RESULTS OF LONGITUDINAL HELICOPTER SYSTEM IDENTIFICATION USING OUTPUT-ERROR AND BOTH GENETIC AND LEVENBERG-MARQUARDT OPTIMIZATION ALGORITHM

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Abstract. *This article presents the results concerning the estimation of the linear longitudinal forward flight derivatives of the "Squirrel" AS 355 F2 helicopter. The identification problem is solved in the time domain using the output error approach, in conjuction with the genetic algorithm optimization and the Levenberg-Marquardt optimization. A flight test compaing was performed and experimental data was obtained forthis analysis. The linear longitudinal equations of motion of the helicopter are used in order to identify the dimensional derivatives in forward flight at 5,000 ft of altitude and with 80 and 40 kts true speed. The maneuvers were specified in accordance with conventional flight test procedures, taking in account thelimitations of the mathematical model and flight safety constraints.*

Keywords: *Helicopter System Identification, Output Error, Genetic Optimization Algorithm, Levenberg-Marquardt Optimization Algorithm*

Nomenclature

a_x, a_y, a_z	accelerometer components along the body axes (longitudinal, lateral and vertical), m/s^2
A, B, H_0, H_1	state-space representation
F_x, F_z	external force (longitudinal and vertical), N
G	gravity acceleration, m/s^2
GW	gross weight, N
$I_{()}$	moment of inertia, Kg.m^2
K_α	scale factor of the angle of attack measured at nose boom
m	mass, kg
M_y	pitching moment, Nm
$M_{()}$	dimensional moment stability derivatives, rad/m.s, rad/m.s, 1/s or rad/s^2.cm for M_u, M_w, M_q and $M_{\delta m}$, respectively
n_x, n_y, n_z	load factors components along the body axes (longitudinal, lateral and vertical)
p, q, r	fuselage angular rates, rad/s
p_{bboom}, q_{bboom}	basic static and dynamic pressure, respectively, measured at nose boom, mbar
$Qcomb_{1/2}$	fuel quantity at engine 1 and 2, Kg
T_{ind}	air external temperature, ºC
u, w	velocity components (longitudinal and vertical), m/s
$\bar{u}$,x, y	state-space representation vectors (input, state and output)
x_a, z_a	accelerometer position relative to center-of-gravity
x_{boom}	nose boom position relative to center-of-gravity
$\dot{x}_{bias}$	acceleration bias vector
y_{ref}	output bias vector
$X_{()}$	dimensional longitudinal force stability derivatives, 1/s, 1/s , m/s.rad or m/s^2.cm for X_u, X_w, X_q and $X_{\delta m}$, respectively
$Z_{()}$	dimensional vertical force stability derivatives, 1/s, 1/s, m/s.rad or m/s^2.cm for Z_u, Z_w, Z_q and $Z_{\delta m}$, respectively
α_g, β_g	angle of attack and angle of sideslip, respectively, measured at nose boom, rad
$\delta m, \delta c$	longitudinal cyclic and collective deflections, respectively, cm
Δ	variation from initial trim condition
θ, ϕ	fuselage attitudes (pitch and roll), rad
Θ	identification vector
ρ_{yysim}	correlation coefficient
τ	time delay, s
$(\ )_0$	initial trim condition

1. INTRODUCTION

System identification techniques applied to aerodynamic parameter estimation of aircraft with fixed and rotary wing are well developed and commonly used in many research centers and aeronautical industries around the world. Among widely available results, the research on parameter estimation of rotary wing aircraft is not as common as in the case of fixed wing aircraft. The dynamic system identification of helicopters is almost exclusively realized in the frequency domain. Tischler and Remple (2006) describe the state-of-the-art of helicopter dynamic systems identification in their book “Aircraft and Rotorcraft System Identification”. In this reference work emphasis is given on engineering methods and interpretations of flight test results using a system identification software package, known as CIFER@ (Comprehensive Identification from Frequency Responses). In the frequency response methodology, accurate mathematical modeling of the aircraft starts with flight test data collected in the time domain. The reliability of the identified model is also proved by a time-domain verification method using different flight test data.

