ON THE MECHANISM OF VORTEX SHEDDING FROM BLUFF BODIES

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Abstract. *The dynamics in the formation region behind two-dimensional bluff bodies is proposed, introducing a consistent topology and postulating a simple basic mechanism of vortex shedding. Trough an analytical development from this model, a theoretical Strouhal number is derived involving only quantities from the free-streamline theory. To validate this theoretical expression, its results are compared with classic experimental data. Based on the proposed dynamics and on the theoretical Strouhal number, some discussions are made about previous concepts that are part of the history of bluff bodies scientific study.*

Keywords: *bluff bodies, formation region, vortex shedding, Strouhal number, free-streamline*

1. INTRODUCTION

Since the early studies of fluid mechanics, a lot of experimental, theoretical and computational investigations have been made, searching for the basic mechanism of vortex shedding from bluff bodies. After a long time without a simple mechanism to explain the phenomenon, the research has been concentrated on instabilities conditions. Deep studies were carried out, almost always with such a complexity level that indicates the need of reconsidering a simpler approach for simple cases. After the free-streamline theory introduced by Helmholtz and extended by Kirchhoff (1869), for the first time the drag coefficient of a body, C_D, was calculated using the potential theory. In the Kirchhoff model, only the potential flow, outside the wake, was regarded. Kármán (1911) presented his vortex street model, that regarded only the wake behind an unknown body, to derive his drag coefficient (Fig. 1(b)).

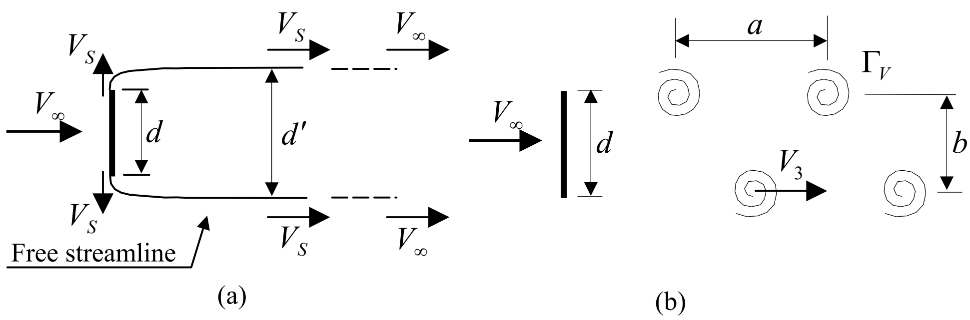

Figure 1. (a) Free-streamline theory; (b) Kármán vortex street theory.

The basic unrealistic assumption of Kirchhoff's model was pointed out by Prandtl when Heisenberg (1922) tried to unify the two theories. Roshko (1954a) made the necessary modification to the free-streamline theory, introducing the possibility of a base pressure lower than the undisturbed flow value. At the same time, Roshko proposed a fixed wake width d' after some distance downstream (Fig. 1(a)), and presented his notched hodograph. The notched hodograph method intents to calculate the wake width d' and the drag coefficient C_D for a given base pressure behind a body of a known shape. Until present date, there is no closed theory to give the main quantities for a flow past a bluff body without some experimental information. The researchers agree that is necessary to understand the dynamics of vortex formation and shedding in the formation region, in order to obtain a closed theory. Despite of the expressive quantity and quality of experimental data, and the facilities of computing resources, there is no satisfactory insight about the formation region to allow the conception of such closed theory.

The present work will state reasonable hypothesis concerning the flow behind a bluff body, based in quantitative and qualitative evidences. The main hypothesis, is a simple mechanism for vortex shedding. From these hypothesis, through a rather easy physical and mathematical modeling, is possible to derive a theoretical expression for the vortex shedding frequency, represented by the Strouhal number. Comparisons with experimental results will show that the proposed mechanism may be correct.

In the following figures, the bluff body is represented by a circular cylinder, but only as illustration. The concepts don't regard a specific shape, so that the comparisons have been made for several different bodies.

2. FUNDAMENTALS

After the boundary layer separation at a side of a bluff body, the free shear layer rolls up in a strong vortex near the base of the body. When the Reynolds number is greater than a characteristic value, the formed vortex gets away from the formation region at a characteristic frequency, alternately from the two sides of the body, giving a regular pattern to the wake.

