EXPERIMENTAL CHARACTERIZATION OF THERMAL RESIDUAL STRESS EFFECTS ON THE STIFFNESS OF COMPOSITE PLATES

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Abstract: *Laminated composite structures have been extensively used in modern engineering, particularly, in the aeronautical industry. The increasing use of composite structures in industrial applications requires the development of analytical and numerical tools for the design of these structures. Cure induced thermal residual stresses originate from the strong anisotropy of the thermal expansion coefficient of unidirectional lamina and the difference between the cure temperature and the operation temperature of the structure. The thermal residual stresses can affect the mechanical properties composite plates; these effects are more pronounced for thin laminates. In this work the influence of the thermal residual stresses on the stiffness of reinforced orthotropic composite plates was experimentally characterized. Carbon/epoxy plates were manufactured using two different techniques: one group of plates was manufactured by co-curing the reinforcers at 184 ^oC and a second group was manufactured by secondary bonding of the reinforcers to the plate at room temperature (22 ^oC); this second group is free of thermal residual stresses at room temperature. A topogrametry equipment was used to evaluate the stiffness of these two groups of composite plates. This equipment was calibrated for determining the surface transverse displacements of the plates when static loads were applied. A numerical model was elaborated using software ABAQUS® for comparison with the obtained experimental results. Good accuracy between experimental and computed displacements was obtained after proper calibration of the numerical model. The co-cured plates present substantially higher stiffness values than those obtained for plates produced with secondary bonding at room temperature. These experimental and numerical results demonstrate the importance of cure induced thermal residual stresses on the stiffness of thin composite laminated plates.*

Keywords*: composites, laminates, thermal stresses, topogrametry, stiffness*

1. INTRODUCTION

Laminated structures have been extensively used in the modern engineering, particularly in the aeronautical industry due to high strength, durability and light weight. They are very useful in civil, mechanical and transport engineering. The adequate utilization of laminates requires the development of analytical and numerical tools for the design of these structures.

The mechanical properties of composites laminates can be affected by thermal residual stresses. These stresses may result from external loading, boundary conditions, environmental conditions, characteristics of the constituent materials, and due to the curing process during manufacturing.

Analytical studies of thermal residual stresses effects in the mechanical properties of laminates can be found in the technical literature. Almeida and Hansen (1999) investigated the effect of manufacturing-introduced thermal residual stresses on the free vibration response of stringer reinforced composites plates. They assumed a number of different configurations for the plates and stringers. The scope of this work was restricted to the numerical analysis of reinforced symmetric laminated plates. The analysis was based on a finite element formulation using a bi-cubic Reissner-Mindlin element. They demonstrated that the natural frequencies might be increased by properly tailoring the residual stresses.

Dano and Hyer (1998) used the Rayleigh Ritz theory for the prediction of the displacements of non-symmetric laminated carbon/epoxy due to the curing process. They characterized the thermal residual stresses in terms of the out of plane displacements induced during manufacturing. Fares, Zenkour, and El-Marghany (2000) presented a refined non-linear first order theory for the determination of thermal effects on the bending response of cross-ply laminated plates. The non-linear governing equations of laminated plates subjected to thermomechanical loadings were derived. The influence of the geometric nonlinearity on the thermal bending response was illustrated.

Nogueira, Neto, and Almeida (1999) performed a preliminary experimental work to verify the numerical results obtained by Almeida and Hansen (1999).

The effect of the thermal residual stresses in the mechanical behavior of laminated plates can be evaluated using optical methods. The application of Moiré technique for the determination of in plane displacements in laminates is presented by Chai et al. (2003). This method is based on an interferometer using two laser beams. The residual in plane deformation in the composite was measured from the Moiré fringes and the corresponding residual in plane strain components ε_x, ε_y, γ_{xy} were calculated. The experimental results demonstrated that this micro-Moiré interferometer could resolve Moiré fringes in a range of ten to few hundreds of micrometers. The measurement technique has a great potential in many other applications.

New methods, such as topogrametry, can be used for the experimental determination of flexural properties of composites plates. Topogrametry is an optical method that combines the topometric and photometric methods for determining displacements on surfaces. This technique is based on the geometric relation between two images of the same object captured from two different perspectives; the orthogonal patterns of a grid projected onto the surface with a light projector is used to map the geometry of the surface.