In this work, on the other hand, it is described an alternative scenario for helicopter parameter estimation using the time-domain output-error approach combined with both genetic and Levenberg-Marquardt algorithm optimization. The proposed method is applied in the identification of the longitudinal aerodynamic derivatives of a well-known helicopter. The rotorcraft identification methodology used in this work is the well-known M^4V methodology, shown schematically in Figure 1. This methodology takes in account the five main elements of rotorcraft system identification, including the rotorcraft excitation maneuvers, rotorcraft attitude measurements, dynamic models for rotorcraft equations of motion, the parameter estimation methods through output-error minimization and finally the validation of the identified rotorcraft model. Each one of these elements will be discussed on the following sections in order to clarify the utilized system identification methodology.

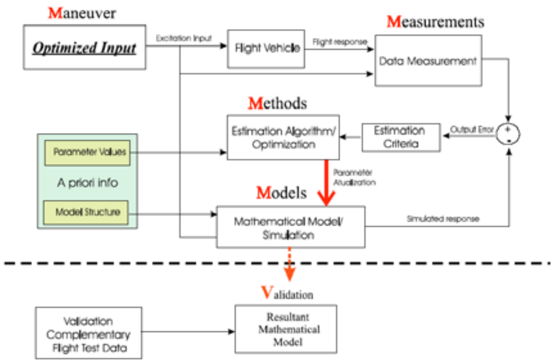

Figure 1. Output error system identification schematic

2. MANEUVERS AND VALIDATION

The choice of the proper flight test maneuvers, by shaping the excitation signals, is of great importance to minimize the uncertainties in the parameter estimation procedures and to maximize the flight test data content. According to Salis Brasil (2005), the optimization of the excitation signal can be realized from the a priori knowledge of the initial dynamic parameters of interest. Cruz *et al.* (2006) show the longitudinal short-period derivatives of the “Squirrel” AS 355 F2 helicopter, however, in order to apply these optimization procedures it would be necessary to know the complete parametric initial estimate, that was not available.

In this way, the maneuvers were specified applying conventional flight test procedures and taking into account the mathematical model constraints. Since this work focuses the determination of the comprehensive linear longitudinal forward flight derivatives, the special sequence of sharp-edged pulses known as the “3-2-1-1” was used at 80 and 40 kt at 5,000 ft of pressure altitude for exciting both the short-period and long-period modes instead of the frequency sweep, used only for short-period mode, as shown by Cruz et al. (2006).

Others special sequences of sharp-edged pulses “3-2-1-1” and “2-1-1-1” was used at the same airspeed and altitude for exciting the longitudinal dynamic modes. The responses obtained were used to validate the longitudinal forward flight derivatives estimation as shown in section 6 below.

3. MEASUREMENTS

The tested helicopter was equipped with an Aydin Vector Data Acquisition System and a flight-test air data system, mounted on a nose boom. All parameters were obtained with a sampling rate of 40 Hz. The output sensors included: fuel quantity from both engines ($Qcomb_{1/2}$), nose boom basic static (pb_{boom}) and dynamic pressures (qb_{boom}), external temperature (T_{ind}), aerodynamic angle of attack (α_g) and sideslip (β_g), angular speeds (p, q and r), load factors (n_x, n_y and n_z), longitudinal and lateral attitudes (θ and ϕ) and, finally, the cyclic, collective and pedal input deflections.

Starting from the fuel quantity distribution, it is possible to determine the mass and inertia parameters of the rotorcraft. The longitudinal and vertical speeds, u and w, respectively, along the body axes X and Z of the helicopter, are obtained from boom static and dynamic pressures, external temperature, aerodynamic angle of attack and boom calibration curves. The pitch rate, Euler angles and accelerometer measurements are collected directly from the flight test instrumentation since the load factors gives accelerometer measurements including gravity effects.

The output observation vector is also subjected to a perturbation vector (bias), which for the longitudinal three-DOF model used in this work, is nominally given by:

$$y = [\ \Delta\alpha_g \quad \Delta q \quad \Delta a_x \quad \Delta a_z \quad \Delta\theta\ ] \tag{1}$$

4. MODEL

The helicopter equations of motion, deduced from direct application of Newton second law for translational and rotational movements, are given by Prouty (1989) and Cooke and Fitzpatrick (2002). This work deals with the estimation of aerodynamic parameters of the longitudinal mode only. In this case, the non-coupled longitudinal dynamic equations are given as:

$$F_X = GW sin\theta + \frac{GW}{m}(\dot{u} + wq) \tag{2}$$

$$F_Z = -GW\, cos\phi\, cos\theta + \frac{GW}{m}(\dot{w} - uq) \tag{3}$$

$$M_Y = I_y \dot{q} \tag{4}$$

The kinematic relation for the pitch rate about the Y-axis is written as, $q = \dot{\theta}\, cos\,\phi$.

