EFFECTS OF NANO-PARTICLES ON SKIN-FRICTION AND HEAT TRANSFER COEFFICIENTS AND BOUNDARY LAYER THICKNESS OF LAMINAR NANOFLUID FLOW OVER A FLAT PLATE

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Abstract. *This paper focuses on the effects of nano-sized particles on the physical properties of the nano-fluid (thermal conductivity, viscosity, density and specific heat) and on the velocity and thermal boundary layers of laminar nano-fluid flow over a uniform temperature flat plate. Different nano-particles materials are investigated. It is shown that not necessarily the highest thermal conductivity nano-particles lead to higher heat transfer, and that the nano-particle heat capacity,* ρc_p *plays a very important role towards the heat transfer enhancement. The result shows that the local Nusselt number is approximately linearly proportional to the volumetric fraction of the nano-particles in the fluids. The linear relation between the local Nusselt number and the volumetric fraction of the nano-particles is presented. Appropriate dimensionless groups and properties ratios are identified and the results are presented in normalized charts.*

***Keywords**: nano-fluid, nano-particle, boundary layer, heat transfer enhancement*

1. INTRODUCTION

In compact structures such as electronic packages, where the availability of space to fit cooling flow paths is limited, there is insufficient room to increase heat transfer area. Nano-fluids are an alternative to enhance the heat transfer rate (Choi, 1995; Triksaksri and Wongwises, 2006; Xuan and Li, 2003; Li and Xuay, 2002; Yang et al., 2005; Wen and Ding, 2004; Maiga et al., 2004; Roy et al., 2004; Xuan et al., 2005). Nano-fluids are described as fluids that contain nano-sized particles (smaller than 100 nm in diameter) (Xuan and Roetzel, 2000). Nano-sized particles are recommended to improve the thermal performance in microscale flow channels because they do not deposit quickly and do not clog the flow paths (Gosselin and da Silva, 2004). The volumetric fraction (ϕ) of nano-particles in the fluid is normally very low ($\phi < 0.05$), hence nano-fluids behave and can be modeled similarly to normal fluids (Xuan and Li, 2000). Nano-particles may be either metallic or non-metallic. Metallic nano-particles are added into the fluid in order to increase thermal conductivity (Xuan and Li, 2000; Lee et al., 1999; Masuda et al., 1993). The most commonly used nano-particles, with high thermal conductivity (k), are Al_2O_3 and Cu (Masuda et al., 1993), probably due to the assumed belief heat transfer would likely increase as a result of thermal conductivity increase in isolation. Whether the high thermal conductivity of nano-sized particles is the only factor in heat transfer enhancement of nano-fluids forms one of the objectives of this letter.

The general objectives of this letter are: i) to study the property effects of nano-particles on velocity and thermal boundary layers, on the local skin-friction coefficient ($C_{f,x}$), and on the local Nusselt number (Nu_x) of laminar nano-fluid flow over a uniform temperature flat plate, and ii) to investigate different materials for nano-particles. The scope of this study is on investigating the fluid mechanics and thermal characteristics of a nano-fluid, not its manufacturing and fabrication process.

2. THEORY

Consider the laminar flow of a nano-fluid over a flat plate with constant temperature T_0 as shown in Fig. 1. The fluid approaches the plate with uniform velocity U_∞, pressure P_∞ and temperature T_∞. The relative properties of nano-particles are defined as

$$\tilde{\rho} = \frac{\rho_p}{\rho_f}, \qquad \tilde{c}_p = \frac{(c_p)_p}{(c_p)_f} \qquad \text{and} \qquad \tilde{k} = \frac{k_p}{k_f} \tag{1}$$

where $\tilde{\rho}$, $\tilde{c}_p$ and $\tilde{k}$ are the relative density, relative specific heat and relative thermal conductivity of the nano-particles as compared to the base fluid properties. The subscript "p" refers to particles and the subscript "f" refers to fluid.

