CHANGES IN THE SEMI-MAJOR AXIS DUE GEOPOTENTIAL PERTUBATION AND 2:1 RESONANCE EFFECTS

Formiga, J. K. S., jkennety@dem.inpe.br.
Vilhena de Moraes, R.,rodolpho@feg.unesp.br.
National Institute for Space Research -INPE-DMC, São José dos Campos, SP
Group of Orbital Dynamics and Planetology, DMA-FEG-UNESP, Guaratinguetá-SP

Abstract. *The motion of an artificial satellite considering geopotential perturbations and resonances between the frequencies of the mean orbital motion and the Earth rotational motion is studied. The behavior of the motion of the satellite is analyzed in the neighborhood of the 2:1 resonances. A suitable sequence of canonical transformations reduces the system of differential equations describing the orbital motion to an integrable kernel. The phase space of the resulting system is studied considering that one resonant angle is fixed. Simulations are presented showing the variations of the semi-major axis of artificial satellites.*

Keywords: *resonance, artificial satellites, celestial mechanics.*

1. INTRODUCTION

Synchronous satellites on circular orbits are extensively used for communication and navigation purposes. This fact justifies the great attention that was given in the literature to orbital resonance which affects these satellites: (i) resonance of the rotation motion of a planet with the translational motion of the satellite (Lima Junior, 1998; Formiga, 2005); (ii) sun-synchronous resonance (Hughes,1980); (iii) spin-orbit resonance (Beletskii,1975; Hamill e Blitzer,1974; Vilhena de Moraes et al,1990), (iv) resonances between the frequencies of the rotational motion of the satellite (Hamill e Blitzer,1974); (v) resonance including solar radiation pressure perturbation (Ferraz-Mello,1979).

In this work, the type of resonance considered is the commensurability between the frequencies of the satellite mean orbital motion and the Earth rotational motion. This case of resonance occurs frequently in real cases. In fact, in a survey from a sample of 1818 artificial satellites, chosen in a random choice from the NORAD 2-line elements (Celestrak, 2004), about 85% of them are orbiting near some a resonance's region.

The system of differential equations describing the orbital motion of an artificial satellite under the influence of perturbations due to the geopotential, can be described in a canonical form. In order to study the effects of resonances, a suitable sequence of canonical transformations can be performed reducing the system of differential equations to an integrable kernel (Lima Jr. P. H. C. L., 1998). This system is here integrated numerically. Simulations can show the behaviour of motion in the neighbourhood of the exact resonance. Some results considering the 2:1 resonances were already known but here, new results are presented.

2. EQUATIONS OF MOTION AND THE CONSIDERED POTENTIAL

Using the classical set of Delaunay variables (L,G,H,l,g,h), the sideral time $\Theta=\omega_e t$, where ω_e is the angular speed of the Earth, and geopotential perturbations, the equations of motion can be written as (Osório, 1973):

$$\frac{d(X,Y,Z,\Theta)}{dt}=\frac{\partial F}{\partial(x,y,z,\theta)}$$
$$\frac{d(x,y,z,\theta)}{dt}=-\frac{\partial F}{\partial(X,Y,Z,\Theta)} \tag{1}$$

were the Hamiltonian F is

$$F=\frac{\mu^2}{2L^2}+R_{\ell mpq} \tag{2}$$

and

$$R_{\ell mpq}=\sum_{\ell l=2}^{\infty}\sum_{m=0}^{\ell}\sum_{p=0}^{\ell}\sum_{q=-\infty}^{\infty}\frac{\mu^2}{L^2}\left(\frac{\mu a_e}{L^2}\right)^{\ell}J_{\ell m}F_{\ell mp}(L,G,H).\,H_q^{-(\ell+1),((\ell-2p)}(L,G)\cos\varphi_{\ell mpq}(\ell',g,h,\Theta) \tag{3}$$

and the argument is given by:

$$\varphi_{\ell mpq}(\ell', g, h, \Theta) = q\ell' + (\ell - 2p)g + m(h - \Theta - \lambda_{\ell m}) + (\ell' - m)\frac{\pi}{2} \quad (4)$$

Also $\mu=GM \approx 3{,}986009\text{x}10^5$ Km^3/s^2 is the gaussian constant; $\lambda_{\ell m}$ is the longitude of the semi major axis of symmetry for the spherical harmonic (ℓ,m); $H_q^{(\ell+1),(\ell-2p)}(e)$ are the Hansen's functions and $F_{\ell mp}(i)$ are the Kaula's inclination functions.