These equations are clearly non-linear, but a meaningful analysis can be realized by converting into linear differential equations, considering only the small-perturbations around trimmed equilibrium points in the flight envelop of the rotorcraft. In matrix notation, the linearized dynamic model is given by Cooke and Fitzpatrick (2002) for a pure cyclic input deflection:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{X_u}{m} & \frac{X_w}{m} & \frac{X_q}{m} - w_0 & -g\,cos\,\theta_0 \\ \frac{Z_u}{m} & \frac{Z_w}{m} & \frac{Z_q}{m} + u_0 & -g\,cos\,\phi_0 sin\,\theta_0 \\ \frac{M_u}{I_y} & \frac{M_w}{I_y} & \frac{M_q}{I_y} & 0 \\ 0 & 0 & cos\,\phi_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} \frac{X_{\delta m}}{m} \\ \frac{Z_{\delta m}}{m} \\ \frac{M_{\delta m}}{I_y} \\ 0 \end{bmatrix} [\Delta\delta m] \tag{5}$$

Tischler and Remple (2006) include a constant and unknown acceleration bias vector in the state equation in order to provide a first-order correction for the trim control value obtained from the average of the initial few seconds at the start of the test input:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{X_u}{m} & \frac{X_w}{m} & \frac{X_q}{m} - w_0 & -g\,cos\,\theta_0 \\ \frac{Z_u}{m} & \frac{Z_w}{m} & \frac{Z_q}{m} + u_0 & -g\,cos\,\phi_0 sin\,\theta_0 \\ \frac{M_u}{I_y} & \frac{M_w}{I_y} & \frac{M_q}{I_y} & 0 \\ 0 & 0 & cos\,\phi_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} \frac{X_{\delta m}}{m} \\ \frac{Z_{\delta m}}{m} \\ \frac{M_{\delta m}}{I_y} \\ 0 \end{bmatrix} [\Delta\delta m] + \begin{bmatrix} \Delta\dot{u}_{bias} \\ \Delta\dot{w}_{bias} \\ \Delta\dot{q}_{bias} \\ \Delta\dot{\theta}_{bias} \end{bmatrix} \tag{6}$$

As previously shown in section 3, the output vector, y, is different from the state vector, x. The formulation of the measurement vector in terms of the H_0 and H_1 matrices allows the Euler and aerodynamic angles and accelerometer measurements to be expressed in terms of the states. In this way, the equations of motion in the state-space form is given by Tischler and Remple (2006),

$$\dot{x} = A\,x + B\bar{u}(t-\tau) + \dot{x}_{bias} \text{ and} \tag{7}$$

$$y = H_0 x + H_1 \dot{x} + y_{ref} \tag{8}$$

where H_0 and H_1 are obtained from the linearized equations presented by Tischler and Remple (2006), considering only longitudinal motion ($p = r = 0$):

$$\Delta\alpha_g = \frac{\Delta w - \Delta q x_{boom}}{u_0} \tag{9}$$

$$\Delta a_x = \Delta\dot{u} + w_0 \Delta q + (g \cos\theta_0)\Delta\theta + z_a \Delta\dot{q} \tag{10}$$

$$\Delta a_z = \Delta\dot{w} - u_0 \Delta q + (g \, sen\,\theta_0)\Delta\theta - x_a \Delta\dot{q} \tag{11}$$

In above equations, the coordinates x_{boom} and x_a represent the nose boom and accelerometer package relative positions measured with respect to the longitudinal center-of-gravity of the rotorcraft.

It must be noted that the time-delay τ is associated with the longitudinal cyclic input only, and it was included to account for time delays associated with unmodeled dynamics, such as actuator dynamics, control linkages and transient rotor dynamics, for instance. The results obtained in this work shows that this parameter can be relevant to match accelerometer measurements, but has little effect on other observed outputs.