Phase displacement technique is used for the determination of four phase maps. They contain the necessary information to relate the images of the two CCD cameras to the physical point to be measured. It is possible to determine three axes coordinate using standard triangulation methods with the application of the photometric technique.

In this work, a topogrametric technique was used for the experimental characterization of the flexibility of carbon/epoxy cross-ply laminates. The spatial distribution of transverse displacement for plates subjected to concentrated loads was measured thus characterizing the flexibility of the plate. Comparison of the measured displacements of plates with and without thermal residual stresses is a direct evaluation of the effect of thermal residual stresses on the flexural behavior of composite plates. The experimental results are compared to numerical models developed using software ABAQUS® for validation.

2. ANALYTICAL MODELING

The first step in the analytical solution of the problem is the numerical evaluation of the magnitude of the thermal residual stresses induced during the curing process. The analysis is usually based on a simplified model that does not account either to viscoelastic effects or to the variation of material properties with temperature (Palerosi and Almeida, 2007). Advanced commercial finite element packages include these features in the analysis, however, the experimental determination of viscoelastic parameters and variation of physical and mechanical properties with temperature is very costly and subject to wide scatter. Thus, thermal residual stresses are usually calculated from linear models. A usual practical procedure to compensate for neglecting these factors is to replace the cure temperature by an equivalent reference temperature chosen as to adjust the numerical and experimental results (Palerosi and Almeida, 2007). Another common hypothesis is to neglect part tool interaction effects and, as a consequence, the laminate is assumed to be unconstrained during the curing process. The stress stiffening is computed from the thermal stresses obtained for the thermal problem.

In summary, the thermal residual stresses are computed based on the following hypotheses:

- the analysis is linear;
- there are no external mechanical loads;
- laminates are symmetric and there is no membrane-bending coupling.

The thermo elastic anisotropic stress-strains relations, Eq. (1), can be written as:

$$\varepsilon_i = S_{ij}\sigma_{ij} + \alpha_i \Delta T \qquad i, j = 1, 2, \ldots 6 \tag{1}$$

where ε_i, S_{ij}, and $\alpha_i \Delta T$ are the total strains, mechanical strains and free thermal strains, respectively. Three dimensional stress-strain relations are determined by inversion, Eq. (2):

$$\sigma_i = Q_{ij}(\varepsilon_j - \alpha_j \Delta T) \qquad i, j = 1, 2, \ldots. 6 \tag{2}$$

where Q_{ij}, are the material stiffness in structural coordinates, α_j are the thermal expansion coefficients and ΔT is the temperature difference (operating temperature minus reference temperature), σ_i are the stresses and ε_j are the engineering strains. The above equations in matrix form become the Eq. (3):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{Bmatrix} \tag{3}$$

Thermal residual stresses, Eq. (4), are determinated by seeking a stationary value of the total potential energy when the system is subject only to a thermal change (Almeida and Hansen, 1999).

$$\phi = \frac{1}{2}\int_A \{e\}^T [D]\{e\} dA - \Delta T \int_A \left(\{\varepsilon\}\{N\}_t + \{\kappa\}\{M\}_t\right) dA \tag{4}$$

where *A* is the area of the plate, $\{N\}_t, \{M\}_t$ are the laminate stress results associated with the thermal loads and [*D*] is the matrix which relates middle surfaces strains to the stress resultants, {ε } is used to represent the vector of in-plane linear strain components and {*e*} is a middle surface linear strain vector.

The second step is the bending analysis of the plate subjected to a concentrated load. A linear static analysis is considerate for these computations. The stress stiffening effects due to the thermal residual stresses, Eq. (5) are incorporated in the analysis by including the geometric stiffness matrix associated with those stresses, that is, the flexural problem assumes the form:

$$\left([K]+[K]_G^{thermal}\right)\{a\}=\{f\} \qquad (5)$$

where [*K*] is the stiffness matrix of the plate, $[K]_G^{thermal}$ is the geometric stiffness matrix associated with the thermal stresses, {*a*} is the vector of structural displacements and {*f*} the vector of concentrated applied loads.

Since the thermal problem is linear, the geometric stiffness matrix $[K]_G^{thermal}$ is proportional to the temperature difference *ΔT*. Therefore, as proposed above, an equivalent reference temperature may replace the cure temperature when computing the temperature difference order to adjust the numerical results to the experimental data.

