INVERSE MODELING AND SHAPE OPTIMIZATION OF AN ENERGY ABSORBER

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Abstract. *This work presents a shape optimization methodology applied to structures submitted to impact loads in order to improve their crashworthiness. To this end, it is necessary to know the structural material properties, which were obtained using a dual experimental–optimization methodology. Optimum parameters are obtained for strain rate sensitive materials described by Johnson-Cook constitutive law that best fit the experimental data. These parameters were then used in the analysis of a complex structure, which is crashworthy optimized using a Response Surface Methodology.*

Keywords: *Material Characterization, Structural Optimization, Inverse Modeling, Structural Impact, Crashworthiness.*

1. Introduction

In recent years, several different optimization schemes combined with finite element modeling have been used to improve the energy absorption of structures. Often these structures are designed for crashworthiness applications. In this work, the shape optimization method is used for crashworthiness design aiming to improve the capability of an object of absorbing energy by plastic deformation.

Shape optimization consists in converting the boundaries of a given geometry into splines and polynomial functions. In this case, the parameters of the splines and polynomial functions are the design variables (Bendsøe and Sigmund 2003). Craig et al. (2002) used shape variables in a vehicle model undergoing a full frontal impact to improve its crashworthiness. Also, Hailiang et al. (2002) optimized the thickness of thin-walled structures to work as energy absorbers in automotive systems. Walter et al. (2003) improved the crashworthiness ability of an automotive cockpit by modifying the size and shape of some ribs structures. Finally, Stander et al. (2003) applied shape optimization in a full vehicle impact model to improve its crashworthiness and test three different metamodeling techniques: Response Surface Method, Neural Networks and Kriging.

Another important point in every finte element model concerns on the material properties and in the present work the inverse modeling technique is applied to obtain the constitutive law parameters that best describe the physical material behavior according to a dual experimental–optimization methodology. It is dealt here with complex geometries and situations that are not possible to perform a traditional material characterization using a standard specimen in a tensile or compression test. In these scenarios, inverse modeling techniques are commonly used like in Kajberg and Lindkvist (2004), Kajberg and Wikman (2007), Mahnken and Stein (1996a), Mahnken and Stein (1996b), Mahnken and Stein (1997), Mahnken and Kuhl (1999) and Stander and Roux (2005).

2. Problem Statement

The main objetive of this work is to study the behavior of a compressor employed in domestic refrigerators submitted to a drop test. After its production, the compressor may be transported to the customer. During transportation the compressor can fall directly on the ground and some of its components may be damaged, causing serious problems in the whole system.

Previous numerical simulations performed in LS-Dyna finite element code and experimental drop tests revealed that the brackets shown in Figure 1 are the most critical parts of the compressor suspension system. Hence, the basic idea was to find a structure capable of absorbing the impact energy as much as possible but keeping the structural integrity of the brackets. It was then explored, as it will be shown, an optimization of these supports, from now on called energy absorber, in order to diminish the damage on the compressor caused by its fall.

(a) Frontal bracket.

(b) Rear bracket.

Figure 1. Metallic brackets of the compressor suspension system.

The energy absorber in the compressor suspension system is shown in Figure 2. It is made of a polymer called poly-terephthalate butylene or simply PTB. Due to the small size of the part, it is not possible to extract a normalized specimen from it and from the economic point of view is not possible to build a specimen using other traditional manufacturing processes. A way found to this problem was found applying the inverse modeling technique demonstrated in next section. With the material constitutive law constants determined by the inverse modeling technique one can develop a shape optimization of the energy absorber to increase its crashworthiness.

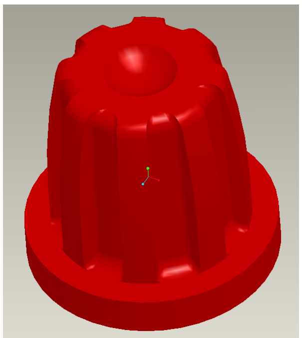


Figure 2. Energy absorber of the compressor suspension system.