It must be observed that, as in the earlier case of the state equation, y_{ref} refers to residual errors from the output trim values and also from the sensor biases, misalignments of the instrumentation and various others influences. Furthermore, Jategaonkar (2006) also recommends scale factors in the sensor model for the angle of attack (K_α) and angle of sideslip measurements. Consequently, the observations vector becomes:

$$\begin{bmatrix} \Delta\alpha_g \\ \Delta q \\ \Delta a_x \\ \Delta a_z \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} K_\alpha & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 0 & 1/u_0 & -x_{boom}/u_0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & w_0 & g\cos\theta_0 \\ 0 & 0 & -u_0 & g\,sen\,\theta_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & z_a & 0 \\ 0 & 1 & -x_a & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} \right) + \begin{bmatrix} \Delta\alpha g_{ref} \\ \Delta q_{ref} \\ \Delta ax_{ref} \\ \Delta az_{ref} \\ \Delta\theta_{ref} \end{bmatrix} \tag{12}$$

Therefore, there are twenty-three parameters to be identified:

$$\Theta = \begin{bmatrix} \frac{X_u}{m}, \frac{X_w}{m}, \frac{X_q}{m}, \frac{Z_u}{m}, \frac{Z_w}{m}, \frac{Z_q}{m}, \frac{M_u}{I_y}, \frac{M_w}{I_y}, \frac{M_q}{I_y}, \frac{X_{\delta m}}{m}, \frac{Z_{\delta m}}{m}, \frac{M_{\delta m}}{I_y}, \Delta\dot{u}_{bias}, \Delta\dot{w}_{bias}, \Delta\dot{q}_{bias}, \Delta\dot{\theta}_{bias}, \\ \Delta\alpha g_{ref}, \Delta q_{ref}, \Delta ax_{ref}, \Delta az_{ref}, \Delta\theta_{ref}, K_\alpha, \tau \end{bmatrix} \tag{13}$$

5. IDENTIFICATION METHODS

5.1. Output error approach

The output error approach was used to estimate the longitudinal forward flight derivatives and a time delay. According to Figure 1, the basic principle of this methodology is to minimize the error between the in flight measured response and the estimated result of the identified mathematical model submitted to the observed inputs. This minimization process is made as function of the parameters of the dynamic model, such as aerodynamic derivatives, sensor bias and sensitivities.

Therefore, it is defined a function that measures the agreement between the real and the simulated data with a certain group of adjustable parameters (the coefficients of the model presented above), in such a way that a low value means good similarity. This group of parameters is then adjusted to minimize the function, known as cost function.

Mathematically, consider Θ as being the vector of M adjustable parameters, $\bar{u}$ the input vector of the system and y the vector of observations. Consider that $y_{sim}(x_i;\ \Theta_1 \ldots\ \Theta_M)$, where $i = 1, \ldots, N$, is the output of the model for a determined group of parameters Θ. Be f the cost function. So, the minimization problem can be enunciated as: "Determine the vector of adjustable parameters Θ that minimizes the cost function given by $f = f(y_{sim}(x_i;\ \Theta_1 \ldots\ \Theta_M), y)$."

There are many methods to compose the cost function, such as maximum likelihood (MLE) and least squares presented by Maine and Lliff (1985). Basically, the least squares composition is a minimum square error adjustment:

$$f = \sum_{i=1}^{N} \left[y_i - y_{sim}\left(x_i;\vec{\Theta}\right)\right]\left[y_i - y_{sim}\left(x_i;\vec{\Theta}\right)\right]^T \tag{14}$$

On the other hand, the MLE method takes into account the measurement noise variance to weight the system outputs during the optimization procedure through the measurements noise covariance matrix, *FF*. Suppose that each value of output y_i has an aleatory measured error with normal distribution around the "true" value $(y_i)_{sim}$. Then, the conditional probability density function $p((y_i - (y_i)_{sim}) \mid \Theta)$ is given by:

$$p((\vec{y} - \vec{y}_{sim}) \mid \vec{\Theta}) = \frac{1}{(2\pi)^{M/2} \mid FF^T \mid^{N/2}} exp\left\{ -\frac{1}{2} \sum_{i=1}^{N} \left[y_i - y_{sim}\left(x_i; \vec{\Theta}\right) \right]^T \left[FF^T \right]^{-1} \left[y_i - y_{sim}\left(x_i; \vec{\Theta}\right) \right] \right\} \quad (15)$$

where *FF* is the covariance matrix given for:

$$FF^T = \frac{1}{N} \sum_{i=1}^{N} \left[y_i - y_{sim}(x_i, \vec{\Theta}) \right] \left[y_i - y_{sim}(x_i, \vec{\Theta}) \right]^T \quad (16)$$