3. METHODOLOGY

Let us represent by n the mean motion of the satellite and by α=q/m the commensurability from resonance condition

$$qn - m\omega_e = 0 \quad (5)$$

were q and m are integers.

The influence of the resonance on the orbital motion can be analyzed integrating a system of differential equations of the following type (Lima Jr. P. H. C. L., 1998; Formiga, 2005):

$$\frac{dX_1}{dt} = -\sum_{p=s}^{\infty} B_{(2p+k)mp(\alpha m)}(X_1, C_1, C_2)sen\varphi^*_{(2p+k)mp(\alpha m)}(x_1, \Theta_1) \quad (6)$$

$$\frac{d\varphi^*_{(2p+k)mp(\alpha m)}}{dt} = m\alpha\frac{\mu^2}{X_1^{\,3}} - m\omega_e - m\alpha\sum_{j=1}^{\infty}\frac{\partial B_{2j,0,j,0}(X_1, C_1, C_2)}{\partial X_1} - m\alpha\sum_{p=s}^{\infty}\frac{\partial B_{(2p+k)mp(\alpha m)}(X_1, C_1, C_2)}{\partial X_1}\cos\varphi^*_{(2p+k)mp(\alpha m)}(x_1, \Theta_1) \quad (7)$$

Explicit functions relating $B_{(2p+k)mp(\acute{\alpha}m)}(X_1, C_1, C_2)$ and $F_{lmpq}(X_1, C_1, C_2)$, with the keplerian elements can be found in Lima Jr. P. H. C. L.(1998) and Formiga (2005).

4. NUMERICAL SIMULATIONS

Since Hansen's coefficients have been used this theory can be applied for orbits of any eccentricity below one. In this section we present simulations considering a few initial conditions arbitrarily chosen.

In what follows, it will be show some examples of the influence of a resonance on the metric orbital elements of orbits whose periods are near to the 2:1 resonances with respect the period of the Earth's rotation. Several resonant harmonics can be considered. Here, for the resonance 2:1 it was considered first the simultaneous influence of the harmonics J_2, $J_{2,2}$ and thus the simultaneous influence of the harmonics J_2, $J_{3,2}$

Let us consider the case e=0,01, i=4°, φ^*=0 (critical angle) and the harmonics J_2 and $J_{2,2}$. Numerical values for the Harmonics coefficients J_2 and $J_{2,2}$ are given by JGM3.

Figure 1 represents the temporary variation of the semi-major axis in the neighbourhood of the resonance 2:1. For several values considered for the semi-major axes, it can be observed distinct behaviors for their temporary variations. The more the satellites approaches the region that we defined as a resonant region, the more the variations increases. It is remarkable the oscillation in the region between by a= 26560,0 km and a=26562,48 km, that characterizes paths for which the effect of the resonance is maximum for the case where e=0,01 and i=4°. For instance, a small variation in the semi-axis, of about 10 m (a = 26562,49 km) an abrupt decrease for a is observed.