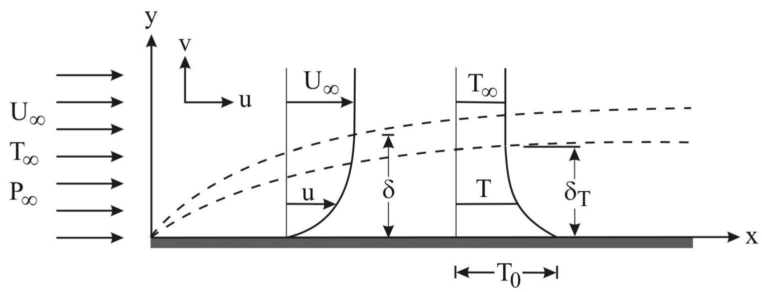

Figure 1. Velocity and thermal boundary layers for laminar flow over a constant temperature flat plate

The density of the nano-fluid is calculated by

$$\frac{\rho_{nf}}{\rho_f} = (1-\phi) + \phi\tilde{\rho} \tag{2}$$

The subscript "nf" refers to nano-fluid. According to Hamilton and Crosser (1962), the thermal conductivity of the nano-fluid is calculated by

$$\frac{k_{nf}}{k_f} = \frac{(n-1)(1-\phi) + (1+(n-1)\phi)\tilde{k}}{(n-1)+\phi+(1-\phi)\tilde{k}} \tag{3}$$

where the empirical shape factor n is defined by

$$n = \frac{3}{\Psi} \tag{4}$$

The sphericity, Ψ, is defined as the ratio between the surface area of a sphere which has the same volume as the particles and the surface area of the particles. Wasp (1977) reported the special case of Eq. (3) when the sphericity of the particles Ψ is 1 (or n=3) as

$$\frac{k_{nf}}{k_f} = \frac{2(1-\phi) + (1-2\phi)\tilde{k}}{2+\phi+(1-\phi)\tilde{k}} \tag{5}$$

By following the viscosity model for two binary solutions proposed by Brinkmann (1952), the dynamic viscosity of the nano-fluid is calculated by

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}} \tag{6}$$

Combining Eq. (2)-(6) with the momentum and energy equations, the governing equations are

$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = 0$$

$$\frac{\partial \hat{u}^2}{\partial x} + \frac{\partial \hat{u}\hat{v}}{\partial y} = \frac{-1}{a_3 \rho_f U_\infty^2}\frac{\partial P}{\partial x} + \frac{a_1 \nu_f}{U_\infty}\frac{\partial^2 \hat{u}}{\partial y^2}$$

and $$\frac{\partial \hat{u}\theta}{\partial x}+\frac{\partial \hat{v}\theta}{\partial y}=\frac{a_2\alpha_f}{U_\infty}\frac{\partial^2\theta}{\partial y^2} \tag{7}$$

where ν_f is the kinematic viscosity of the fluid and the fluid thermal diffusivity is $\alpha_f = k_f/(\rho c_p)_f$. The dimensionless velocity and temperature are defined by

$$\hat{u},\hat{v}=\frac{(u,v)}{U_\infty} \qquad and \qquad \theta=\frac{T_0-T}{T_0-T_\infty} \tag{8}$$

and the parameters a_1, a_2 and a_3 are related to the volumetric fraction (ϕ), and the relative properties of the nano-particles as

$$a_1=\frac{1}{(1-\phi)^{2.5}\left((1-\phi)+\phi\tilde{\rho}\right)}$$

$$a_2=\frac{(n-1)(1-\phi)+(1+(n-1)\phi)\tilde{k}}{\left((1-\phi)+\phi\tilde{\rho}\tilde{c}_p\right)\left((n-1)+\phi+(1-\phi)\tilde{k}\right)}$$

and $$a_3=(1-\phi)+\phi\tilde{\rho} \tag{9}$$

The governing equations in Eq. (7) are similar to the momentum and energy equations of a regular fluid. The relations for velocity boundary layer thickness δ and skin-friction coefficient $C_{f,x}$ of a nano-fluid can be obtained by substituting the effective properties of nano-fluid, Eqs. (2) - (6) into the integral solutions of a regular fluid (Xuan and Roetzel, 2000; Bejan, 1995) and can be written as

$$\frac{\delta}{x}=4.641\,\mathrm{Re}_{x,f}^{-1/2}(1-\phi)^{-1.25}\left((1-\phi)+\phi\tilde{\rho}\right)^{-1/2} \tag{10}$$

