\ T_p \end{cases} \tag{1}$$

Where:

F_m : maximum value of the Fourier series, given by Equation (4);
f_{mi} : factor of heel-impact;
T_p : step period;
C_1 : coefficients given by Equation (5);
C_2 : coefficients given by Equation (6).

$$F_m=P\left(1+\sum_{i=1}^{nh}\alpha_i\right) \tag{2}$$

$$C_1=\left(\frac{1}{f_{mi}}-1\right) \tag{3}$$

$$C_2=\begin{cases} P\left(1-\alpha_2\right) & \text{if } nh=3 \\ P\left(1-\alpha_2+\alpha_4\right) & \text{if } nh=4 \end{cases} \tag{4}$$

Table 2. Forcing frequencies (f_s), dynamic coefficients (α_i) and phase angles (Φ_i).

Harmonic i	Person Walking		
	f_s (Hz)	α_i	Φ_i
1	1.6 - 2.2	0.5	0
2	3.2 - 4.4	0.2	$\pi/2$
3	4.8 - 6.6	0.1	π
4	6.4 - 8.8	0.05	$3\pi/2$

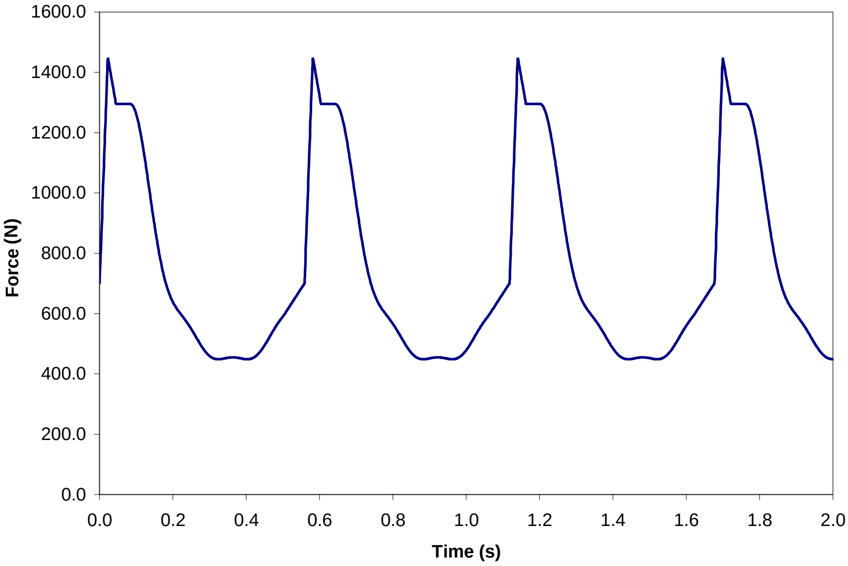


Figure 2. Dynamic loading function for one person walking at 5.37Hz (3 x 1.79Hz = 5.37Hz).

The following strategy was adopted: the step distance corresponding to the third harmonic with step frequency of 1.79Hz is equal to 0.65m, as shown in Table 2. The step period is equal to 1/f = 1/1.79Hz = 0.558s, corresponding to the distance of 0.65m. This way, the modelling considered six forces to model one human step and each one of the loads P1, P2, P3, P4, P5 and P6 were applied on the structure during 0.558/5 = 0.1116s, corresponding the contact time of each dynamical load, see Figure 3.

However, the dynamical forces were not applied simultaneously. The first applied load would be P1, according with Equation (2), by 0.1116s, and at the end of this period of time, the load P1 becomes zero and the load P2 is then applied for 0.1116s. This process occurs successively and all dynamical loads are applied along the structure, as presented in Figure 3. It is noticed that all the dynamical actions associated to the time function will be applied correctly on the structural system.