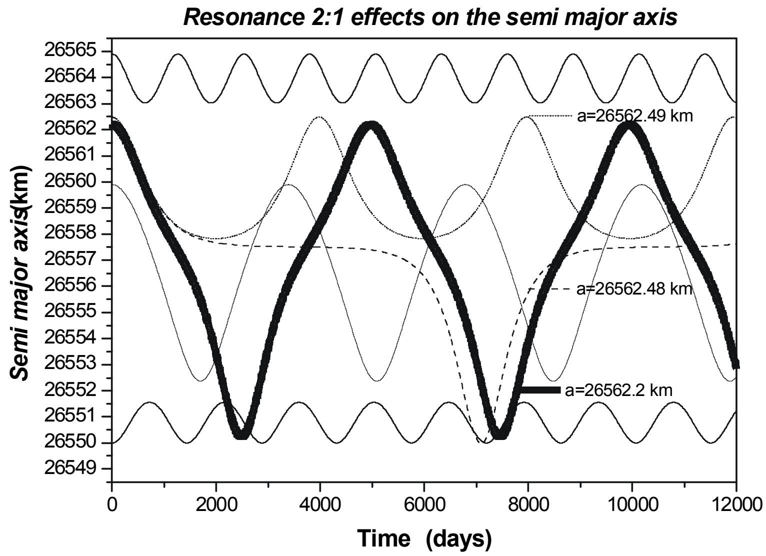

Figure1- Variation of orbits in the time: e=0,01, i=4°

Table1 contains the amplitude and period of the variations of orbital elements for hypothetical satellites considering low eccentricity, small and high inclination and the influence of the harmonics J_2 and $J_{2,2}$. Table 2 considers the influence of the harmonics J_2 and $J_{3,2}$.

For several values of semi-major axes considered, it can be observed distinct behaviors of temporary variations. The more the satellites approach the region that was defined as a resonant region, the more the variations increases. For the case e=0.01 and 4° it is remarkable the oscillation in the region between by *a*= 26560.00 km and *a*=26562.50 km. For instance, a variation of about 10m in the initial semi-major axis, *a* = 26562.48 km, brings up a variation of about more than 12 km in its amplitude in a period of about 7000 days.

For the considered cases and for the same regions, the influence of the harmonic $J_{3,2}$ is smaller than the influence of the harmonic $J_{2,2}$ as can be seen by Table 2.

Also, when the inclination increases the resonance's effect decreases but is still noticeable. This can be observed considering, for instance, e = 0.01 and i=55°.

Table 1.Amplitude and period of perturbations due to the 2:1 resonance: J_2+$J_{2,2}$

Orbital elements			**Amplitude**	Period
a a_o**=26561,770 km**	***e***	***i***	Δa_{max}	**T (days)**
a_o +0.430	0,01	4°	12 km	2500
a_o +0.718	0,01	4°	12,5 km	7000
a_o +0.290	0,01	4°	4,5 km	2000
a_o +0.500	0,01	4°	3 km	1000
a_o –2,5	0,01	4°	6,5 km	1700
a_o –5	0,01	4°	1,4 km	500
a_o	0,05	55°	1,8 km	3200
a_o +0.718	0,005	55°	2,75 km	3900
a_o –1,770	0,005	55°	1,2 km	2000
a_o +5,129	0,005	55°	900 m	1500
a_o +3,729	0,005	55°	1,8 km	2500
a_o	0,05	63,4°	7,5 km	1000
a_o –2	0,05	63,4°	4 km	1000
a_o	0,05	87°	7 km	2000
a_o –2	0,05	87°	3,5 km	1900

The temporary variations, due to the 2:1 resonance, of the semi-major axis of hypothetical satellites with high orbital eccentricity (e = 0.7) and inclination i=55° are shown in "Fig. 2" (considered harmonics J_2 and $J_{3,2}$). It is also remarkable here that for an initial semi-major axis of 26560.0 the resonance causes a variation of about 10 km in semi-major axis with a period of about 500 days but if we shift 10km in the initial condition (*a*= 26550.0) the amplitude increases to about 30km with the same period.