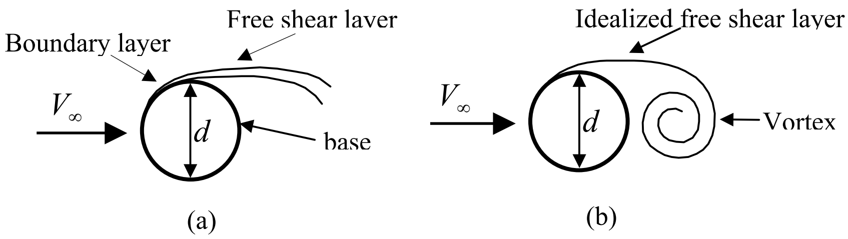

Figure 2. Vortex formation.

The frequency f for the vortex shedding from one side, is given, dimensionless, by the Strouhal number

$$St = \frac{fd}{V_\infty} \quad . \tag{1}$$

In the Kirchhoff's theory modified by Roshko, all vorticity in the flow is between the two idealized free-streamlines. The largest velocity near the separation point, defined by Roshko as the separation velocity

$$V_S = k\,V_\infty , \tag{2}$$

is assumed to remain constant along the free-streamline near the body, with the base pressure coefficient, therefore, given by $C_{Pb} = 1 - k^2$. If k is known, one can calculate the drag coefficient C_D and the wake width d' trough the notched hodograph. Roshko applied his hodograph to the cases of normal flat plate, circular cylinder and ninety degree edge. Abernathy (1962) extended the method to the inclined flat plate.

3. FORMATION REGION MODEL

The present model is a two-dimensional one. Thus, three-dimensional concepts will be eventually replaced by two-dimensional concepts. To begin with, it will be supposed that the free shear layer rolls up, giving a circular shape to the fluid that contains vorticity (Fig. 2(b)). The circular shape for a vortex is a simplification that will be made in the present idealized model. Once the formation of a vortex near the boundary layer separation point has begun, the vortex center moves fast to the center of the body base, where it grows receiving all the circulation produced in the boundary layer (Fig. 3).

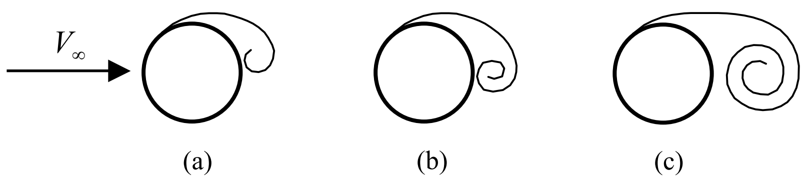

Figure 3. The new vortex runs toward the base center, where it grows.

Then, in this idealized model, the two growing vortices, the 'near' and the 'far', have their centers over the wake center line (Fig. 4(a)). This is only an approximation based on a large number of experimental and computational works, by several authors. Beyond the formation region (Fig. 4(c)) the 'free' vortices will leave the center line, opening the wake as two lines of opposite vorticities, resulting the Kármán vortex street layout.

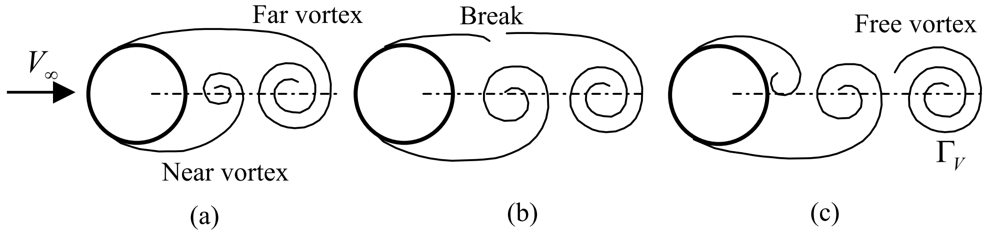

Figure 4. Formation and shedding of vortices.

At the right moment, the 'far' vortex is shed with disruption of the free shear layer that was feeding it (Fig. 4(b)). This 'far' vortex assumes, now, the condition of 'free' vortex (Fig. 4(c)). The 'near' vortex, that now assumes the condition of the 'far' one, growing, allows the 'free' vortex to get farther from the body. The 'remaining' part of the broken free shear layer (Fig. 4(b)), starts rolling up toward the body base, beginning the formation of a new 'near' vortex, that moves the former to the 'far' vortex condition (Fig. 4(c)). We may assume that these 'remaining' parts are always linked to the body, and do not integrate the vortices formed and shed.