3. Inverse Modeling Technique

The inverse modeling technique presented by Kajberg and Lindkvist (2004), Kajberg and Wikman (2007), Mahnken and Stein (1996a), Mahnken and Stein (1996b), Mahnken and Stein (1997), Mahnken and Kuhl (1999) and Stander and Roux (2005), Figure 3 is described below

1. A parametrization of the system, which is a complete description of a physical system $\mathcal{M}$ using a minimal set $\mathcal{P}$ of model parameters;

2. So-called forward modeling, which is to find physical laws that, within a given set of model parameters, predict measured quantities belonging to $\mathcal{M}$;

3. Finally, inverse modeling, where measured quantities belonging to $\mathcal{M}$ are used for deducing values of the model parameters.

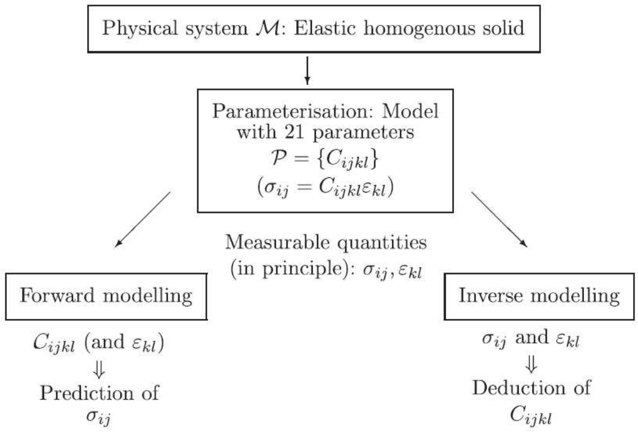


Figure 3. Example of a scientific study of a physical system according to Kajberg and Lindkvist (2004).

3.1 Objetive Function

The objective function chosen by Anghileri et al. (2005), Avalle et al. (2005), Croix et al. (2005) and Monacelli et al. (2005) is a least-square functional with residuals based on the difference between experimental and FE-calculated data. The least-square criterion is justified based on the hypothesis that the sum of several different contributions will tend to be normally distributed, irrespective of the probability distribution of the individual contributions.

The data consists of two quantities measured and evaluated at time instants t. These are the axial displacements $X_i(t)$ and $x_i(t)$, respectively experimental and numerical values. And the loading forces $F_j(t)$ and $f_j(t)$, respectively experimental and numerical values. However, the measured quantities are of different physical dimensions and therefore some kind of normalization has to be performed before summing all squares. Further, the influence of each quantity has to be tuned in order to get sum of squares of the same magnitude. Therefore, all residuals are normalized by basically scaling them with the difference between the maximum and mean value of the actual quantity. The sums of squares based on each individual quantity were studied and mutually compared after the parameter estimations had been performed. They were all of same magnitude. The objective function $\mathcal{F}$ also known as *Standard Composite* is given by

$$\mathcal{F} = \sqrt{\sum_{j=1}^{m} W_j \left[\frac{f_j(x) - F_j}{\sigma_j}\right]^2 + \sum_{i=1}^{m} w_i \left[\frac{x_i - X_i}{\chi_i}\right]^2}, \tag{1}$$

where σ e χ are scale factors, W and w being weights.

3.2 Response Surface Methodology

To perform the optimization scheme in LS-OPT optimization comercial package, the Response Surface Methodology was chosen according to exemples demonstrated by Stander and Roux (2005) in foam material characterization.

Response Surface Methodology (or RSM) requires the analysis of a predetermined set of designs. A design surface is fitted to the response values using regression analysis. Least squares approximations are commonly used for this purpose. The response surfaces are then used to construct an approximate design ”subproblem” which can be optimized.