$$C_{f,x}=\frac{\tau}{\frac{1}{2}\rho_f U_\infty^2}=0.6466\,\mathrm{Re}_{x,f}^{-1/2}\frac{\left((1-\phi)+\phi\tilde{\rho}\right)^{1/2}}{(1-\phi)^{1.25}} \tag{11}$$

where $\mathrm{Re}_{x,f}$ is the local Reynolds number of a pure fluid ($\mathrm{Re}_{x,f}=xU_\infty/\nu_f$). Equations (10) and (11) are valid when the profile shapes of the dimensionless velocity (Bejan, 1995) is $\hat{u}=\frac{\xi}{2}\left(3-\xi^2\right)$ where $\xi=y/\delta$.

3. RESULTS AND DISCUSSION

Figure 2 shows the effects of added nano-sized particles of gold, tungsten, silver, lead, copper and aluminum oxide in water at 298 K and 1 atm $(\mathrm{Pr}_f = 6.21)$ on the velocity boundary layer thickness and on the skin friction coefficient. The relative properties of the six nano-particles are reported in Table 1. Figure 1 shows that by adding nano-particles, the nano-fluid becomes heavier than the pure fluid, so that the velocity boundary layer thickness becomes thinner. The more nano-particles that are added, the denser the nano-fluid will become and the thinner the velocity boundary layer will be. The skin-friction coefficient is inversely proportional to the velocity boundary thickness; hence the skin-friction coefficient of the nano-fluid is higher than the pure fluid. The nano-fluid with the heaviest nano-particles (in this case, gold and tungsten) has the thinnest velocity boundary layer thickness and requires more pumping work than the pure fluid.

The thermal boundary layer thickness is separated into two cases, i.e., when $\delta_T < \delta$ and $\delta_T > \delta$. For the first case when $\delta_T < \delta$, the thermal boundary layer thickness and the local Nusselt number of the nano-fluid are expressed, respectively as

$$\frac{\delta_T}{\delta}=0.9757\,\mathrm{Pr}_f^{-1/3}\,\Pi_1$$

$$Nu_x=\frac{hx}{k_f}=0.3321\,\mathrm{Pr}_f^{1/3}\,\mathrm{Re}_{x,f}^{1/2}\,\Pi_2 \tag{12}$$

where $$\Pi_1 = (1-\phi)^{5/6}\left[\frac{(n-1)(1-\phi)+(1+(n-1)\phi)\tilde{k}}{(n-1)+\phi+(1-\phi)\tilde{k}}\right]^{1/3}\left[\frac{(1-\phi)+\phi\tilde{\rho}}{(1-\phi)+\phi\tilde{\rho}\tilde{c}_p}\right]^{1/3}$$

and $$\Pi_2 = (1-\phi)^{5/12}\left[(1-\phi)+\phi\tilde{\rho}\tilde{c}_p\right]^{1/3}\left[(1-\phi)+\phi\tilde{\rho}\right]^{1/6}\left[\frac{(n-1)(1-\phi)+(1+(n-1)\phi)\tilde{k}}{(n-1)+\phi+(1-\phi)\tilde{k}}\right]^{2/3}$$

Table 1. The relative properties of nano-particles in water at 298 K and 1 atm.

Material	$\tilde{\rho}$, Eq. (1)	$\tilde{k}$, Eq. (1)	$\tilde{c}_p$, Eq. (1)	ε, Eq. (13)
Gold	19.3	525	0.03	3.985
Tungsten	19.3	298	0.03	3.975
Lead	11.3	58	0.03	2.881
Silver	10.5	711	0.05	2.880
Cu	8.9	668	0.09	2.751
Al_2O_3	3.9	58	0.21	1.652

The only differences between the solutions for the nano-fluid and the pure fluid are the factors Π_1 and Π_2. The factors Π_1 and Π_2 are functions of the volumetric fraction and the relative properties of the nano-particles.

