ON THE MODELLING OF NATURAL CONVECTION IN ADJACENT FLUID AND POROUS LAYERS

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Abstract. *This study deals with the onset of thermal natural convection in a system consisting of a fluid layer overlying a homogeneous porous medium. One of the fundamental open questions concerning two-layer systems is the modelling of the interface and its consequences on transport phenomena. Two different formulations are generally adopted. In the one-domain approach, a unique set of conservation equations is written for the entire domain, thus avoiding the explicit formulation of boundary conditions at the interface. In the two-domain approach, conservation equations in the fluid and in the porous region are coupled by the appropriate set of interfacial conditions. One-domain results are compared with those obtained with the Darcy and the Darcy-Brinkman formulations of the two-domain approach. Important discrepancies are observed between one- and two-domain models. It is shown that, when the transition between the two regions is characterized by discontinuous spacial variations of the macroscopic properties, the one-domain approach formulation has to be modified by taking the derivatives in the sens of distributions. In this way, one- and two-domain formulations lead to the same stability thresholds.*

Keywords: *natural convection, superposed layers, interfacial modelling, linear stability analysis.*

1. INTRODUCTION

Convective heat and species transport at the interface between a fluid and a porous region can be encountered in numerous industrial processes (solidification, filtration, catalytic reactor, drying, etc) or environmental situations (geothermal systems, ground water pollution, etc) and therefore, transport phenomena analysis in such configurations has been the subject of particular attention in the last decades (Nield and Bejan, 2006). Nevertheless, one of the fundamental open questions concerns the modelling of the fluid/porous interface and its consequences on transport phenomena.

Two different formulations are generally adopted. In the *one-domain approach*, the porous layer is considered as a pseudo fluid and the whole cavity as a continuum (Arquis and Caltagirone, 1984). In that case, heat and mass transfer are governed by a unique set of conservation equations valid in both the fluid and porous regions thus avoiding the explicit formulation of boundary conditions at the interface. The momentum conservation equation is a modified Navier-Stokes equation, which naturally incorporates the viscous diffusion contribution (Brinkman term) in the porous medium.

In the *two-domain approach*, conservation equations in the fluid and in the porous region are coupled by the appropriate set of interfacial conditions. For momentum transport, these conditions mainly depend on the order of the momentum equation in the porous medium which choice has been widely commented since the pioneering study by Beavers and Joseph (1967). In this study, Beavers and Joseph considered a one dimensional flow parallel to the fluid/porous interface. Since the flows in the fluid and porous layers are described by the Stokes and Darcy equations, respectively, a semi-empirical slip boundary condition was proposed at the interface

$$\left.\frac{\partial u}{\partial z}\right|_{z=int} = \frac{\alpha}{\sqrt{K}}(u_{int} - U) \tag{1}$$

where u_{int} is the fluid velocity at the interface, U is the seepage velocity, K is the permeability of the homogeneous porous material and α is an *empirical* dimensionless slip coefficient. The agreement between the experimental data provided in the work by Beavers and Joseph (1967) and the analytical solution is obtained by adjusting the values of α between 0.1 and 4, depending on the nature of the porous layer. This parameter has been found to be strongly dependent on the structure of the porous interface, but not on the nature of the fluid. Nield (1977) was the first to use the slip condition in the stability analysis of superposed fluid and porous layers. Poulikakos *et al.* (1986) also reported a numerical study of high Rayleigh number convection in superposed layers, using the Beavers and Joseph condition. A generalization of the slip condition for multidirectional flows was proposed by Jones (1973). An interesting comparison between the linear stability results obtained using both the Beavers and Joseph and the generalized Jones condition is shown in the work by Taslim and Narusawa (1989).

An alternative solution to the problem of matching the flow equations in the two regions is to use the Brinkman correction to the Darcy law (Brinkman, 1947). Therefore, momentum equations in both regions are of the same differential order and continuity of both velocity and shear stress can be satisfied. In this case, Neale and Nader (1974) have shown that the analytical solution is equivalent to the solution of Beavers and Joseph (1967) if $\alpha = \sqrt{\mu_{eff}/\mu}$ (μ_{eff} being the effective viscosity involved in the Brinkman term). Finally, when important spatial variations of the porous structure are present at the fluid/porous inter-region, a macroscopic stress jump boundary condition has been derived in the context of volume averaging (Ochoa-Tapia and Whitaker, 1995a, 1995b). This representation, based on the Darcy-Brinkman momentum equation, involves an adjustable stress jump coefficient which has been found to be explicitly dependent on the continuous spatial variations of the effective properties at the inter-region (Goyeau *et al.*, 2003; Chandesris and Jamet, 2006). The influence of the stress jump coefficient on the stability of natural convection in superposed fluid and porous layers was investigated in Hirata *et al.* (2007a).