Table 2.Amplitude and period of perturbations due to the 2:1 resonance: $J_2+J_{3,2}$

Orbital elements			**Amplitude**	**Period t(days)**
a_o=26561.770 km	*e*	*i*	$\Delta a_{máx}$	
a_o –3.77	0,01	4°	120 m	500
a_o +0.718	0,01	4°	245 m	1000
a_o +0.929	0,01	4°	110 m	500
a_o +5	0,01	4°	100 m	250
a_o –2,5	0,005	4°	100 m	500
a_o +0.429	0,005	4°	700 m	1500
a_o +0.929	0,005	55°	1,9 km	16000
a_o +1,73	0,005	55°	250 m	16000
a_o +5	0,005	55°	30 m	1500
a_o –3,77	0,005	55°	47 m	500
a_o –0.429	0,005	87°	110 m	1800
a_o –2,23	0,005	87°	45 m	800
a_o –3,77	0,005	87°	20 m	500

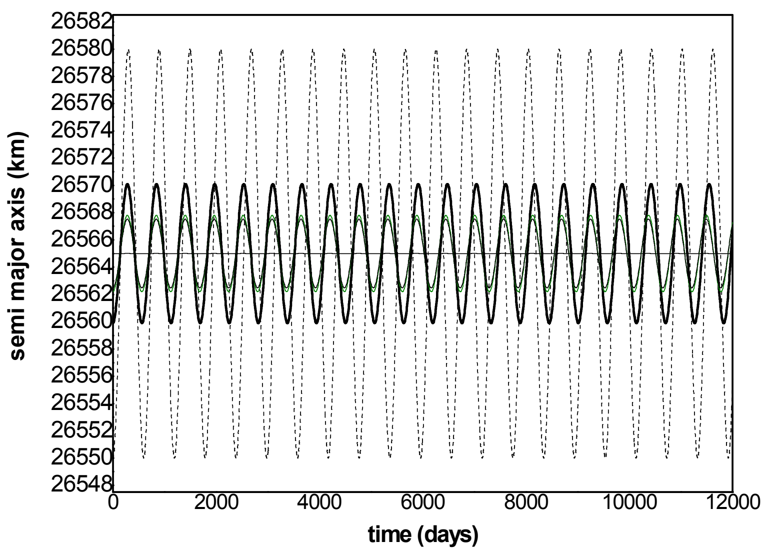

Figure 2- Ressonance effects on the semi-major axis ($J_2+J_{2,2}$), e= 0.7 and i=55°

Figure 3 shows the variation of the semi-major axis for the same previous case but now considering the influence of the harmonics J_2 and $J_{3,2}$.

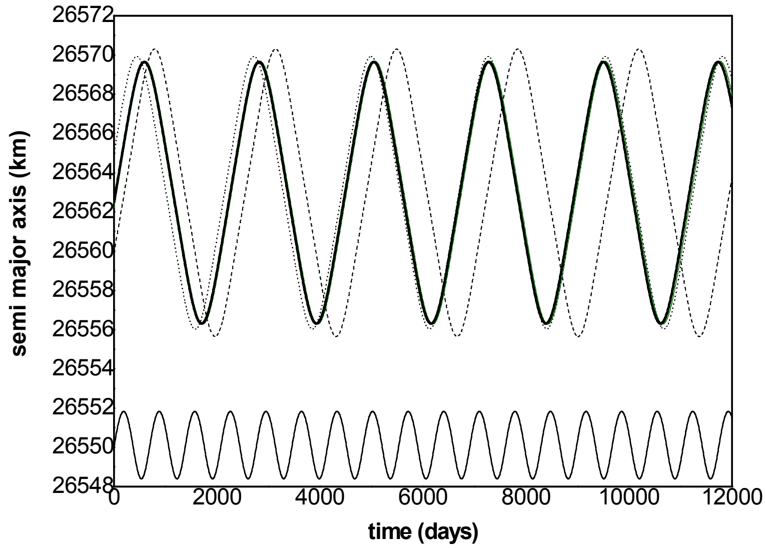

Figure 3- Ressonance effects on the semi-major axis (J_2+$J_{3,2}$) e= 0.7 and i=55°