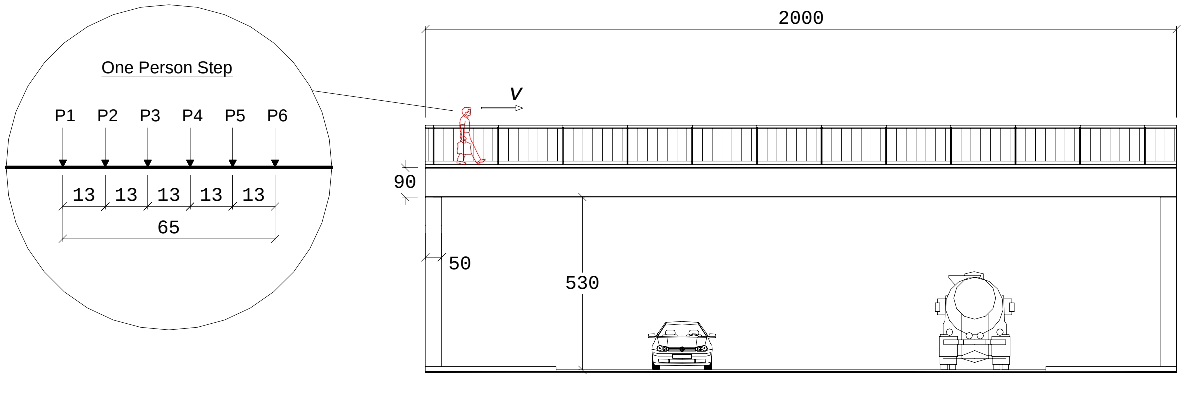


Figure 3. People walking on the footbridge (dimensions in cm).

3. STRUCTURAL SYSTEM

The outdoor footbridge studied in this work, simply supported by columns at its extremities, spanning varying from 10m to 35m by 2.5m, is currently used for crossing of pedestrians. The structural system is constituted of composite girders and a 100mm thick concrete slab, as presented in Figure 4.

The steel sections used were welded wide flanges (WWF) made with a 300MPa yield stress steel grade. A $2.05x10^5$ MPa Young's modulus was adopted for the steel beams. The concrete slab has a 30MPa specified compression strength and a $3.84x10^4$MPa Young's Modulus. Table 3 depicts the geometrical characteristics of all the steel sections used in the structural model. It is also assumed that an individual human weight was equal to 700N (0.7kN) (Murray *et al.*, 1997). In this investigation a damping ratio, ξ, equal to 0.005, 0.0075 and 0.01 (ξ = 0.5%, 0.75% and 1%) was adopted in all the structural systems (Murray *et al.*, 1997) based on the Rayleigh proportional damping formulation (Clough and Penzien, 1993).

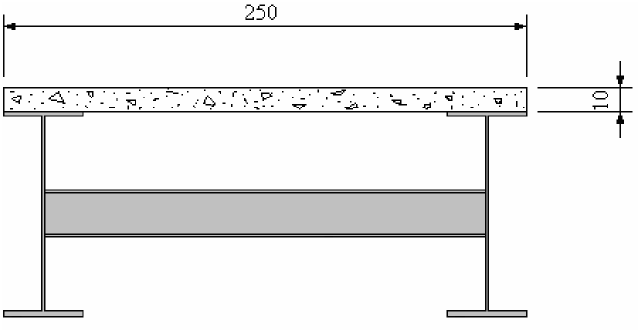


Figure 4: Standard outdoor footbridges cross section.

Table 3. Geometrical characteristics of the beams steel sections.

Main Span (m)	Beams	Height (mm)	Flange Width (mm)	Top Flange Thickness (mm)	Bottom Flange Thickness (mm)	Web Thickness (mm)
10.0	400x58	400	200	12.5	12.5	6.3
12.5	500x73	500	250	12.5	12.5	6.3
15.0	550x100	550	250	19.0	19.0	6.3
17.5	600x140	600	300	22.4	22.4	8.0
20.0	700x154	700	320	22.4	22.4	8.0
22.5	800x173	800	320	25.0	25.0	8.0
25.0	900x191	900	350	25.0	25.0	8.0
27.5	1000x201	1000	400	22.4	22.4	8.0
30.0	1100x235	1100	400	25.0	25.0	9.5
32.5	1200x244	1200	450	22.4	22.4	9.5
35.0	1200x307	1200	450	31.5	31.5	9.5

4. COMPUTATIONAL MODELLING

The proposed computational model, developed for the composite slab dynamic analysis, adopted the usual mesh refinement techniques present in finite element method simulations implemented in the ANSYS program (ANSYS, 1998). In this computational model, floor steel girders are represented by three-dimensional beam elements, where flexural and torsion effects are considered. The concrete slab is represented by shell finite elements, as presented in Figure 5. The computational model considered a full interaction between steel and concrete simulating a composite structural system.

5. DYNAMICAL ANALYSIS

For practical purposes, a linear time-domain analysis was performed throughout this study. This section presents the evaluation of the structural systems vibrations levels when submitted to dynamic excitations coming from human walking. The outdoor footbridges dynamic responses were determined through an analysis of its natural frequencies, displacements, velocities and accelerations. The results of the dynamic analysis were obtained from an extensive numerical analysis, based on the finite element method using the ANSYS program (ANSYS, 1998).