From the equation of conditional probability density function, it can be obtained a cost function, when applying the negative of the natural log on both sides:

$$f(\vec{\Theta}) = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left[y_i - y_{sim}(x_i, \vec{\Theta}) \right]^T \left[FF^T \right]^{-1} \left[y_i - y_{sim}(x_i, \vec{\Theta}) \right] + ln \left| FF^T \right| \right\}, \quad (17)$$

what shows that the parameters identification problem became a quadratic function minimization problem, since to maximize the conditional probability density function is equal to minimize the previous equation. Taking in account that the covariance matrix is constant, the cost function can be written as:

$$f(\vec{\Theta}) = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left[y_i - y_{sim}(x_i, \vec{\Theta}) \right]^T \left[FF^T \right]^{-1} \left[y_i - y_{sim}(x_i, \vec{\Theta}) \right] \right\} \quad (18)$$

It should be noticed that to determine a vector Θ that maximizes the conditional probability density function is equivalent to say that such vector of parameters is that which maximizes the probability of $y_i = (y_i)_{sim}$, $i = 1, \ldots, N$. Comparing Eq. (14) and (18), it can be concluded that the least squares cost function, except for the multiplication factor, is given by Eq. (18), replacing *FF* by the identity matrix. Therefore, in order to implement the weigthed least-square cost function, it is enough to replace *FF* by a diagonal matrix to consider different weights to each state. In this work, it was developed a Matlab program to minimize the cost function defined by both weighted least square and MLE errors.

5.2. Levenberg-Marquardt optimization algorithm

Several methods are of traditional application for the minimization problem: Levenberq-Marquadt, Gauss-Newton, Newton-Raphson, among others. This article utilizes the Levenberg-Marquardt optimization algorithm proposed by Mulder *et al.* (1999) and Press *et al.* (1988). Therefore, the cost function given by Eq. (18) may be minimized iteratively according to:

$$\hat{\Theta}^{(j)} = \hat{\Theta}^{(j-1)} + \lambda^{(j-1)} \xi^{(j-1)} \quad (19)$$

where $\hat{\Theta}^{(j)}$ represents the estimate vector at *j*th iteration, $\lambda^{(j-1)}$ is the scalar chosen in accordance with the algorithm proposed by Press *et al.* (1988) for a reduced value of $f(\vec{\Theta})$ and $\xi^{(j-1)}$ is based on information about the cost function acquired at previous iterations. Numerically, $\xi^{(j-1)}$ is often obtained using the values of the first- and the second-order gradients of $f(\vec{\Theta})$ with respect to parameters vector Θ, according to:

$$\xi^{(j-1)} = -\left[f''(\vec{\Theta}^{(j-1)}) \right]^{-1} f'(\vec{\Theta}^{(j-1)}) = -\left[\frac{\partial^2 f(\vec{\Theta})}{\partial \vec{\Theta} \, \partial \vec{\Theta}^T} \right]^{-1} \left. \frac{\partial f(\vec{\Theta})}{\partial \vec{\Theta}} \right|_{\vec{\Theta} = \vec{\Theta}^{(j-1)}} \quad (20)$$

A difficulty with the Eq. (20) is that Hessian matrix $f''(\vec{\Theta})$ may not be positive definite and thus not point in a "downhill" direction. Mulder *et al.* (1999) propose to replace the $f''(\vec{\Theta})$ by its expectation *R*, known as the Fisher information matrix that can be shown to be non-negative definite. Therefore, the Eq. (19) becomes:

$$\hat{\Theta}^{(j)} = \hat{\Theta}^{(j-1)} - \lambda^{(j-1)} R^{-1}(\hat{\Theta}^{(j-1)}) f'(\hat{\Theta}^{(j-1)}) \quad (21)$$

The first-order gradient of the likelihood function in Eq. (21) can be derived from Eq. (18) as:

$$f'(\hat{\Theta}^{(j-1)}) = \sum_{i=1}^{N} \frac{\partial\left[y_i - y_{sim}(x_i,\vec{\Theta})\right]^T}{\partial\vec{\Theta}}\Bigg|_{\vec{\Theta}=\hat{\Theta}^{(j-1)}} \left[FF^T(\hat{\Theta}^{(j-1)})\right]^{-1}\left[y_i - y_{sim}(x_i,\hat{\Theta}^{(j-1)})\right] \tag{22}$$