3. EXPERIMENTAL PROCEDURES

Symmetric reinforced square carbon/epoxy plates were used in this work to assess the effect of thermal residual stresses on the plate stiffness. The laminate for the analysis was $[0/90]_s$, with two carbon/epoxy fabric layers. The reinforcement was placed around the plate perimeter, forming a symmetric frame. Two different types of plates were considered, according to the adopted manufacturing process. For the first type of plates, the base plate and the reinforcement were manufactured separately and then bonded at room temperature (secondary bonding process). The adopted nomenclature for this kind of plate was *FB* (after "Frame reinforcement, Bonded"). This first set of plates is virtually free of thermal residual stresses at room temperature. In the second type of plates, the base plate and reinforcement were simultaneously cured in a specially designed mold. In this case, thermal residual stresses are introduced in the laminates during cooling down from the cure temperature to the room temperature. The nomenclature for this second type of plate is *FC* (after "Frame reinforcement, Co-cured"). All plates were cured at 177°C. The dimensions of the plates were shown in Fig 1. Lamina orientation is presented in Table 1. Two plates of type 1 and six plates of type 2 were manufactured and tested.

The mechanical properties of the materials (Tape and Fabric carbon/epoxy pre-impregnated materials provided by Hexcel Composites) used in manufacturing the plates are given in Table 2.

Table 1. Laminate orientation.

Laminate	
L1	$[(0,90)^{CF}]_S$
L2	$[(0/90)_2^{CT}/(0,90)^{CF}]_S$
L3	$[90_4^{CT}/(0,90)^{CF}]]_S$
L4	$[0_4^{CT}/(0,90)_2^{CF}]_S$

CT = carbon/epoxy unidirectional tape
CF = carbon/epoxy fabric

Table 2. Material properties.

Properties	Tape	Fabric
Longitudinal modulus of elasticity E_1, *MPa*	130100	66600
Transverse modulus of elasticity E_2, *MPa*	9400	66600
In plane Poisson's ratio υ_{12}	0.300	0.050
In plane shear modulus G_{12}, *MPa*	5800	4600
Transverse shear modulus G_{13}, *MPa*	5800	4600
Transverse shear modulus G_{23}, *MPa*	3360	3360
Longitudinal thermal expansion coefficient α_1, $^0C^{-1}$	-0.17e-06	0.350
Transverse thermal expansion coefficient α_2, $^0C^{-1}$	23.10e-06	1.79e-06
Ply thickness *t, mm*	0.170	1.79e-06
Density *ρ, g/cm*^{3}	1.56	1.56

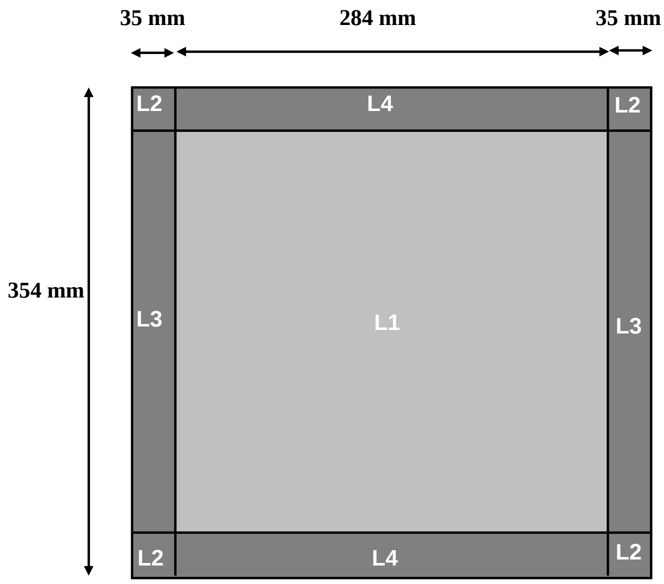


Fig. 1. Square plates dimensions.

The bending tests consisted in mounting the plate in a specially designed test fixture such that the plate is simply supported at each of the four vertices, as shown in as shown in Fig. 2. A dead weight was applied as a concentrated load to the plate. The measure of the plate transverse structural displacements allows the experimental characterization of the plate flexibility and as a consequence, its stiffness. The loads were applied at two different points and two values of loads were used to evaluate if the structural behavior of the plate was linear. Just the results obtained for loads applied at the center of the plate are presented in this work.