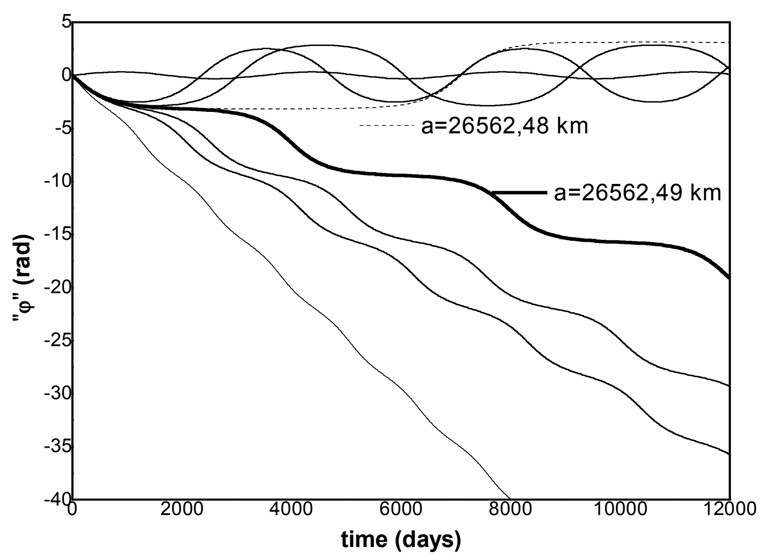

Figure 4- Temporal evolution of the resonant phase angle φ (J_2+$J_{2,2}$) e= 0.01 and i=4°

The temporal evolution of the resonant angle φ is shown in "Fig. 4". The resonance area, shown "Fig. 1", is characterized here by a mean value of the resonant phase angle equal to zero, and corresponds to initial semi-major in the region between by a= 26560.00 km and a=26562.50 km

5. CONCLUSIONS

In this work, based on Lima Junior's theory a integrable kernel was found. The theory, valid for any type resonance p/q, and was here applied for 2:1 resonant case.

The motion near the region of the exact resonance, is extremely sensitive to the small alterations considered. This can be and indicative that these regions are chaotic.

The preliminary results indicate that orbits lifetimes for orbits near orbits resonant can be significantly changed due resonant effects.

This work provides a good approach for long period orbital evolution studies for satellites orbiting in regions where the influence of the resonance is more pronounced.

6. ACKNOWLEDGEMENTS

This work was supported by CNPq and by Brazilian Institute for Space Research.

7. REFERENCES

Beletskii, V. V., Resonance Phenomena at Rotations of Artificial and Natural Celestial Bodies. In: GIACAGLIA, G.E.O.(Ed).**Satellites dynamics**. Berlin. Verlang, 1975.

CELESTRAK, 2004 "NORAD 2-line Elementes". 10 Ago. 2004, <http://www.celestrak.com>.

Ferraz Melo, S. Periodic orbits in a region of instability created by independent small divisors. In: NAGOZY, E; Ferraz Melo, S.(Eds). Natural and artificial satellite motion**.** Austin :University of Texas Press, 1979. p. 283-292.

Formiga, J. K. S., 2005, Estudo de ressonâncias no movimento orbital de satélites artificiais. 2005. 133f. Dissertação (Mestrado em Física) – Faculdade de Engenharia do Campus de Guaratinguetá, Universidade Estadual Paulista, Guaratinguetá, São Paulo, Brazil.

Hamill, P. J.; Blitzer, L. Spin-orbit coupling: a unified theory of orbital and rotational resonance. **Celestial mechanics**, v.9, p.127-146, 1974.

Hugues, S. Earth satellite Orbits with Resonant Lunisolar Perturbations. Resonances dependent only inclination. Proceedings of the Royal society of London serie A. London, v. 372, n. 1745, p. 243-264, 1980.

Lima Jr. P. H. C. L., 1998, Sistemas ressonantes a altas excentricidades no movimento de satélites artificiais. Tese (Doutorado em 1998), Instituto tecnológico de aeronáutica, São José dos Campos, São Paulo, Brazil.

Osório, J. P., 1973, Perturbações de órbitas de satélites no estudo do campo gravitacional terrestre. Porto: Imprensa Portuguesa.

Vilhena de Moraes, R. Silva, P. A. F. Influence of the resonance in gravity- gradient stabilized satellite**.** Celestial mechanics and dynamical astronomy**.** v.47**,** p. 225-243, 1990.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.