With the objective of evaluating quantitative and qualitatively the obtained results according to the proposed analysis methodology, the outdoor footbridge peak accelerations were calculated using the loading model previously described. These vales were compared with the results supplied by current criteria for structural design (Murray *et al.*, 1997), (ISO 2631/2, 1989). This comparison was performed in order to evaluate a possible occurrence of unwanted excessive vibration levels and human discomfort.

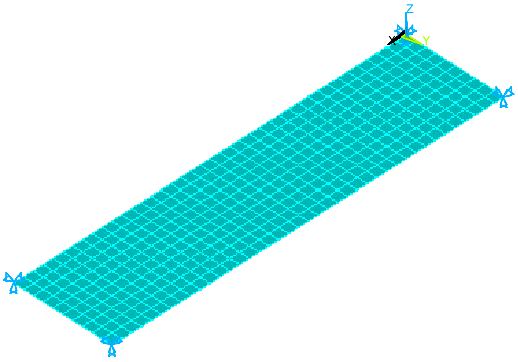


Figure 5: Finite element model.

5.1. Natural frequencies and vibration modes

The footbridges natural frequencies were determined with the aid of the finite element method simulations (ANSYS, 1998), as illustrated in Table 4. These results were compared to those obtained according to the design criteria (Murray *et al.*, 1997) and were used to evaluate only the footbridges fundamental frequencies.

When the footbridges freely vibrate in a particular mode, it moves up and down with a certain configuration or mode shape. Each footbridge natural frequency has an associated mode shape. It was observed in all investigated structural models that flexural effects were predominant in the fundamental mode vibrations. However it is important to observe that torsional effects are present starting from the second vibration mode, see Figure 6. Figure 6 illustrates the mode shapes corresponding to the first six natural frequencies of the pedestrian footbridge with main span equal to 20m.

The numerical results for the footbridges fundamental frequency, with the main span varying from 10m to 35m, were in accordance to the literature values (Murray *et al.*, 1997). It could be clearly observed, as expected, that as the structural span is increased the footbridge fundamental frequency decreases, see Table 4. This fact also serves to demonstrate that the developed models are coherent to the theory.

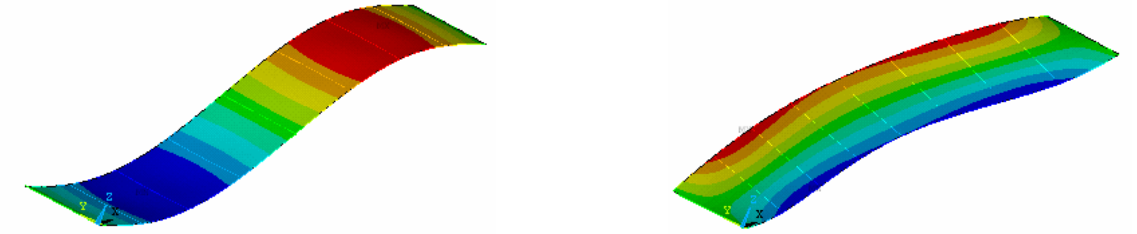
Table 4 indicated that the difference between the footbridge fundamental frequency evaluated by the developed model or by the AISC recommendations (Murray *et al.*, 1997) is proportional to the footbridge span. Considering that the finite element model (FEM) was developed to determine accurate results and the AISC equations (Murray *et al.*, 1997) are in fact based on simplified models associated to a single degree of freedom system (SDOF) it is reasonable to accept that the numerical results determined in the present study are closer to the actual value.

Table 4. Footbridges natural frequencies.

Main Span	Natural Frequencies f_{0i} (Hz)						AISC*	Differences
(m)	f_{01}	f_{02}	f_{03}	f_{04}	f_{05}	f_{06}	f_{01} (Hz)	(%)
10,0	9.04	19.52	30.58	53.31	53.76	62.87	8.58	5.14
12,5	7.72	17.83	26.66	46.31	46.88	50.53	7.23	6.26
15,0	6.63	16.19	22.85	36.76	39.87	45.98	6.03	9.03
17,5	5.91	15.07	20.07	29.98	35.32	42.12	5.23	11.55
20,0	5.37	14.60	18.23	24.87	32.95	39.16	4.74	11.87
22,5	4.99	14.11	16.83	21.28	30.87	36.73	4.35	12.66
25,0	4.65	13.51	15.63	18.79	28.96	34.50	4.04	13.13
27,5	4.31	12.61	14.45	17.11	27.04	32.17	3.74	13.17
30,0	4.11	11.47	13.59	16.19	24.86	30.52	3.52	14.48
32,5	3.84	10.36	12.67	15.51	22.99	28.58	3.28	14.55
35,0	3.55	9.45	11.53	14.68	21.07	26.21	2.96	15.41
*AISC: (Murray *et al.*, 1997)								