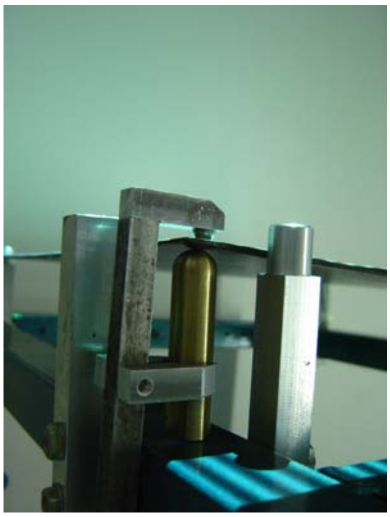


Fig. 2 Detailed view of the plate boundary conditions at the vertices

An optical system, provided by Photonita, formed by two CCD cameras and a projector was assembled for measuring the transverse displacement in the bending tests. The system based in topogrametric methods (Fantin, 1999) yields a map of the surface within accuracy of 0.02 mm. Figure 3 presents a photo of the system set up and just one of the CCD cameras is visible in the picture.

In order to measure the plate transverse due to the applied the loads it is necessary to map the plate surface before and after the load is applied and computing the difference. It does not suffice to map the surface after the load is applied due to the initial imperfections of the plate and imperfect alignment of the measuring system and the normal direction to the plate. The difference between the measured surfaces was computed before and after loading compensate for those effects allowing and accurate measurement of the transverse displacements due to the applied load. The TPLA40012 software provided by Photonita is used for mathematical computations on the measured surfaces. Fig. 4 illustrates typical contour plots of the surface before and after applying the load and the resulting subtraction operation on the measured surfaces. It was necessary an initial calibration tests to adequate the use of optical system.

In order to provide better image contrast, one of the faces of each plate was painted white once the original black color of the carbon/epoxy plates are not appropriate for projecting the grid. A metallic piece was glued at each loading application points. Loads of 4.9 N and 9.8 N were applied in the metallic piece for assessing the linearity of the bending behavior of the plates.