The response surface method relies on the fact that the set of designs on which it is based is well chosen. Randomly chosen designs may cause an inaccurate surface to be constructed or even prevent the ability to construct a surface at all. Because simulations are often time-consuming and may take days to run, the overall efficiency of the design process relies heavily on the appropriate selection of a design set on which to base the approximations. For the purpose of determining the individual designs, the theory of experimental design (Design of Experiments or DOE) is required. Several experimental design criteria are available but one of the most popular for an arbitrarily shaped design space is the D-optimality criterion. This criterion has the flexibility of allowing any number of designs to be placed appropriately in a design space with an irregular boundary. The understanding of the D-optimality criterion requires the formulation of the least squares problem.

Consider a single response variable y dependent upon a number of variables x. The exact functional relationship between these quantities is

$$y = \eta(x) \tag{2}$$

The exact functional relationship is now approximated (e.g. polynomial approximation) as

$$\eta(x) \approx f(x) \tag{3}$$

The approximating function f is assumed to be a summation of basis functions:

$$f(x) = \sum_{i=1}^{L} a_i \phi_i(x), \tag{4}$$

where L is the number of basis functions ϕ_i used to approximate the model.

The constants **a**=$[a_1,a_2,...a_L]^T$ have to be determined in order to minimize the sum of the square error:

$$\sum_{p=1}^{P}\{[y(x_p) - f(x_p)]^2\} = \sum_{p=1}^{P}\left\{[y(x_p) - \sum_{p=1}^{L} a_i\phi_i(x_p)]^2\right\} \tag{5}$$

where P is the number of experimental points and y is the exact functional response at the experimental points $\mathbf{x}_i$.

The solution to the unknown coefficients is:

$$a = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y \tag{6}$$

where **X** is the matrix:

$$\mathbf{X} = [\mathbf{X}_{ui}] = [\phi(x_u)] \tag{7}$$

The next critical step is to choose appropriate basis functions. A popular choice is the quadratic approximation

$$\phi = [1, x_1, \ldots, x_n, x_1^2, x_1x_2, \ldots, x_1x_n, \ldots, x_n^2]^T \tag{8}$$

but any suitable function can be chosen. LS-OPT allows linear, elliptical (linear and diagonal terms), interaction (linear and off-diagonal terms) and quadratic functions.

3.3 Results

Five specimens were tested in a Instron machine under compression load. The results of each specimen were recorded in a equal interval of 1 second resulting in about 241 values of force and 241 values of axial displacement. The mean force versus displacement curve was then calculated.

After that, the optimization of the constants of the Ramberg-Osgood constitutive law were found applying the optimization scheme using the Response Surface Methodology and Equation 1. The Ramberg-Osgood constitutive law is given by

$$\sigma_s = (\sigma_0 + K\epsilon^n), \tag{9}$$

where σ_s is the quasi-static stress, σ_0 is the quasi static yield stress, K and n are material hardening constants.

The chosen design variables were the Young Modulus (E) and the constants of Equation 9 σ_0, K and n. Figure 4 shows the comparison between the force versus displacement curve obtained by experimental tests and by optimization of these material parameters. Table 1 presents the histories of the design variables and their bounds.

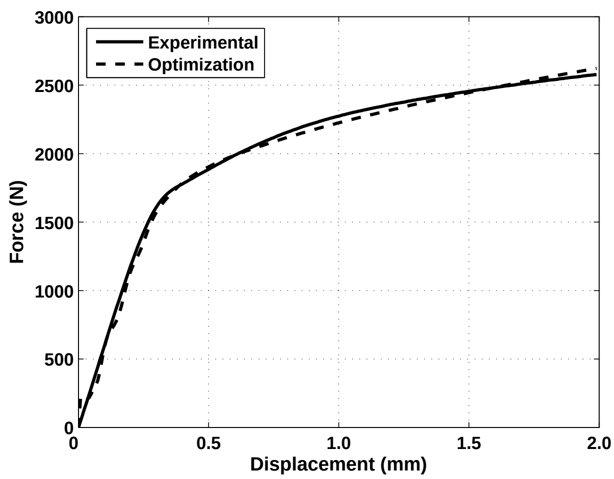


Figure 4. Comparison between the force versus displacement curve of the energy absorber obtained by experimental tests and by optimization of material parameters.