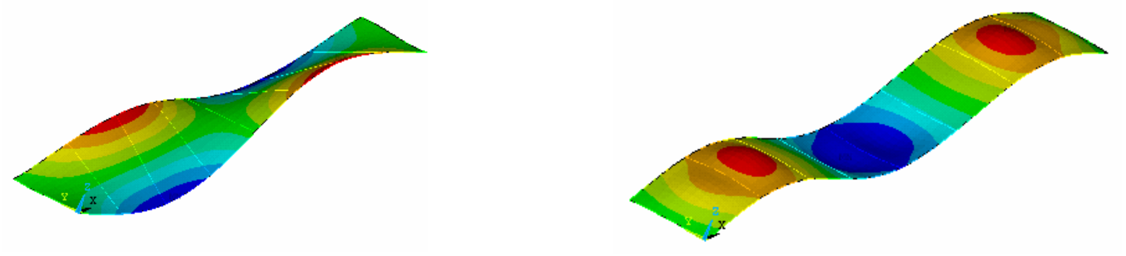
(a) Mode shape associated with the 1^{st} natural frequency: f_{01}=5.37Hz.

(b) Mode shape associated with the 2^{nd} natural frequency: f_{02}=14.60Hz.



(c) Mode shape associated with the 3^{rd} natural frequency: f_{03}=18.23Hz

(d) Mode shape associated with the 4^{th} natural frequency: f_{04}=24.87Hz.



(e) Mode shape associated with the 5^{th} natural frequency: f_{05}=32.95Hz.

(f) Mode shape associated with the 6^{th} natural frequency: f_{06}=39.16Hz.

Figure 6. Footbridge vibration modes. Main span equal to 20m.

5.2. Peak accelerations

The present analysis proceeds with the evaluation of the footbridges performance in terms of vibration serviceability due to human activities when submitted to dynamic forces induced by pedestrians walking. The first step of this investigation concerns in the determination of the footbridge peak accelerations. The footbridge peak accelerations were determined based on the developed finite element model (FEM). These maximum accelerations are then compared with results supplied by design criteria (ISO 2631/2, 1989), see Table 5.

It must be emphasized that the excitation frequency has great importance in the definition of the dynamic loading. Experimental studies already accomplished previously (Bachmann and Ammann, 1987), (Ellis, 2000), (Murray *et al*, 1997), have considered a frequency band ranging from 1.5Hz to 2.5Hz for the forcing frequencies induced by people walking, with the medium reference value equal 2.0Hz. On the other hand, the forcing frequencies induced by people running can generate periodic forces with associated frequencies ranging from 2.4 to 2.7 Hz (Bachmann and Ammann, 1987), (Ellis, 2000), (Murray *et al*, 1997).

The peak accelerations values presented in the Table 5 have shown that for all analyzed outdoor footbridges the peak accelerations were higher than those proposed by the design criteria (Murray *et al.*, 1997) and code recommendations (ISO 2631/2, 1989), violating human comfort criteria. This fact emphasizes that when the position of the dynamical loading corresponding to the excitation induced by people walking was changed and at the same time the human heel effect was incorporated in the analysis there is a substantial increase in the structure dynamical response, see Table 5.

It must be emphasized that the footbridges structural damping and heel impact coefficients considered in this investigation were in accordance with current design recommendations (Murray *et al.*, 1997). On the other hand, the calculated peak accelerations were higher than those proposed by code recommendations (Murray *et al.*, 1997), (ISO 2631/2, 1989), as presented in Table 5. Such fact is relevant, because the limit states related to excessive vibrations was violated and the human comfort was compromised when the transient effect due to the human heel impact was considered in the analysis.

The results presented in Table 5 have shown that when the structural damping, ξ, decrease the peak accelerations values increase and when the heel impact coefficient, f_{mi}, increase the peak accelerations values increase. However, when the peak acceleration values recommended by Murray *et al.* (1997) and ISO 2631/2 (1989) were investigated, all the structural models indicated possible problems related to human comfort for the load model, as illustrated in Table 5.