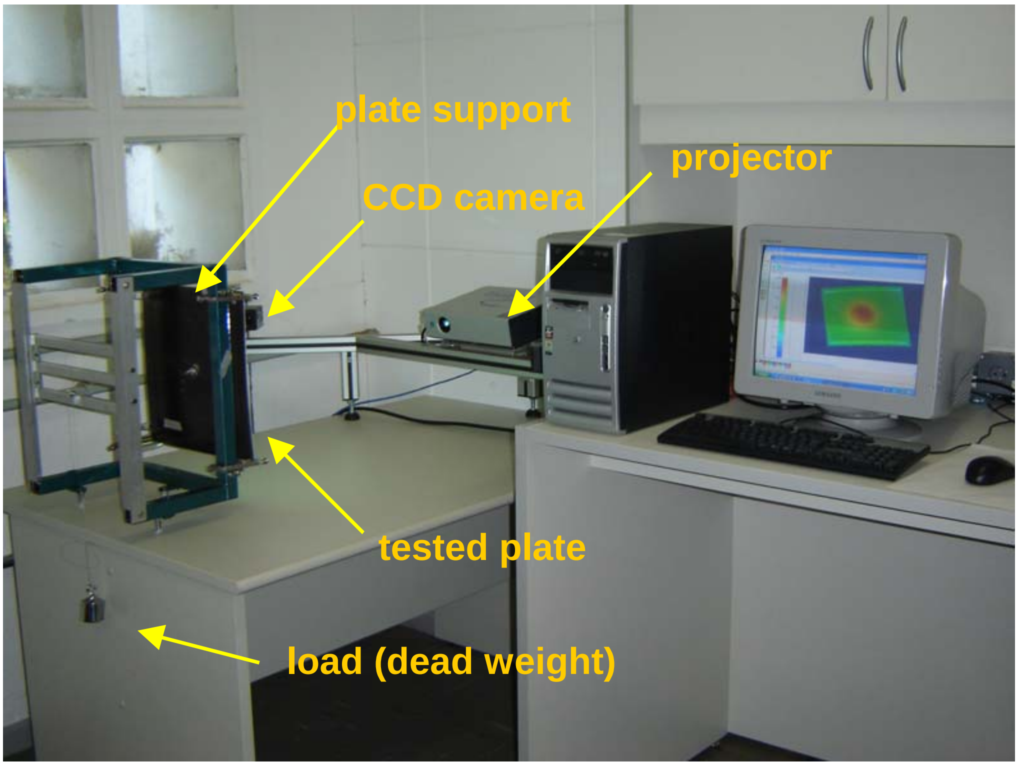
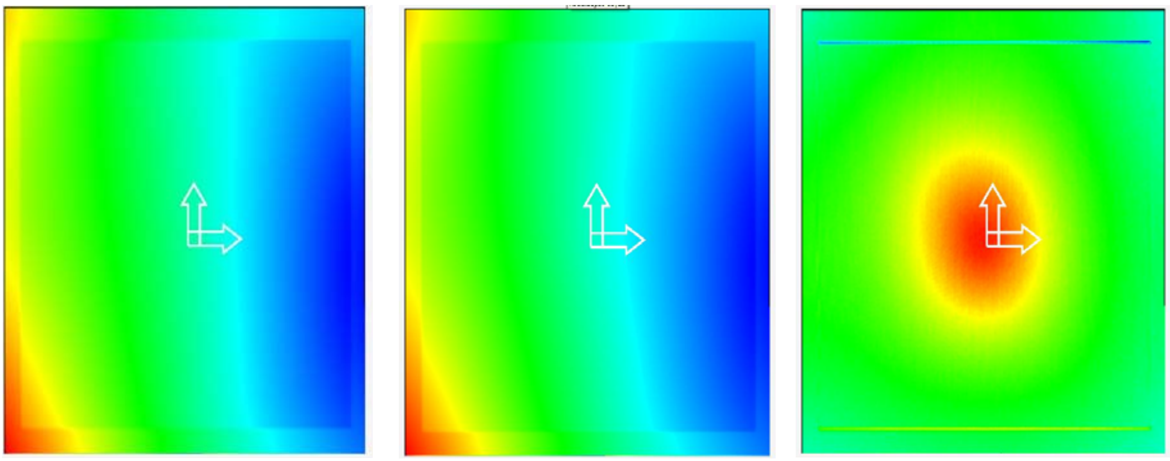


Fig. 3. Loading and measuring system for plate bending tests.

Two images were obtained for each applied loads using the TPLA40012 software. The first image was taken for unloaded plate whereas the second was captured after applying the loads. The subtraction of the two images was done using the software. In the third image could visualize the plate displacement at the surface due to loading applied. The example depicted in Fig. 4 demonstrates that only after subtraction of the surfaces the expected symmetric displacement pattern is obtained. Notice that the color codes are different in Fig. 4(a)-4(c). This fact combined with misalignment of the topogrametric system makes Fig. 4(a) and 4(b) look very similar.



a) Unloaded plate b) Loaded plate c) Image subtraction

Fig. 4. Images from bending tests.

4. RESULTS AND DISCUSSION

The transverse displacement profiles along a horizontal line across the center of the plate were determined from the obtained images in the bending tests for a concentrated load applied at the center of the plate. A numerical model in software ABAQUS® was elaborated to assess the experimental results. This model considered the standard analysis, mesh with 4-node quadrilateral plate elements of doubly curved thin shell. The element formulation uses reduced integration, hourglass control, and finite membrane strains.

The experimental and numerical spatial distributions of transverse displacements for the plates manufactured with secondary bonding (free from thermal stresses at room temperature) are compared in Figs. 5 and 6 for 4.9 N and 9.8 N loads, respectively.

The experimental results demonstrate the existence of geometrically non linear effects in the plate structural behavior in the absence of thermal effects. The obtained experimental values are in reasonably good agreement with the values obtained from the adjusted theoretical model.