From the situation shown in Fig. 4(b), after half a period of vortex shedding from one side, $\tau/2$, the new 'far' vortex will be shed, and the new 'near' vortex will have the circulation separated from the body at certain rate, along this time. t will be taken as the time since the disruption of a free shear layer. Thus, the new 'near' vortex starts its formation at $t = 0$, represented by Fig. 4(b). Then, at $t = 0$ the body is moving upstream at velocity V_∞, relative to the fluid, dragging the wake containing the under formation vortices and the free vortices. At this time the 'near' vortex starts getting circulation from its free shear layer.

After Fage and Johansen (1928), the mean rate at which the circulation is leaving the separation point is known as $\frac{\mathrm{d}\Gamma_T}{\mathrm{d}t} = \frac{V_S^2}{2}$. Here, the T index indicates 'total from the boundary layer'. This rate expression will not be considered here, because it was demonstrated assuming a linear profile of velocity in the free shear layer. To get generality, a general profile shall be taken, giving an unknown mean velocity $\overline{V}$ to the actual free shear layer. This is an average quantity that includes all the effects not explicitly mentioned here, due to, for instance, other vortices and their images. In this case, following the same analytical procedure as Fage and Johansen, we get

$$\frac{\mathrm{d}\Gamma_T}{\mathrm{d}t} = V_S\,\overline{V} \tag{3}$$

that can be integrated, resulting

$$\Gamma_{T,t} = V_S\,\overline{V}\,t\,. \tag{4}$$

At $t = \tau/2$, the 'near' vortex will have the circulation $\Gamma_{T,\tau/2} = \frac{V_S\,\overline{V}\,\tau}{2}$, or $\Gamma_{T,\tau/2} = \frac{V_S\,\overline{V}}{2f}$, in which replacing f from Eq. (1) and V_S from Eq. (2), give us

$$\Gamma_{T,\tau/2} = \frac{k\,\overline{V}\,d}{2\,St}\,. \tag{5}$$

We'll postulate that, until this time, the loss of circulation, mentioned by Prandtl (1922) and observed by Fage and Johansen (1928), is not significant yet for the 'near' vortex. Is postulated, finally, as the main mechanism for free shear layer disruption and vortex shedding, that the disruption happens when the total circulation of the 'near' vortex induces a velocity that cancels the mean velocity $\overline{V}$ in the 'far' vortex free shear layer. This hypothesis, formulated independently, does not invalidate the mechanism proposed by Gerrard (1966), but is a quantitative way to define the moment of disruption. For compatibility with Kirchhoff-Roshko's model, this free shear layer is represented by the free-streamline. It will be supposed that the 'near' vortex center is already at a downstream position where the wake width is the final constant value d'. This situation is sketched on Fig. 5.

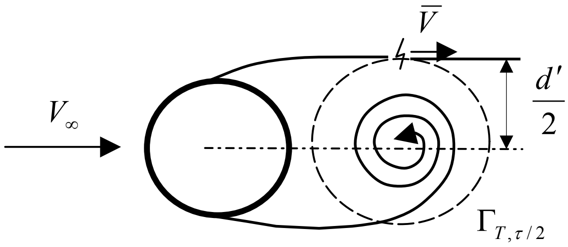

Figure 5. Vortex shedding mechanism.

Taking these hypothesis and supposing that the free shear layer mean velocity did not change, except due to the 'near' vortex induction effect, to cancel this mean velocity, we shall write

$$\Gamma_{T,\tau/2} = 2\pi\left(\frac{d'}{2}\right)\overline{V}, \tag{6}$$

Comparing Eq. (5) and Eq. (6), we get

$$\frac{k\overline{V}d}{2St} = 2\pi\left(\frac{d'}{2}\right)\overline{V}$$

and, from this, finally,

$$St = \frac{k}{2\pi\left(d'/d\right)} \tag{7}$$

The 'far' vortex, for its configuration, shall not play a significant role in this mechanism, least of all the 'free' vortices.

4. COMPARISON WITH EXPERIMENTS

As mentioned early, there is a lot of experimental and numerical results of flow past bluff bodies. As the present study was strongly based on the efforts of the classic researchers, we selected some of these inspiring works to demonstrate the agreement between theoretical and experimental results.