Table 5. Outdoor footbridges peak accelerations at resonance.

Main Span (m)	Heel Impact Coefficient (f_{mi})	Structural Damping (ξ)		
		0.50%	0.75%	1.00%
10.0	1.12	1.060	1.010	0.967
12.5		1.660	1.580	1.510
15.0		0.934	0.892	0.851
17.5		1.670	1.590	1.520
20.0		0.969	0.926	0.883
22.5		1.200	1.140	1.090
25.0		0.481	0.460	0.439
27.5		1.390	1.330	1.270
30.0		0.675	0.645	0.615
32.5		1.570	1.500	1.430
35.0		0.559	0.531	0.509
Limit acceleration: a_{lim} = 0.490 m/s^2 (Murray *et al.*, 1997), (ISO 2631/2, 1989).				

6. FINAL REMARKS

This paper presents a contribution for the evaluation of the structural behaviour of footbridges. The present investigation is carried out based on a more realistic loading model developed in order to incorporate the dynamical effects induced by people walking.

The proposed analysis methodology consider the investigation of the dynamic behaviour, in terms of serviceability limit states, of several outdoor footbridges made with a composite slab system with welded wide flange (WWF), steel beams and a 100mm thick concrete slab.

Computational models, based on the finite element method, developed using the ANSYS program. These models enabled a complete dynamical evaluation of the investigated composites floors systems especially in terms of human comfort and its vibration serviceability limit states.

The outdoor footbridges dynamic response in terms of peak accelerations was obtained and compared with the limit values proposed by Murray *et al.* (1997) and ISO 2631/2 (1989). The obtained results in this investigation have shown that when the position of the dynamical loading was changed according to the individual position and at the same time the human heel effect was incorporated in the analysis the peak accelerations were higher than Murray *et al.* (1997) and ISO 2631/2 (1989) limit values.

It must be emphasized that the loading model used in this investigation incorporated a more realistic load where the dynamic action position was changed according to the individual position on the structural model. Another important point is related to the fact that the generated time function has a space and time description and the load model also considered the human heel impact effect. On the other hand, the AISC recommendations (Murray *et al.*, 1997) only considered a single harmonic applied at the pedestrian footbridge midspan, without varying the load position.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

ANSYS, 1998, "Swanson analysis systems", Inc., P.O. Box 65, Johnson Road, Houston, PA, 15342-0065, version 5.5, Basic analysis procedures, Second edition.

Bachmann, H. and Ammann, W., 1987, "Vibrations in structures induced by man and machines", IABSE Structural Engineering Document 3E, International Association for Bridges and Structural Engineering, ISBN 3-85748-052-X.

Clough, R.W. and Penzien, J., 1993, "Dynamics of Structures", McGraw-Hill, 634 pgs.

Ellis, B.R., 2000, "On the response of long-span floors to walking loads generated by individuals and crowds", The Structural Engineer, Vol. 78, 17-25.

Figueiredo, F.P., 2005, Estudo do comportamento dinâmico de passarelas devido ao caminhar de pedestres, Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, PGECIV, Universidade do Estado do Rio de Janeiro, UERJ, Rio de Janeiro, Brasil, 122 pgs.

International Standards Organisation/ISO 2631-2, 1989, "Evaluation of human exposure to whole-body vibration. Part 2: Continuous and shock-induced vibration in buildings (1-80Hz)".

Lima, N.L. de A., 2007, Efeito da Modelagem do Carregamento, do Impacto do Calcanhar Humano e do Amortecimento Estrutural na Resposta Dinâmica de Passarelas Mistas, Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, PGECIV, Universidade do Estado do Rio de Janeiro, UERJ, Rio de Janeiro, Brasil, 138 pgs.

Murray, T.M., Allen, D.E., Ungar. E.E., 1997, "Floor vibrations due to human activity", Steel Design Guide Series, American Institute of Steel Construction, AISC, 70 pgs.

Pimentel, R.L., Pavic, A., Waldron, P., 2001, "Evaluation of design requirements for footbridges excited by vertical forces from walking", Canadian Journal of Civil Engineering, Vol. 28(5), 769-776.

Varela, W.D., 2004, "Modelo teórico-experimental para análises de vibrações induzidas por pessoas caminhando sobre lajes de edifícios", Ph.D. Thesis (In Portuguese), Federal University of Rio de Janeiro, Civil Engineering Department, COPPE/UFRJ, Rio de Janeiro, Brazil, 309 pgs.

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