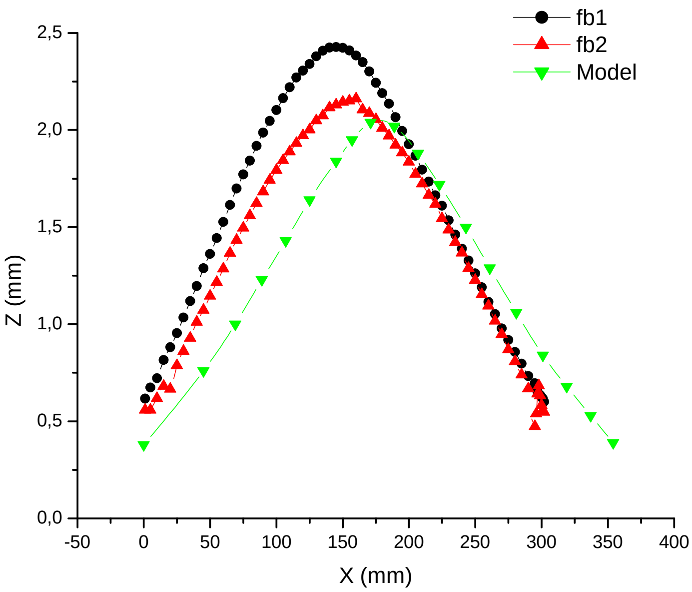


Fig. 5. Spatial distribution of transverse displacements for FB (secondary bonding) plates for 4.9 N load.

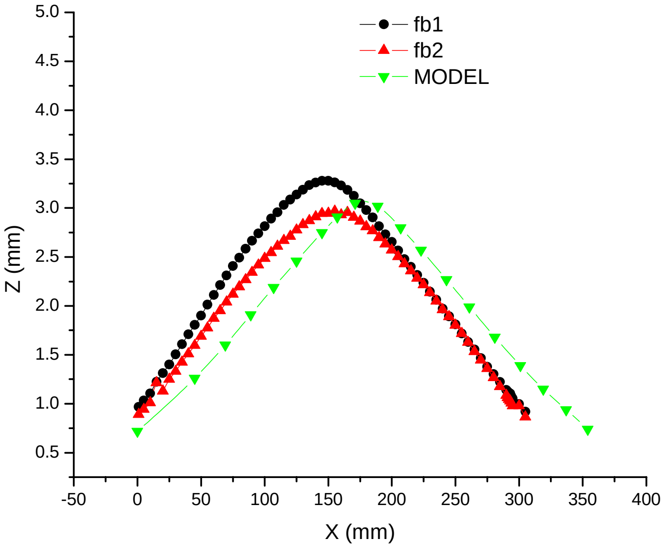


Fig. 6. Spatial distribution of transverse displacements for FB (secondary bonding) plates for 9.8 N load.

In order to evaluate the effect of the thermal residual stresses on the stiffness of the composites plates, the co-cured reinforced plates cured at 184°C square were tested. The flexural behavior of these plates was characterized using the same procedure, applying a concentrated load at the center of the plate. The spatial distribution of the transverse displacements along a horizontal line across the center of the plates is presented in Figs. 7 and 8, for the type FC plates (co-cured). For the analytical model, the thermal problem is first solved to compute the thermal residual stresses. The second step, the structural displacements including the stress stiffening effects due to the thermal stresses are computed according to Eq. (5). It should be noted that the experimental results in Figs. 6 and 7 include both, the effect of geometrically non-linear effects due to the plate boundary conditions and the stress stiffening effect introduced by the thermal residual effects. The reference temperature was chosen to be 184 °C to yield best agreement with experimental results. Therefore, the temperature gradient was set to 162 °C for room temperature of 22 °C. By comparison of Figs. 4 and 5 with Figs. 6 and 7, a significant increment in the stiffness of the reinforced plates is observed due to the thermal stresses introduced during the manufacturing process.