Applying the following equivalency presented by Goodwin and Payne (1977)

$$E\left\{f''\left(\hat{\Theta}^{(j-1)}\right)\right\} = E\left\{f'\left(\hat{\Theta}^{(j-1)}\right)\left[f'\left(\hat{\Theta}^{(j-1)}\right)\right]^T\right\} \tag{23}$$

the Fisher information matrix may be obtained as:

$$R(\hat{\Theta}^{(j-1)}) = E\{f''(\hat{\Theta}^{(j-1)})\} = \sum_{i=1}^{N} \frac{\partial\left[y_i - y_{sim}(x_i,\vec{\Theta})\right]^T}{\partial\vec{\Theta}}\Bigg|_{\vec{\Theta}=\hat{\Theta}^{(j-1)}} \left[FF^T(\hat{\Theta}^{(j-1)})\right]^{-1} \frac{\partial\left[y_i - y_{sim}(x_i,\vec{\Theta})\right]}{\partial\vec{\Theta}}\Bigg|_{\vec{\Theta}=\hat{\Theta}^{(j-1)}} \tag{24}$$

The iteration starts with initial estimates of all unknown parameters and computes parameter updates according to Eq. (21) based on Eq. (22) and (24). Assuming initially a modest value for λ, as 0.001, on Eq. (21), the cost function is evaluate and if its value is greater than or equal to the last value, so λ is increased by a factor of 10 and the program returns to evaluate the cost function until to obtain a value less than the last value. Also necessary is a condition for stopping, so all these procedures repeat until that the cost function has been reached at least five significant digits equals to the last value.

5.3. Genetic optimization algorithm

This work also describes an application of the MATLAB Genetic Algorithm (GA) and the Direct Search Toolbox to parameter estimation of a rotorcraft. Based on biological evolution principles, the genetic algorithms are structured random search techniques for optimization. Therefore, they utilize concepts such as population size, individual characterization and processes related to selection, reproduction, crossover and mutation of the individuals.

The GA optimization process starts by creating a matrix, representing an initial population *IP*, with each line formed by randomly chosen individual (n_{ind}) with fixed-length character strings (n_θ), as shown below,

$$IP = \begin{bmatrix} \theta_1^1 & \theta_2^1 & \theta_3^1 & \cdots & \theta_{n_\theta}^1 \\ \theta_1^2 & \ddots & & & \\ \theta_1^3 & & \ddots & & \\ \vdots & & & \ddots & \\ \theta_1^{n_{ind}} & \theta_2^{n_{ind}} & \theta_3^{n_{ind}} & \cdots & \theta_{n_\theta}^{n_{ind}} \end{bmatrix}. \tag{25}$$

In this work, the fixed-length character string (n_θ) represents a vector with twenty-three components as shown in Eq. (13). The following step is to assign a fitness value to each individual in the population using a fitness measure, represented by the cost function of Eq. (18). A new population is created by applying three genetic operations, depicted in Figure 2, to each individual string of the current generation. The individuals generated by the genetic processes are:

- Elite: formed by individuals in the current generation that are copied to the new generation based on their fitness properties;
- Crossover: new individuals, created by genetically recombining two substrings, using the crossover operation at a randomly chosen crossover point. The newly created individual inherits the characteristics from the original chosen pair; and
- Mutation: new individuals are created by randomly changing a few strings chosen at random positions in the original string.

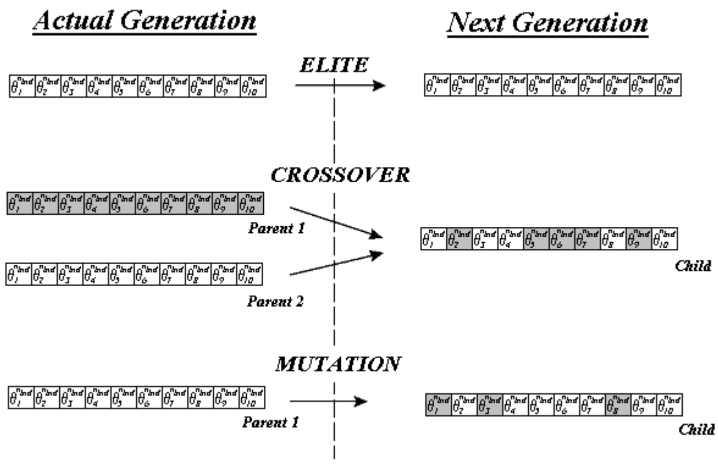

Figure 2. Reproduction Instruments

The GA iteratively performs the above steps until a termination criterion is satisfied. In this work, the maximum number of generations was used for this purpose.