A large majority of studies on linear stability analysis for the onset of thermal convection in superposed fluid and porous layers has been performed using a two-domain approach with the Darcy equation for the momentum transport in the porous region (Nield, 1977; Chen and Chen, 1988; Carr and Straughan, 2003). The first stability analysis based on the one-domain modelling in this stratified configuration has been proposed by Zhao and Chen (2001). The comparison between their results and those obtained with the two-domain approach (Chen and Chen, 1988) shows a qualitative agreement of the marginal stability curves, while the critical values of the Rayleigh number may significantly differ (up to 40%).

Recently, a systematic comparison was provided (Hirata *et al.*, 2007b) between linear stability results of the one-domain approach (1Ω) and the Brinkman-extended two-domain model ($2\Omega_{DB}$). The results were also compared with those obtained using the classical Darcy's formulation of the two-domain model ($2\Omega_D$) (Carr and Straughan, 2003). As in the work of Zhao and Chen (2001), important discrepancies were observed between results of one and two-domain approaches. The marginal stability curves of the $2\Omega_{DB}$ model presented better agreement with the $2\Omega_D$ curves than with those of the 1Ω approach, indicating that the mathematical formulation was responsible for the discrepancies, while the inclusion of the Brinkman term played a secondary role. In this paper, it is shown that when the transition between the fluid and the porous regions is characterized by discontinuous spacial variations of the macroscopic properties, the one-domain approach formulation has to be modified by taking the derivatives in the sense of distributions. In this way, one and two-domain formulations lead to the same stability thresholds.

This paper is organized as follows. After presenting the geometrical configuration of the problem, the governing equations of the Brinkman-extended two-domain approach ($2\Omega_{DB}$) are presented. Then, the formulation of the one-domain approach (1Ω) used by Zhao and Chen (2001) and Hirata *et al.* (2006) is presented. For conciseness, the linear stability analisys and the application of the Generalized Integral Transfom Technique (Cotta, 1993) to the resulting eigenvalue problem will not be described in the present paper. The details concerning its applications to one- and two-domain approaches can be found in Hirata *et al.* (2006) and (2007b), respectively. In section 3. the numerical results of models $2\Omega_{DB}$ and 1Ω are compared with those obtained with the classical Darcy formulation of the two-domain approach ($2\Omega_D$) (Carr and Straughan, 2003). It is shown that the marginal stability curves of $2\Omega_{DB}$ present a good agreement with those obtained using $2\Omega_D$, especially for high α values. Nevertheless, important discrepancies are found with the 1Ω model. This is due to the fact that the one-domain approach formulation has to be modified by taking the derivatives in the sense of distributions. The corrected formulation of the one-domain approach ($1\Omega_{NEW}$) is then presented. The results obtained with the corrected formulation are compared with those obtained with the Brinkman-extended formulation of the two-domain approach.

2. GOVERNING EQUATIONS

The system under consideration consists of a horizontal porous layer of thickness d_m^* underlying a fluid layer of thickness d_f^*, with a total thickness $d^* = d_m^* + d_f^*$, as shown in Fig. (1). The upper and lower walls are impermeable and are kept at temperatures T_u^* and T_b^*, respectively. The porous medium is saturated by the same fluid which fills the rest of the domain, and is supposed to be in thermal equilibrium with the fluid. The fluid is assumed to be Newtonian and to satisfy the Boussinesq approximation:

$$\rho(T^*) = \rho_0(1 - \beta_T(T^* - T_0^*)). \tag{2}$$

2.1 The two-domain approach ($2\Omega_{DB}$)

In this section, we briefly recall the two-domain Brinkman-extended formulation presented by Hirata *et al.* (2007b). This formulation is different from the classical Darcyt's formulation of the two-domain approach (Chen and Chen, 1988; Carr and Straughan, 2003), since viscous diffusion is included in the momentum equation for the porous medium.

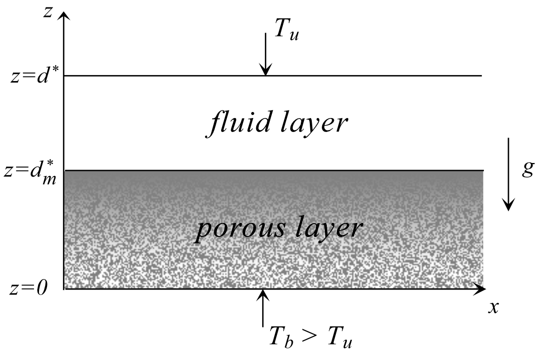


Figure 1. Geometric configuration of the problem.