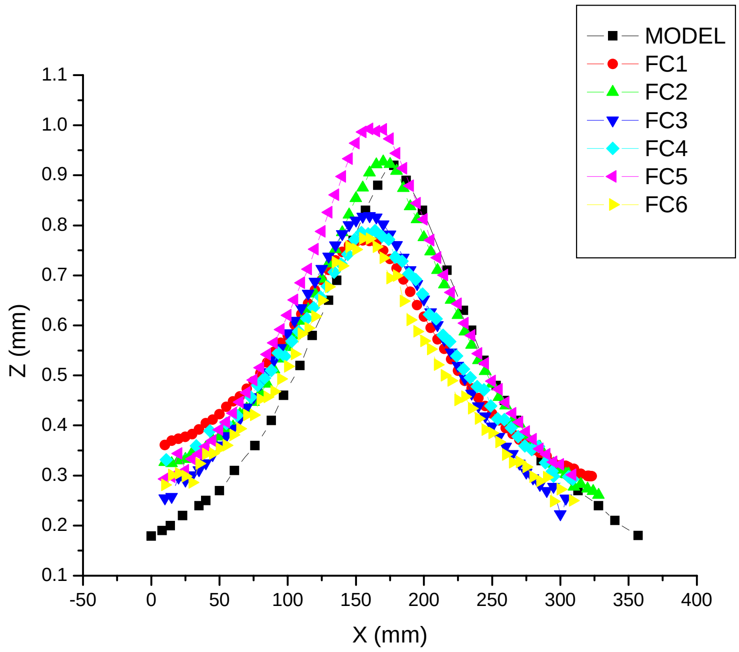


Fig. 7. Spatial distribution of transverse displacements for FC plates (co-cured) for 4.9 N load.

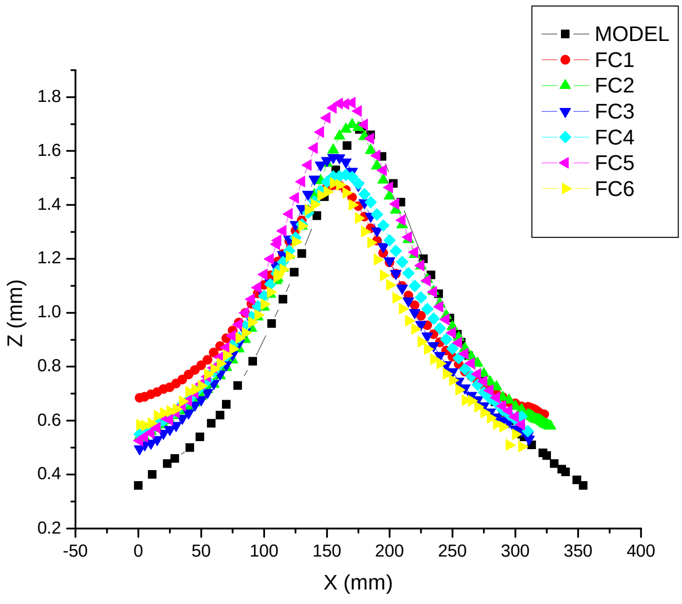


Fig. 8. Spatial distribution of transverse displacements for FC plates (co-cured) for 9.8 N load.

Comparing the displacements obtained for the plates without thermal residual stresses (Figs. 5 and 6 – secondary bonding plates) to the results obtained for the plates with thermal residual stresses (Figs 7 and 8 – co-cured plates) it can be concluded that the thermal stresses affect not only the magnitude of the displacements but they also affect their shape of distribution. It should also be noted that these conclusions are valid for both the numerical and experimental results. The numerical model also accurately characterizes the stress stiffening effect observed in the experimental measurements.

The experimental and numerical transverse displacements obtained at the center of the two types of plates are summarized in Tab. 3 and 4 for comparison purposes. The results in Tab. 3 correspond to the plates produced by secondary bonding, that is, nearly free from thermal residual stresses at room temperature. In this table, it is observed that when the load is multiplied by a factor of two the displacements increase by about 50%. This trend is valid for the experimental and numerical results and is associated with geometrical non-linearities introduced by the boundary conditions of the test set-up and the magnitude of the displacements. Figures 5 and 6 indicate that the shape of the distribution along the horizontal centerline of the plate could be approximated by a parabola.

Table 3. Transverse displacements at the center of the plates produced by secondary bonding (FB plates)

Load (N)	Transverse displacements at the center of the plate (mm)			
	FB1	FB2	Average	Model
4.9	2.43	2.22	2.32	2.04
9.8	3.27	3.01	3.14	3.05