Concerning the GA implementation in the MATLAB environment, the GA population is composed of 2 elite individuals, with the rest of the population formed by crossover and mutation processes in the proportion of 80% and 20%, respectively.

The functions that create the initial population and produce mutation children were developed from a uniform distribution of random numbers, generated on a specified interval, as presented in section 6 below. The output function is evaluated over a subdomain of individuals that attends the limits associated with bounds and stability constraints, since the aerodynamic parameters should be on a specified interval and the longitudinal dynamic mode is known to be stable on this helicopter at 5,000 ft level flight with 80 kt.

It should be empathized that the application of GA facilitates the introduction of constraints to the parameter estimation process, including the maximum and minimum values of the parameters. Another interesting characteristic is that the solution obtained by the genetic algorithm is globally optimal. Besides, the parameters estimation procedure becomes less sensitive to the initial values, which is especially important, when a priori knowledge about the parameters is not available. Therefore, the methodology adopted in this work was to initially estimate the forward longitudinal derivatives using the genetic optimization algorithm and then the Levenberg-Marquardt algorithm in order to improve the results.

6. RESULTS

The initial population for the genetic algorithm was generated without a priori knowledge about all the parameters. Some of them, regarding to longitudinal short-period mode, were known from the previous work, presented by Cruz *et al*. (2006) However, physical knowledge of the aerodynamic properties of the rotorcraft was used to define the signs of those stability and control derivatives, and their limiting values. With this in mind, an initial population for the GA was created such that:

$$\begin{cases} -10 \le \dfrac{X_u}{m} \le 10 \quad 0 \le \dfrac{X_w}{m} \le 100 \quad 0 \le \dfrac{X_q}{m} \le 100 \quad 0 \le \dfrac{Z_u}{m} \le 100 \quad -100 \le \dfrac{Z_w}{m} \le 0 \\ 0 \le \dfrac{Z_q}{m} \le 100 \quad 0 \le \dfrac{M_u}{I_y} \le 100 \quad -100 \le \dfrac{M_w}{I_y} \le 0 \quad -100 \le \dfrac{M_q}{I_y} \le 0 \quad -10 \le \dfrac{X_{\delta m}}{I_y} \le 10 \\ 0 \le \dfrac{Z_{\delta m}}{m} \le 10 \quad 0 \le \dfrac{M_{\delta m}}{I_y} \le 10 \quad -2 \le \dot{x}_{bias} \le 2 \quad -2 \le y_{ref} \le 2 \quad 0 \le K_\alpha \le 2 \quad 0 \le \tau \le 1 \end{cases} \tag{26}$$

The limits shown above were not only used to generate the initial population, but were also used as limiting restrictions by the GA process.

The first flight test point was accomplished at 80 kt and 5,000 ft, using the special sequence 3-2-1-1 for exciting the longitudinal dynamics modes. Figure 3 shows the longitudinal cyclic inputs and the measured (full lines) and estimated (dotted lines) responses of the angle of attack at nose boom α_g, pitch rate q, longitudinal acceleration a_x, vertical acceleration a_z and longitudinal attitude θ.