The dimensionless conservation equations for the fluid layer are given by:

$$\nabla.\mathbf{u} = 0 \tag{3}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} = -\nabla P + \nabla^2\mathbf{u} + Gr_T T\mathbf{e}_z \tag{4}$$

$$\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T = \frac{1}{Pr_f}\nabla^2 T \tag{5}$$

While assuming that the porous medium is isotropic and homogeneous, the dimensionless equations for the porous layer can be written as:

$$\nabla.\mathbf{u}_m = 0 \tag{6}$$

$$\frac{1}{\phi}\frac{\partial \mathbf{u}_m}{\partial t} = -\nabla P_m + \frac{1}{Da}\,\mathbf{u}_m + \eta\nabla^2\mathbf{u}_m + Gr_T T_m\mathbf{e}_z \tag{7}$$

$$\frac{(\rho_0\ C_p)_m}{(\rho_0\ C_p)_f}\ \frac{\partial T_m}{\partial t} + \ \mathbf{u}_m.\nabla T_m = \frac{1}{Pr_m}\nabla^2 T_m \tag{8}$$

The reduced viscosity in eq. (7) was taken as $\eta = \mu_{eff}/\mu = 1/\phi$ (Whitaker, 1998). The Darcy number $Da = K/d^{*2}$ is the dimensionless permeability, and the other dimensionless parameters are the Grashof and the Prandtl numbers, defined as: $Gr_T = g\beta_T\Delta T^* d^{*3}/\nu^2$, $Pr_f = \nu/\alpha_{Tf}$, $Pr_m = \nu/\alpha_{Tm}$, where $\alpha_{Tf} = k_f/(\rho_0\ C_p)_f$, and $\alpha_{Tm} = k_m/(\rho_0\ C_p)_f$.

The nondimensional boundary conditions at the the top and bottom walls are:

$$T(1) = \frac{T_u^* - T_0^*}{\Delta T^*},\ \mathbf{u}(1) = 0,\ \text{ and }\ \mathrm{T_m}(0) = \frac{\mathrm{T_l^* - T_0^*}}{\Delta\mathrm{T}^*},\ \mathbf{u}_\mathrm{m}(0) = 0. \tag{9}$$

At the interface, $z = d_m^*/d^* = d_m = 1/(1+\widehat{d}\ )$, where $\widehat{d}$ is the depth ratio defined as $\widehat{d} = d_f/d_m$. The dimensionless conditions of continuity of temperature, heat flux, velocity, normal stress and tangential stress across the interface take the form:

$$T = T_m \tag{10}$$

$$\frac{\partial T}{\partial z} = \frac{1}{\varepsilon_T}\frac{\partial T_m}{\partial z} \tag{11}$$

$$\mathbf{u} = \mathbf{u}_m \tag{12}$$

$$-P + 2\frac{\partial w}{\partial z} = -P_m + 2\eta\frac{\partial w_m}{\partial z} \tag{13}$$

$$\frac{\partial u}{\partial z} = \eta\frac{\partial u_m}{\partial z} \tag{14}$$

where $\varepsilon_T = \alpha_{Tf}/\alpha_{Tm}$ stands for the thermal diffusivity ratio.

It is important to remark that, when using the Darcy formulation of the two-domain approach, the continuity of normal stress does not include the viscous contribution in the porous region, and the continuity of tangential stress is substituted by the Beavers and Joseph boundary condition (eq. (1)).

2.2 The one-domain approach (1Ω)

The one-domain formulation presented by Hirata *et al.* (2006) is now recalled. This description consists of combining the governing equations for the two regions into a unique set of equations, valid for the entire domain. The momentum conservation equation is a modified Navier-Stokes equation, and thus incorporates the Brinkman extension of Darcy law in the porous medium. As shown in Hirata *et al.* (2006), the dimensionless governing equations valid in the two regions are:

$$\nabla.\mathbf{u} = 0 \tag{15}$$

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{\phi}\right) + \frac{1}{\phi}\left(\mathbf{u}.\nabla\frac{\mathbf{u}}{\phi}\right) = -\nabla P - \frac{1}{Da}\,\mathbf{u} + \eta\,\nabla^2\mathbf{u} + (Gr_T T)\mathbf{e}_z \tag{16}$$

$$\frac{(\rho_0 C_p)_m}{(\rho_0 C_p)_f}\,\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T = \frac{1}{Pr\;\alpha_{Tf}}\nabla.(\alpha_T \nabla T) \tag{17}$$

where $\alpha_T = \alpha_{Tf}$ in the fluid region and $\alpha_T = \alpha_{Tm}$ in the porous medium. The momentum equation (16) continuously evolves from the Darcy-Brinkman equation ($\phi \neq 1$) in the porous region, to the Navier-Stokes equation ($\phi = 1, Da \to \infty$) in the fluid region.