In Fig. 6, the theoretical Strouhal (Eq. (7)) is compared with results for flat plates from Fage and Johansen (1927). For this purpose, d'/d has been calculated by matching the C_D from Fage and Johansen, corrected for blockage, with that obtained trough Abernathy's hodograph (1962). This is possible since Abernathy showed that d'/d is practically independent on blockage effects.

Table 1. Experiments by Fage and Johansen, without correction for blockage.

Incidence angle (deg)	k	d'/d theor.	St	St theor.	% err.
30	1.39	0.732	0.306	0.302	1.2
40	1.45	0.980	0.232	0.236	1.5
50	1.49	1.21	0.195	0.196	0.5
60	1.53	1.41	0.172	0.173	0.4
70	1.54	1.59	0.156	0.154	1.2
80	1.54	1.70	0.152	0.144	5.1
90	1.54	1.74	0.146	0.141	3.5

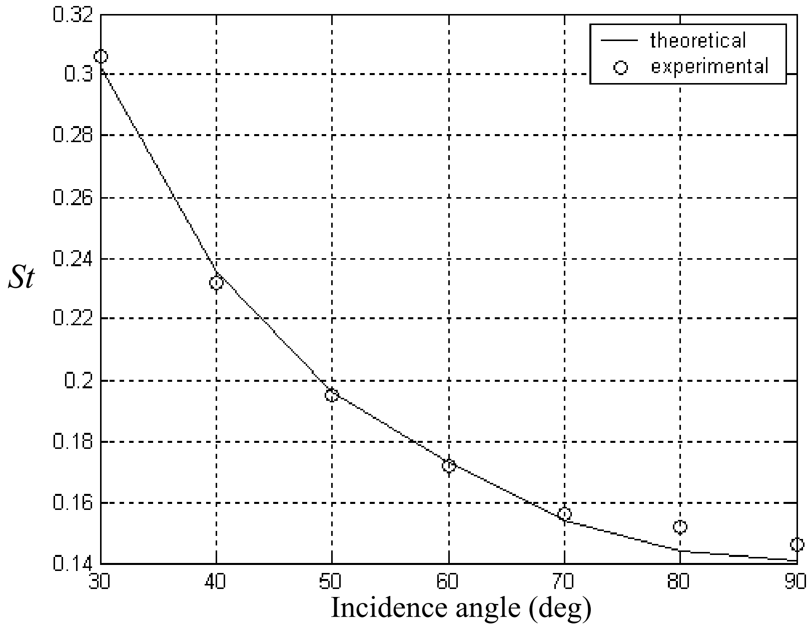

Figure 6. Experiments by Fage and Johansen, without correction for blockage.

In Fig. 8, the theoretical Strouhal (Eq. (7)) is compared with results from Simmons, (1977) who measured d'/d at a distance from the base defined by Roshko (1954b) as the end of the formation region. Simmons used several body shapes, varying the separation angle α , as shown in Fig. 7.

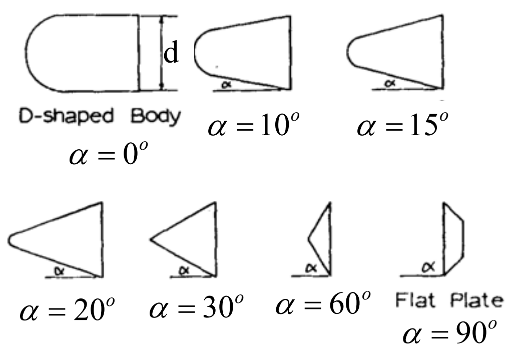

Figure 7. Two-dimensional body shapes used by Simmons (1977),
varying the separation angle α .

Table 2. Experiments by Simmons, without correction for blockage.