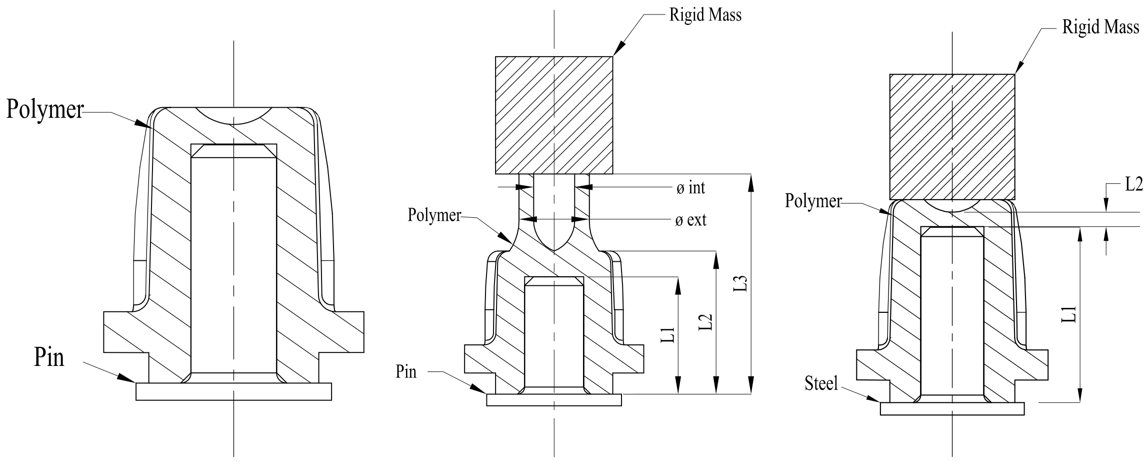
Table 1. Bounds and optimum values of the design variables.

Variable	Lower Bound	Optimum Value	Upper Bound	Unit
E	0.50	3.6088	5.00	GPa
σ_0	30.00	32.00	32.00	MPa
K	25.00	71.65	100.00	MPa
n	0.00	0.1108	0.20	dimensionless

4. Shape Optimization

Now with the correct material properties it is possible do develop the shape optimization of the energy absorber shown in Figure 2. The Response Surface Methodology was used again. The mesh, shape variables and optimization routine were created in Altair HymperMesh, Altair HyperMorph and Altair HyperStudy, respectively.

Two different shapes were proposed. The first model has many geometry modifications compared with the original one, as can be seen in Figure 5.b. On the other hand, the second model received simple modifications, the most significant being the length of the internal pin in Figure 5.c. Both models were impacted by a 1 Kg rigid mass at velocity of 3.41 m/s. Table 2 shows the histories of the design variables and their bounds for each model in Figure 5. These bounds are necessary due to geometric and manufacturing constraints. Figure 6 presents a comparison between kinetic energy absorbed in each case.



(a) Original Model. (b) Model 01. (c) Model 02.

Figure 5. Original model of the assembly steel pin / polymer energy absorber and its proposed shapes.

Table 2. Histories of the design variables and their bounds for each model in Figure 5.

Model	Variable	Initial Value (mm)	Lower Bound (mm)	Upper Bound (mm)	Optimum Value (mm)
Fig. 5.b	Ø ext	6.00	5.00	7.00	5.56
	Ø int	3.50	3.50	4.50	3.50
Fig. 5.c	L1	14.00	10.00	14.00	10.00
	L2	2.15	1.15	2.15	1.15

5. Applications of the optimized shapes

After the material parameter and shape optimization of the energy absorber were performed, the suspension system of the domestic refrigerator compressor was analyzed. The domestic refrigerator compressor is a complex assembly (see Figure 7.a) and an idealized finite element model (see Figure 7.b) was developed the represent the real geometry. Since the interest of this work was only on the suspension system and energy absorbers, all the body of the compressor was represented by a single rigid mass.