The nondimensional boundary conditions at the external walls are:

$$\mathbf{u}(1) = 0,\; T(1) = \frac{T_u^* - T_0^*}{\Delta T^*} \tag{18}$$

$$\mathbf{u}(0) = 0,\; T(0) = \frac{T_l^* - T_0^*}{\Delta T^*} \tag{19}$$

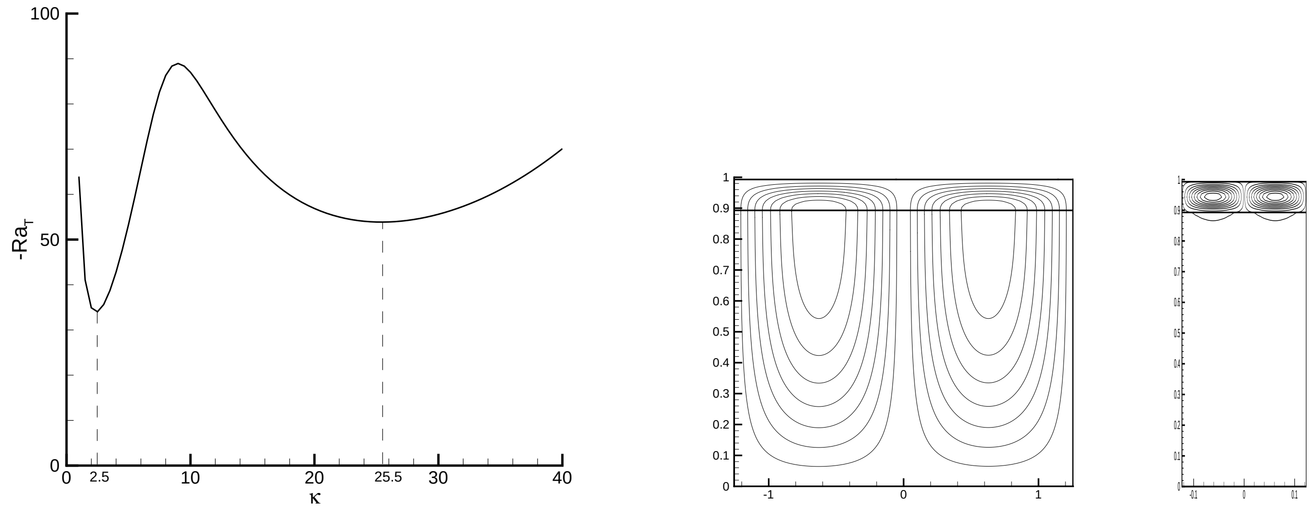
Systems (3)-(14) and (15)-(19) were linearized in the usual way. The eigenvalue problems resulting from the stability analysis of models 1Ω and $2\Omega_{DB}$ were solved using the Generalized Integral Transform Technique (GITT) (Cotta, 1993). The basic concept of this hybrid numerical-analytical approach consists of finding auxiliary problems that will form a basis for the proposed eigenfunction expansion. Upon integral transformation of the original partial differential system, a coupled system of ordinary differential equations for the transformed potentials then results, which is numerically solved to yield the expansion coefficients.

Auxiliary eigenvalue problems were chosen for the temperature and velocity fields of 1Ω and $2\Omega_{DB}$ models. Contrary to the 1Ω model, the auxiliary problems of the $2\Omega_{DB}$ system presented different eigenfunctions for each layer, coupled by interfacial conditions, as proposed by Mikhailov and Ozisik (1984) for the treatment of composite media. The computational costs for the numerical resolution of the final $2\Omega_{DB}$ system were much higher than for the resolution of the trasformed 1Ω system. In some cases, the computational time required for a $2\Omega_{DB}$ run could be twice the time used for the same run with the 1Ω model. The details concerning the application of the GITT to one- and two-domain approaches can be found in Hirata *et al.* (2006) and (2007b), respectively.

3. RESULTS AND DISCUSSION

The porous medium is supposed to be isotropic and homogeneous, and as in Chen and Chen (1988), Carr and Straughan (2003), Zhao and Chen (2001), Hirata *et al.* (2006), $Pr_f = 10$, $\varepsilon_T = 0.7$, and $\eta = (1/0.39)$. Let us recall that the characteristic parameters obtained with our formulation are the thermal Grashof number Gr_T, and the Darcy number Da. Nevertheless, for the sake of comparison with previous works, the marginal stability curves are presented in terms of the Rayleigh number $Ra_T = Gr_T\;Pr_f\;Da$, and according to Chen and Chen (1988), the parameter δ is fixed at 0.003. This parameter is actually a combination of Da and the depth ratio ($\delta^2 = (1+\widehat{d})^2\;Da$).