It is interesting to compare these results to those of the co-cured plates that are subjected to the effect of the thermal residual stresses. The results, listed in Tab. 4, demonstrate that when the load is doubled, the displacements increase by about 80%. Therefore, it can be stated that the stress stiffening effects are less pronounced for the plates with thermal residual stresses. This may be explained by two factors: (a) the magnitude of the displacements is smaller for the plates with thermal residual stresses, and (b) the stress stiffening effect introduced by the thermal residual stresses is larger than the one caused by the boundary conditions; therefore, the mechanical behavior of the plates is dominated by this factor. Examining the effect of the thermal residual stresses on the magnitude of the displacements, it can be observed that the displacement at the center of the plate is reduced by a factor larger than two. This demonstrates that the stiffness of the co-cured plates is indeed larger than that of the plates produced by secondary bonding. However, it is misleading to characterize the plate stiffness based only on this figure. It should be noticed in Fig. 7 and 8 that the shape of the distribution of displacements is similar to that of a Gaussian curve. That is, the thermal residual stresses do not only affect the magnitude of the displacements but also affect their entire distribution along the plate.

Table 4. Transverse displacements at the center of the plates produced by co-cured (FC plates)

Load (N)	Transverse displacements at the center of the plate (mm)							
	FC1	FC2	FC3	FC4	FC5	FC6	Average	Model
4.9	0.77	0.92	0.81	0.75	0.99	0.77	0.88	0.92
9.8	1.47	1.69	1.57	1.50	1.77	1.47	1.57	1.69

5. CONCLUSIONS

The experimental studies showed that thermal residual stresses originated during the manufacture process affected considerably the bending properties of reinforced plates. The co-cured plates presented substantially higher stiffness values than those obtained for plates produced with secondary bonding at room temperature. These plates are essentially identical except for the manufacturing induced stresses. Co-cured plates have thermal residual stresses whereas the plates produced by secondary bonding at room temperature are nearly free from thermal residual stresses. An important contribution of this paper is that the experimental results obtained herein corroborate the numerical predictions of previous works (Almeida and Hansen, 1999). To the best of the authors' knowledge, this is the first paper in the open literature that brings an experimental confirmation of this important aspect.

Also, a geometric non-linearity due to the plate boundary conditions is also observed. This effect is more pronounced for the plates produced by secondary bonding because of the larger magnitude of the displacements. A second factor that explains this observation is that the stress stiffening caused by the thermal residual stresses is the dominant factor in the co-cured plates.

Finally, another important contribution of this paper is the use of the topogrametric technique to measure the distribution of transverse displacements. This technique presents enormous advantages when compared to the traditional approach of using displacement sensors such as LVDT's. The topogrametric technique is non-contact avoiding friction and mechanical loading on the plate. Also, it allows accurate measurement of transverse displacements on a very large number of points over the surface of the plate; thus capturing the deformed shape of the plate. Using a reduced number of displacement sensors along the plate would provide a poor description of the distribution of displacements. The adopted procedure of taking images before and after loading accurately compensates for misalignments and imperfections on the plate surface. Due to the fast processing of images, it was possible to test a large number of plates, with different magnitudes and location of the load. In each test, the topogrametric procedure measured the displacements on a very large number of points in a short interval of time with good accuracy.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Almeida, S.F; Hansen, J.S., 1999, "Natural frequencies of composites plates with tailored thermal residual-stresses". International Journal of Solids and Structures, v.36, pp 3517-3539.

Chai, G.B. et al., 2003, "Residual Interlaminar Deformation Analysis in the Carbon/epoxi Composites Using Micro-Moiré Interferometry". Composites Sciences and Technology, v.63, pp171-175.

Dano, M.L; Hyer, M.W. 1998, "Thermally-induced deformation behavior of unsymmetric laminates". International Journal of Solis and Structures, v.35, pp 2101-2120.

Fantin, A.V., 1999 "Medição de formas livres tridimensionais por topogrametria". Dissertação de Mestrado. Florianópolis, 90p (in Portuguese).

Fares, M.E.; Zenkour, A.M.; El-Marghany, M. Kh., 2000, "Non linear thermal effects on the bending response of cross-ply laminated plates using refined first-order theory". Composites Structures v.49, pp 257-267

Nogueira Neto, Z.S.; Almeida, S.F.M., 1999 "Efeito das tensões térmicas residuais de cura no comportamento mecânico de laminados carbono/epoxi." In: Encontro Técnico Cientifico de Carbono/Grafite, São Jose dos Campos (in Portuguese).

Palerosi, A. C., Almeida, S. F. M., 2007 "Thermoelastic Evaluation of Composite Laminates Using Digital Imaging Processing", Composites Part A: Manufacturing, in print.

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