The material of the brackets in Figure 1 were characterized quasi-statically and dynamically using the same experimental-optimization procedure as in Børovik et al. (2005) to obtain the material parameters of John-Cook constitutive law where use was made of a pressure pulse bar capable to load the specimens covering strain rates from 100 to 2000/s. The well-know John-Cook constitutive law is given by

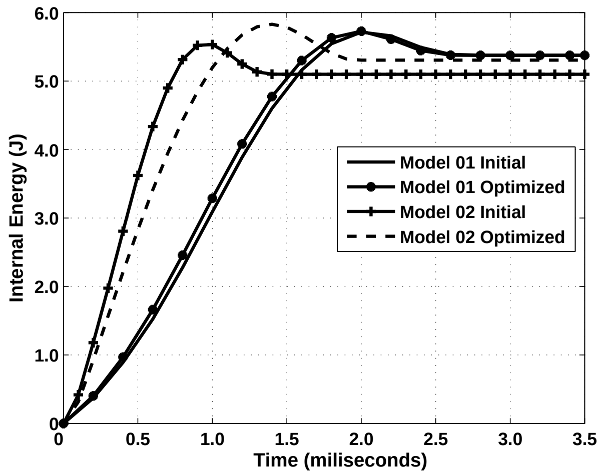


Figure 6. Comparison between kinetic energy absorbed in each model in initial and optimized situations.

$$\sigma = [A + B\epsilon^n][1 + C \ln \dot{\epsilon}^*][1 - T^{*m}], \tag{10}$$

where A is the material yield stress, B and n are material hardening constants, C represents the strain rate effects and m the temperature effects (not considered in this work). The obtained material parameters are shown in Table 3.

Table 3. Johnson-Cook material parameters obtained with a experimental-optimization procedure as in Børovik et al. (2005).

Bracket Type	A (MPa)	B (MPa)	n	C
Figure 1.a	250.15	416.96	0.5224	0.0533
Figure 1.b	300.00	597.92	0.4902	0.0190

After that, two finite element model were simulated using LS-Dyna finite element comercial code: a model without shape optimization of the energy absorber in Figure 5.a. (No Optimization) and a model including the optimized shape of the model in Figure 5.c. (Model 02 Optimized). Due to the increase in the product cost and change in its manufacturing process, the model of Figure 5.b was not considered in this analysis. Also this model did not presented a significant increase in the initial and optimized absorbed kinetic energy as it can be seen in Figure 6.

Four measures were controlled in the FE simulations. Each measure corresponds to the distance from an edge of the bracket to an edge in the rigid mass shown in Figure 8. The difference between the displacement of a node of the bracket and a node of the rigid mass gives the deformation of the bracket. Table 4 shows the comparison between the deformations in the no optimized (No Optimization) and optimized (Model 02 Optimized) models.

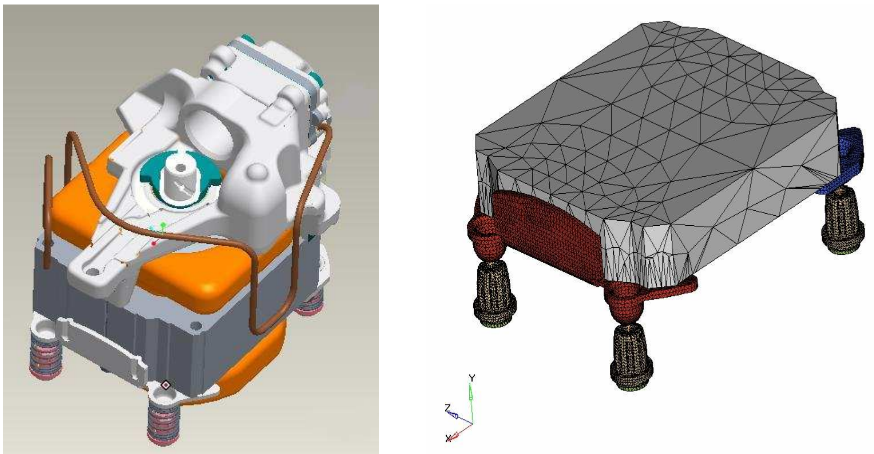
Table 4. Controlled measures in FE simulations according to Figure 8.