As already observed by previous authors, the stability curves can present a bimodal behaviour depending on the values of the characteristic parameters. Fig. (2a) shows the bimodal nature of the stability curve obtained with the two-domain Darcy-Brinkman model, for $\widehat{d} = 0.12$. The critical values for the first minimum are $-Ra_{Tcr,1} = 34.03$ and $\kappa_{cr,1} = 2.5$; and for the second one, $-Ra_{Tcr,2} = 53.87$ and $\kappa_{cr,2} = 25.5$. Each minimum of the curve corresponds to a different mode of natural convection. A "fluid mode" (corresponding to perturbations of large wave numbers), where the convective flow is mainly confined in the fluid layer; and a "porous mode" (corresponding to perturbations of small wave numbers), where the convective flow occurs in the entire porous region. In order to illustrate these two modes, the streamline patterns obtained for $\kappa = 25.5$ and $\kappa = 2.5$ are shown in Figs. (2b) and (2c), respectively. In Fig. (2b), a perturbation of large wave number is introduced, resulting in convection cells mainly confined in the fluid layer with some flow penetration in the upper region of the porous layer. Fig. (2c) shows the convection pattern obtained for a perturbation of small wave number, corresponding to a large wavelength. In this case, fluid motion is present in the entire porous layer.



(a) (b) (c)

Figure 2. (a) Bimodal nature of the stability curve obtained with the $2\Omega_{DB}$ model, for $\widehat{d} = 0.12$ and $\delta = 0.003$. The corresponding streamline patterns for the two minima are shown in (b) $\kappa = 2.5$ ($\Psi_{max} = \pm 0.4386$; $\Delta\Psi = 0.0627$) and (c) $\kappa = 25.5$ ($\Psi_{max} = \pm 0.166$; $\Delta\Psi = 0.02$). The thick horizontal line represents the fluid/porous interface.

Figs. (3a)-(3d) show a comparison of the marginal stability curves for four values of $\widehat{d}$ at a fixed value of $\delta = 0.003$, obtained with the different models, namely: the one-domain approach (1Ω) (Hirata *et al.*, 2006); the two-domain approach using Darcy's formulation ($2\Omega_D$) for different values of the adjustable slip coefficient α (Carr and Straughan, 2003); and the two-domain approach using Brinkman's formulation ($2\Omega_{DB}$). Let us remark that the $2\Omega_D$ curves were obtained by Carr and Straughan (2003), who adopted a equation of state which expresses the fluid density as a quadratic function of temperature. For all values of $\widehat{d}$, it may be noticed that the $2\Omega_{DB}$ curves are located between the curves obtained using the $2\Omega_D$ model, for $\alpha = 1$ and $\alpha = 4$. The stability curves obtained using the 1Ω model present a quite different behavior. These results show that the Brinkman term does not play a crucial role in the stability of the system for the adopted values of the Darcy number ($Da \approx 10^{-5}$). As a consequence, it may be induced that the discrepancies are due to the different mathematical formulation used in one- and two-domain approaches.

As shown in Hirata *et al.* (2006), our 1Ω curves present a good agreement with the results of Zhao and Chen (2001). These authors claim a qualitative agreement for the one- and two-domain approaches. Nevertheless, the comparison only concerns the critical values, and not the entire stability curves. They do not mention such important discrepancies as those displayed in Figs. (3a)-(3d) for large values of κ. The critical Rayleigh numbers and the associated wave numbers for the stability curves in Figs. (3a)-(3d) are shown in Table 1. For all values of $\widehat{d}$ studied, the $2\Omega_{DB}$ and $2\Omega_D$ curves are bimodal. If one now considers the curves obtained using the 1Ω approach, they also present a bimodal behavior for $\widehat{d} = 0.08$ and $\widehat{d} = 0.10$, but not for $\widehat{d} = 0.12$ and $\widehat{d} = 0.14$, which present only one minimum (Figs. (3c) and (3d)). We can observe an important change in the critical wave number of the 1Ω curves between $\widehat{d} = 0.10$ and $\widehat{d} = 0.12$ (see Table 1), which corresponds to the change of the critical convection mode. For the $2\Omega_{DB}$ and $2\Omega_D$ curves, the mode switching occurs between $\widehat{d} = 0.12$ and $\widehat{d} = 0.14$.