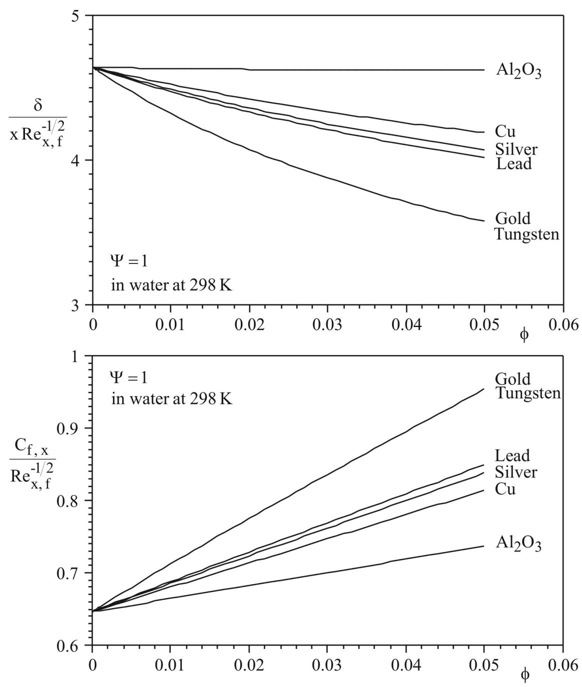

Figure 2. The velocity boundary layer thickness and the skin-friction coefficient of a nano-fluid composed of nano-particles in water at 298 K and 1atm

Figure 3 shows that the nano-fluid with the highest thermal conductivity nano-particles (in this case silver) is not necessarily the one that produces the largest heat transfer enhancement. Beside the thermal conductivity ($\tilde{k}$), the density $(\tilde{\rho})$ and heat capacity $(\tilde{c}_p)$ play an important role on the thickness of the boundary layer and on the heat transfer enhancement of the nano-fluid. The thickness of the thermal boundary layer increases with the volumetric fraction (ϕ) of the nano-particles in the fluid.

Copper and Aluminum oxide are commonly used as nano-particles. Figure 3 shows that nano-particles of both copper and aluminum oxide improve the heat transfer coefficient as previously reported in the literature (Maiga et al., 2004). Nano-particles of gold and tungsten show significant improvement in heat transfer. Due to the current availability in the market and price, nano-particles of tungsten seem to be the most attractive among the six investigated materials in this letter, to increase the thermal performance of the pure fluid.

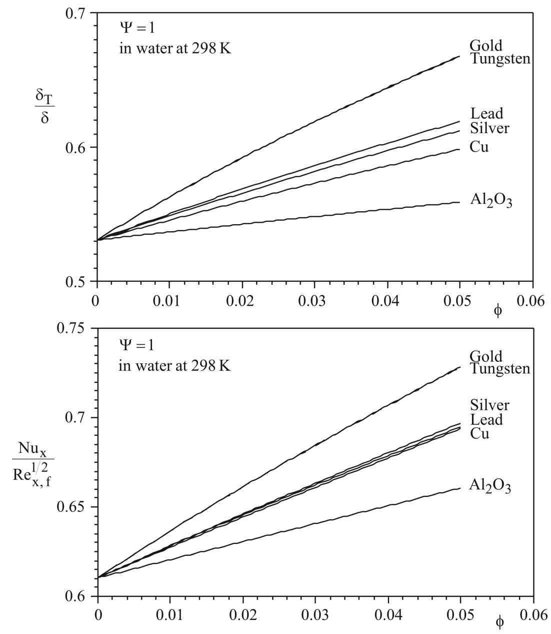

Figure 3. The thermal boundary layer thickness and the Nusselt number of a nano-fluid composed of nano-particles in water at 298 K and 1atm

Figure 3 also shows that the local Nusselt number of nano-fluids is approximately linear-proportional to the volumetric fraction of the nano-particles (ϕ) in the fluids and can be estimated by

$$Nu_x = 0.3321 \mathrm{Pr}_f^{1/3}\, \mathrm{Re}_{x,f}^{1/2} \left(1 + \varepsilon\phi\right) \tag{13}$$

where ε is equal to 3.985 for gold, 3.975 for tungsten, 2.880 for silver, 2.811 for lead, 2.751 for copper and 1.652 for aluminum oxide in water at 298 K and 1 atm. The linear relation in Eq. (13) provides the approximated values of the local Nusselt number within 1 percent of the values obtained by the analytical relation in Eq. (12).