Separation angle α (deg)	k	d'/d	St	St theor.	% err.
0	1.32	0.851	0.255	0.246	3.5
10	1.44	0.970	0.249	0.236	5.1
15	1.46	1.00	0.240	0.232	3.5
20	1.48	1.04	0.230	0.226	1.8
30	1.49	1.15	0.208	0.206	1.0
60	1.53	1.49	0.166	0.163	1.8
90	1.55	1.60	0.143	0.154	7.9

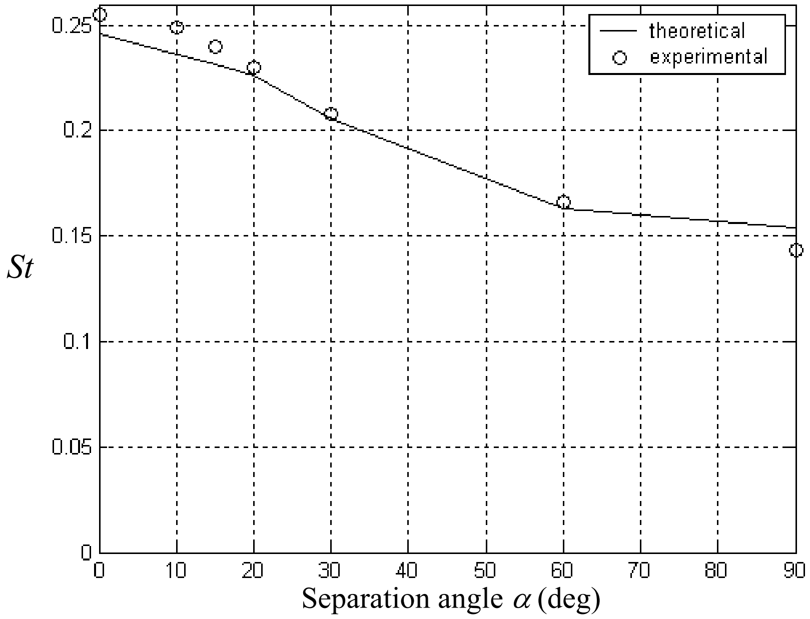

Figure 8. Experiments by Simmons, without correction for blockage.

5. DISCUSSION

The hypothesis made, arose from insights over flow visualizations generated by numerical methods, as discrete vortex and direct numeric simulation, and by traditional visualization methods. The central position of the two vortices in the formation region has been observed by several experimental researchers, but, as this region was obscure, it has almost always been erroneously depicted, showing the vortices growing at the two sides of the body, near the separation points.

It is interesting to note that the shedding frequency, represented here by the Strouhal number, involves only quantities from the free-streamline theory. Also can be noted that Eq. (7) has been derived without regards about blockage effects, and than, shall be valid for wind tunnel experiments without correction. The independence of Eq. (7) on blockage is verified trough the comparison made with data from Fage and Johansen (figure 6), affected by blockage.

The influence of the fluid viscosity is represented by the parameter k introduced by Roshko (1954a).

Figures 6 and 8, show that the theoretical results are in good agreement with experimental data. Thus, the model seems to be a good one.

Despite of not shown, similar comparison was made with Abernathy's data (1962), who measured d'/d at a position where the median lines of the free shear layers became parallel. Simmons and Abernathy (1962) used different experimental definitions for the lateral position at which evaluate the wake width. That from Simmons seems to be in good agreement with the theoretical definition for d'.

These comparisons show that the Roshko's experimental definition for the length of the formation region, used by Simmons, is compatible with our idealized model. This means that Roshko's definition corresponds to the position of our 'near' vortex center, at the shedding moment (Fig. 4(b)).

The wake Strouhal number S^*, defined by Roshko (1954b) as $S^* = \dfrac{fd'}{V_S} = St\dfrac{(d'/d)}{k}$, was found to be approximately equal to 0.16, from a lot of experimental data. Using Eq. (7) we get the theoretical value $S^* = \dfrac{1}{2\pi} \cong 0.159$, which compares very well with the experimental one.

6. CONCLUSION

The two-dimensional model proposed succeeded in derive a good expression for the Strouhal number. The central position for the vortices in the formation region, the predominant action of the under formation vortex and the early concepts from the free-streamline theory, demonstrated to be fundamental to this quantitative approach. The theoretical expression derived agrees very well with early conclusions on fluid mechanics of bluff bodies, and is blockage independent. The model, with its hypothesis, opens new horizons to theoretical researches, and we believe to be the so searched way that will lead to a closed simple theory for two-dimensional bluff bodies. An extension for this model, with this purpose, is already under development trough a large review on the fundamental works. The results achieved, until present state, are encouraging. Despite of its simplicity the model has conceptual innovations that may be applied to other vortices interactions, even on tree-dimensional cases.

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