The $2\Omega_D$ model requires the specification of the empirical slip parameter α in the Beavers and Joseph boundary condition (eq. (1)). Contrarily to the findings of Carr and Straughan (2003), Chen and Chen (1988) mention that their solution “is quite insensitive to α". As shown in Figs. (3a)-(3d), this remark is relevant only for small values of the wave number κ, corresponding to the first minimum of the curves. In the $2\Omega_{DB}$ model, on the contrary, there is no adjustable parameter, and therefore only one stability curve is provided. For the porous mode of instability (first minimum of the curves), all the two-domain curves predict the same critical conditions. This means that when the onset of convective motion occurs within the porous layer, the upper interfacial condition does not play an important role. Concerning the

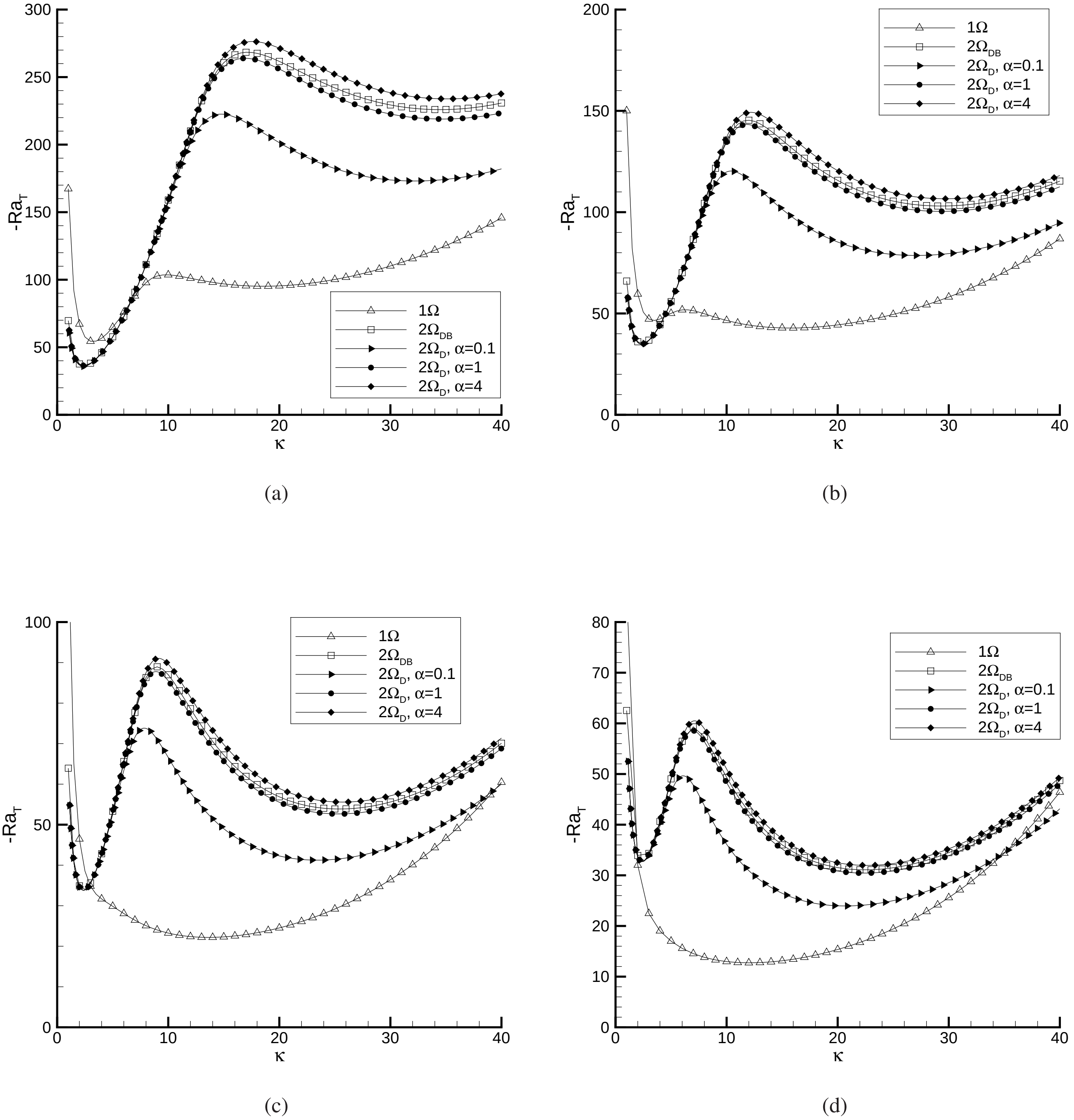


Figure 3. Marginal stability curves obtained with models 1Ω (Hirata *et al.*, 2006), $2\Omega_D$ (Carr and Straughan, 2003), and $2\Omega_{DB}$, for $\delta = 0.003$ and (a) $\widehat{d} = 0.08$, (b) $\widehat{d} = 0.10$, (c) $\widehat{d} = 0.12$, and (d) $\widehat{d} = 0.14$.