The thermal boundary layer thickness and local Nusselt number of a nano-fluid when $\delta_T > \delta$ can be calculated by

$$\frac{\delta_T}{\delta} = 0.6096 \mathrm{Pr}_f^{-1/2}\, \Pi_3$$

$$Nu_x = \frac{hx}{k_f} = 0.5303 \mathrm{Pr}_f^{1/2}\, \mathrm{Re}_{x,f}^{1/2}\, \Pi_4 \tag{14}$$

where $$\Pi_3 = (1-\phi)^{1.25}\left[\frac{(n-1)(1-\phi)+(1+(n-1)\phi)\tilde{k}}{(n-1)+\phi+(1-\phi)\tilde{k}}\right]^{1/2}\left[\frac{(1-\phi)+\phi\tilde{\rho}}{(1-\phi)+\phi\tilde{\rho}\tilde{c}_p}\right]^{1/2}$$

and $$\Pi_4 = \left[(1-\phi)+\phi\tilde{\rho}\tilde{c}_p\right]^{1/2}\left[\frac{(n-1)+\phi+(1-\phi)\tilde{k}}{(n-1)(1-\phi)+(1+(n-1)\phi)\tilde{k}}\right]^{1/2}$$

Equation (14) shows the effect of the nano-particles on the thermal boundary thickness and the Nusselt number when $\delta_T > \delta$. Equations (12) and (14) are valid when the profile shape of the dimensionless temperature is $\theta = \frac{\xi_T}{2}\left(3 - \xi_T^2\right)$ where $\xi_T = y/\delta_T$.

In the previous analysis ($\delta_T < \delta$), Fig. 3 showed that the thickness of the thermal boundary layer increases monotonically with the volumetric fraction ϕ of the added metallic nano-particles. Therefore, it is important to have a well defined condition to select between the set of Eqs. (12 or 13) and (14) in order to predict the heat transfer coefficient of a nano-fluid. By taking the limit $\delta_T \approx \delta$ of Eq. (12), it is possible to show that the thickness of thermal boundary layer of the nano-fluid is approximately the same as the thickness of the velocity boundary layer when

$$\left(1-\phi^*\right)^{2.5} \frac{\left(1-\phi^*\right)+\phi^*\tilde{\rho}}{\left(1-\phi^*\right)+\phi^*\tilde{\rho}\tilde{c}_p} \frac{(n-1)\left(1-\phi^*\right)+\left(1+(n-1)\phi^*\right)\tilde{k}}{(n-1)+\phi^*+\left(1-\phi^*\right)\tilde{k}} \approx 1.05\,\mathrm{Pr}_f \tag{15}$$

where ϕ^* is the volumetric fraction of the nano-particles in the fluid when $\delta_T \approx \delta$. For example, the limit to use Eq. (12) or (13) to predict the heat transfer coefficient of Tungsten nano-particles in water at 298 K and 1 atm is $\phi^* \approx 0.277$.

4. CONCLUSIONS

In this paper we considered the effect of nano-particles on the local skin-friction coefficient and local Nusselt number of a nano-fluid over a flat plate with constant temperature. The mathematical models to predict the local Nusselt number of a nano-fluid for both cases when $\delta_T < \delta$ and $\delta_T > \delta$ were presented. The effects of six different kinds of nano-particles on skin-friction and heat transfer of a nano-fluid were investigated. The result shows the linear relation between the local Nusselt number and the volumetric fraction (ϕ) of nano-particles in the fluids. The linear relation in Eq. (13) can provided the approximated values of the local Nusselt number within 1 percent of the analytical solution in Eq. (12). Gold and tungsten both show significant improvement in heat transfer capability of water. Considering the current market price, tungsten seems to be more economically feasible than gold. Even though this letter presented the heat transfer enhancement of nano-particles in water at 298 K and 1 atm; all the relations were presented in normalized terms, thus the relations presented in this letter could be used to predict the pressure drop and heat transfer enhancement for other kinds of nano-fluids.

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