Measure Number	**Displacement (δ) (mm) No Optimized**	**Displacement (δ) (mm) Optimized**
1	2.68	0.00
2	2.51	1.88
3	0.75	0.04
4	1.07	0.05

The kinetic energy absorbed by the energy absorbers of both models are plotted in Figure 9. As it can be seen in Figure 9, the crashworthiness ability of the optimized model increased when compared to the original one, indicating that the effective plastic strain of the brackets in Figure 2 can be well reduced.

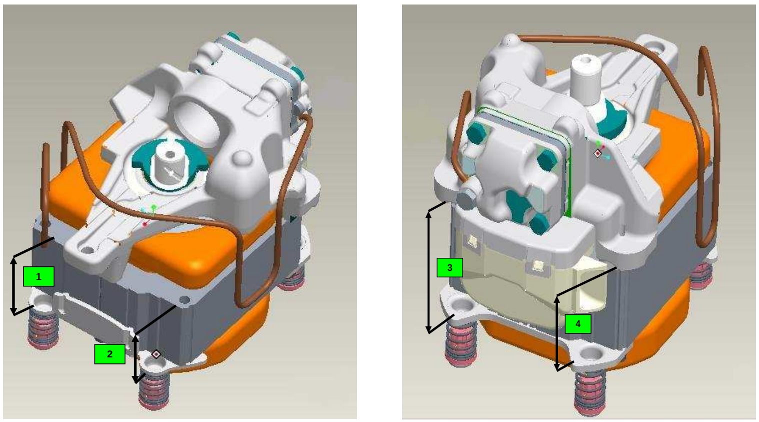
6. Conclusions

The Inverse Modeling Technique reveled to be a powerful tool when a standard specimen is not available for tensile-compression tests. Even few iterations and a linear approach in the optimization scheme leads to small errors and the numerical results of the force versus displacement curve in Figure 4 when compared with the experimental one.



(a) CAD Model. (b) FE Model.

Figure 7. CAD model of the compressor without cover and FE model with 71192 elements and 22753 nodes.



(a) Measures 1 and 2. (b) Measures 3 and 4.

Figure 8. Controlled measures in FE simulations.

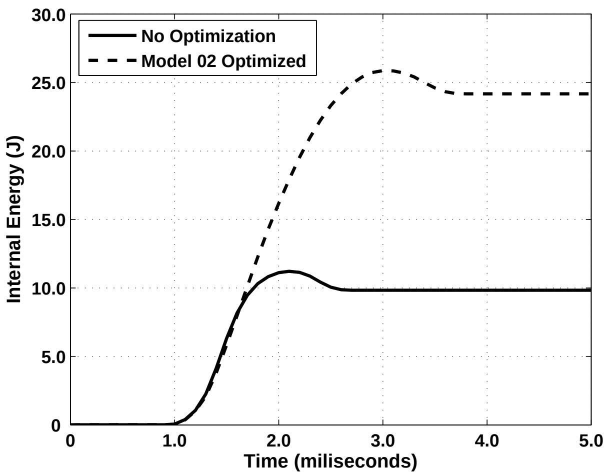


Figure 9. Kinetic energy absorbed by the energy absorbers during the simulation of the compressor system.

The shape optimization of the energy absorber also seems to be a powerful tool since with few shape variables was possible to improve the crashworthiness ability of the energy absorbers and the loss in system kinetic energy, as it can be seen in Figure 9 of Model 02. On the other hand, Model 01 of Figure 5 did not revealed the same good increase in the energy absorption, maybe because the shape variables or the initial shape were not well chosen. Furthermore, the effective plastic strains of the brackets in Figure 1 were dramatically reduced in the optimized model according to the respective values in Table 4 when compared with the model without optimization.

7. Acknowledgements

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