fluid mode of instability, the $2\Omega_{DB}$ curve is systematically located between the $2\Omega_D$ curves for $\alpha = 1$ and $\alpha = 4$. A possible explanation for this fact can be found in the value adopted for the reduced viscosity η. It has been shown that, for one-dimensional flows, the $2\Omega_{DB}$ analytical solution is similar to the $2\Omega_D$ solution by Beavers and Joseph provided that $\alpha = \sqrt{\eta}$ (Neale and Nader, 1974). This is consistent with our results since the porosity of the porous medium ($\phi = 0.39$) is such that $\sqrt{\eta} \cong 1.6$ lies between 1 and 4.

Table 1. Critical Rayleigh numbers and corresponding wave numbers for the stability curves in Figs. (3a)-(3d).

$Model$	$\widehat{d} = 0.08$		$\widehat{d} = 0.10$		$\widehat{d} = 0.12$		$\widehat{d} = 0.14$	
	κ_{cr}	$-Ra_{Tcr}$	κ_{cr}	$-Ra_{Tcr}$	κ_{cr}	$-Ra_{Tcr}$	κ_{cr}	$-Ra_{Tcr}$
1Ω	3.0	54.42	3.5	46.55	13.5	22.19	12.0	12.73
$2\Omega_{DB}$	2.5	36.55	2.5	35.15	2.5	34.03	22.0	31.09
$2\Omega_D,\ \alpha = 0.1$	2.4	35.91	2.3	34.81	2.4	33.82	20.5	23.96
$2\Omega_D,\ \alpha = 1$	2.4	36.30	2.4	34.91	2.4	33.81	22.5	30.45
$2\Omega_D,\ \alpha = 4$	2.4	36.37	2.4	34.93	2.4	33.91	22.5	31.96

The corrected formulation of the one-domain approach ($1\Omega_{\text{NEW}}$)

As shown in the previous section, one- and two-domain approaches lead to different stability thresholds when the transition between the fluid and the porous regions is characterized by a discontinuous variation of the physical properties. This is due to the fact that discontinuous functions can not be differentiated in the ordinary sense of a function. The results presented in this section show that in taking the derivatives in the sense of distributions (Schwartz, 1965), the one-domain approach leads to the same stability results as two-domain approaches. The corrected formulation of the one-domain approach is now presented.

Supposing that the porous medium is homogeneous, the reduced viscosity η, the thermal diffusivity α_T, and the porosity ϕ are represented by step functions. In the corrected formulation of the one-domain approach, these functions should be kept inside the divergence operator as follows:

$$\nabla.\mathbf{u} = 0 \tag{20}$$

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{\phi}\right) + \frac{1}{\phi}\left(\mathbf{u}.\nabla\frac{\mathbf{u}}{\phi}\right) = -\nabla P - \frac{1}{Da}\,\mathbf{u} + \nabla.(\eta\;\nabla\mathbf{u}) + (Gr_T T)\mathbf{e}_z \tag{21}$$

$$\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T = \frac{1}{Pr_f\;\alpha_{Tf}}\nabla.(\alpha_T \nabla T) \tag{22}$$

where the derivatives of all discontinuous functions should be taken *in the sense of distributions*.

The first results obtained with the corrected formulation of the one-domain approach ($1\Omega_{\text{NEW}}$) are now compared with those of the two-domain extended-Brinkman formulation ($2\Omega_{DB}$). The marginal stability curves obtained with the four values of the depth ratio previously analysed are shown in Figs. (4a)-(4d). In all figures, a very good agreement is observed between the $1\Omega_{\text{NEW}}$ and $2\Omega_{DB}$ results, for the entire range of wave numbers analyzed.

4. CONCLUSIONS

A comparison between the linear stability results of thermal natural convection in superposed fluid and porous layers has been carried out, using one- and two-domain approaches.

The marginal stability curves of the Brinkman-extended two-domain model presented a good agreement with the classical Darcy two-domain curves, indicating that the inclusion of the Brinkman term plays a secondary role for the values of the Darcy number used in this work. However, the one-domain curves presented a rather different behavior. It has been shown that, when the transition between regions is characterized by discontinuous variations of the macroscopic physical properties, the derivatives of the discontinuous functions must be taken in the sense of distributions. The one-domain formulation was then corrected, and the results obtained with the new formulation presented a good agreement with those of the Brinkman-extended two-domain approach. It is important to remark that, when the spatial variations of the physical properties are continuous, the terms derived from the distribution theory are null, and therefore, the corrected one-domain formulation is the same as the one presented in section 2.2

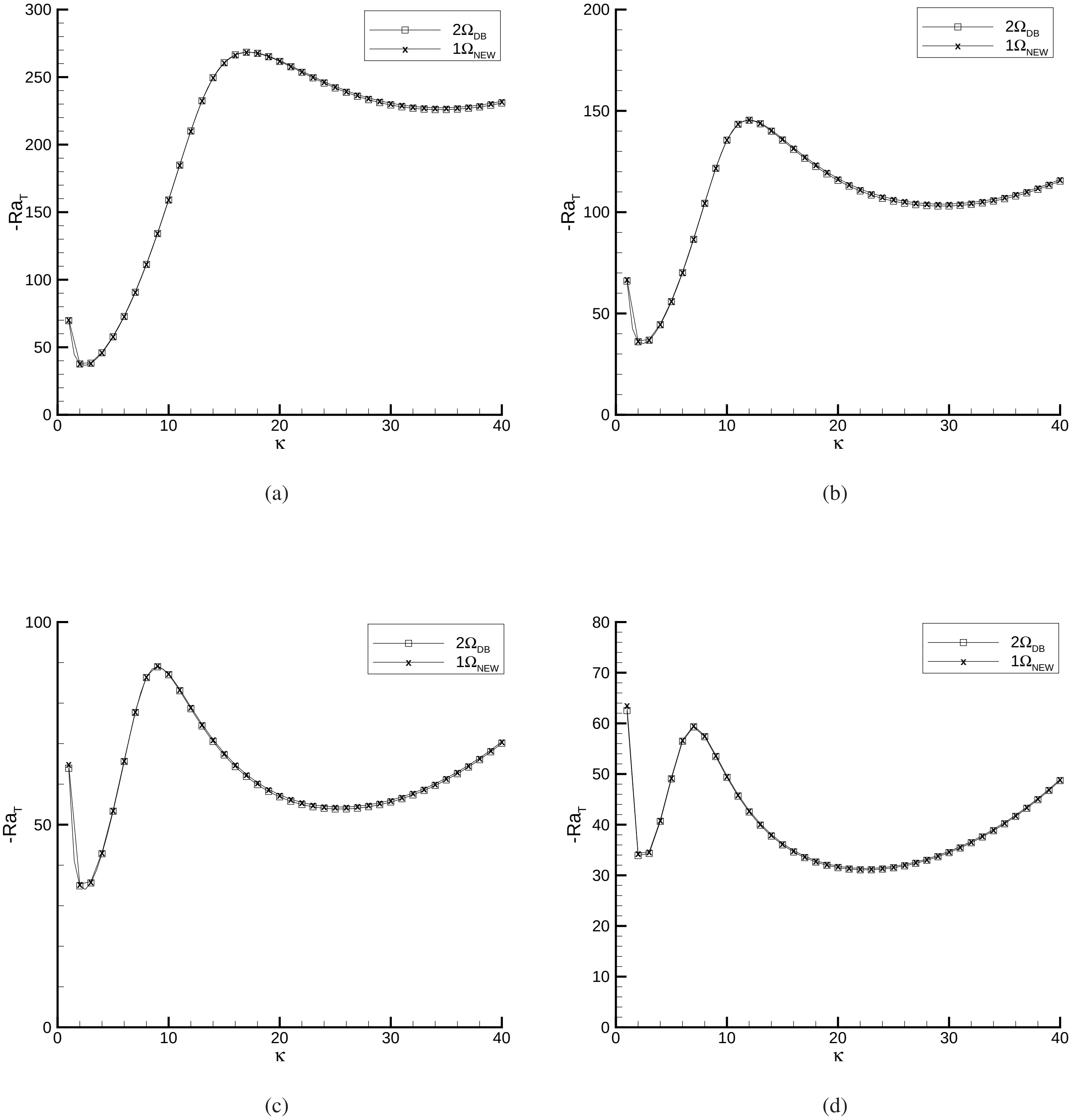


Figure 4. Marginal stability curves obtained with the $2\Omega_{DB}$ model and the corrected formulation of the one-domain approach $1\Omega_{\mathrm{NEW}}$, for $\delta = 0.003$ and (a) $\widehat{d} = 0.08$, (b) $\widehat{d} = 0.10$, (c) $\widehat{d} = 0.12$, and (d) $\widehat{d} = 0.14$.

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7. Responsibility notice

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