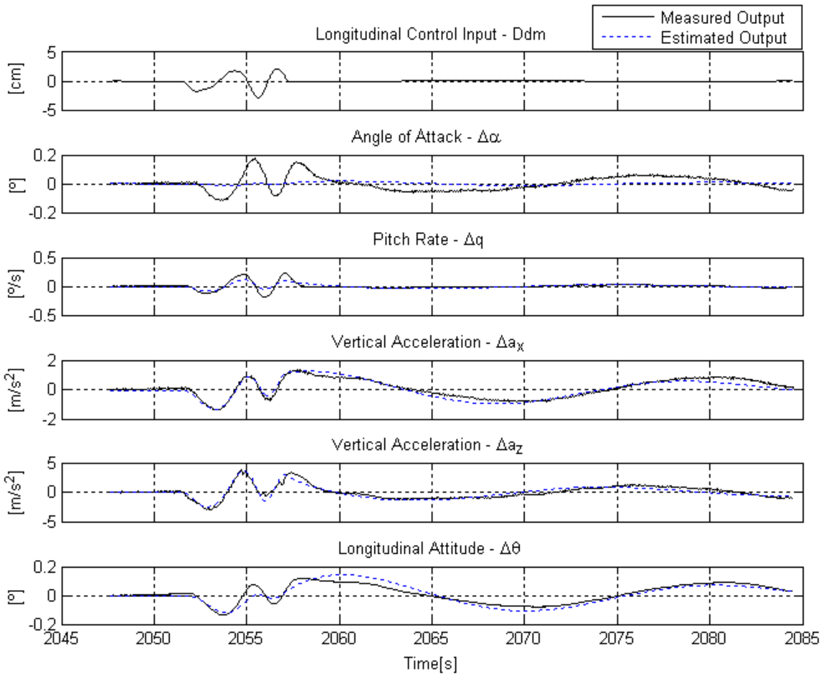

Figure 3. Results for 80 kt and 5,000 ft

The correlation coefficient between measured (y) and simulated data (y_{sim}), defined as the normalized cross-covariance function $\rho_{yy_{sim}}$, as given by Bendat and Piersol (2000),

$$\rho_{yy_{sim}} = \frac{\sum_{i=1}^{N}\left[\left(y_i(t) - \frac{1}{N}\sum_{i=1}^{N} y_i(t)\right)\left(y_{sim_i}(t) - \frac{1}{N}\sum_{i=1}^{N} y_{sim_i}(t)\right)\right]}{\sqrt{\sum_{i=1}^{N}\left[\left(y_i(t) - \frac{1}{N}\sum_{i=1}^{N} y_i(t)\right)^2\right]}\sqrt{\sum_{i=1}^{N}\left[\left(y_{sim_i}(t) - \frac{1}{N}\sum_{i=1}^{N} y_{sim_i}(t)\right)^2\right]}} \tag{27}$$

can be used to estimated how well the estimated signals can reproduce the measured data. If the correlation coefficient is close to unity, one may conclude that the estimation algorithm can provide a good fit to the experimental data, but on other hand, if the coefficient is close to 0, the estimation was poor.

The fitting index, represented by the normalized correlation coefficient, is depicted in Figure 3. In this figure, each group of parameters represents the fitting index of the estimated outputs (α, q, a_x, a_z, θ), obtained with three different types of maneuver at same airspeed and altitude: (a) 3-2-1-1 maneuver used to estimate the forward flight derivatives; (b) another 3-2-1-1 sequence; and (c) 2-1-1-1 maneuver. The (b) and (c) groups of fitting index was used to validate the estimated parameters.

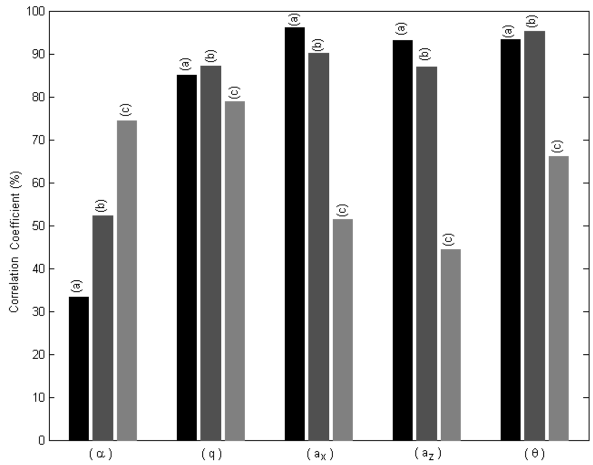

Figure 4. Correlation coefficients for each estimated response at 80 kt and 5,000 ft

Figure 4 shows that the calculated correlation coefficients, except for the aerodynamic angle of attack (α_g), were satisfactory considering the assumptions and limitations of the theoretical helicopter dynamic model. However, the 2-1-1-1 maneuver shows a considerable degradation in the accelerometers and longitudinal attitude estimated responses. The estimated aerodynamic derivatives of the helicopter longitudinal motion are:

$$\begin{cases} \frac{X_u}{m} = 0.0751573 & \frac{X_w}{m} = 0.0765187 & \frac{X_q}{m} = 9.86644 & \frac{Z_u}{m} = 0.144033 & \frac{Z_w}{m} = -0.0586837 \\ \frac{Z_q}{m} = 29.3713 & \frac{M_u}{I_y} = 0.0523414 & \frac{M_w}{I_y} = -0.0158462 & \frac{M_q}{I_y} = -1.01816 & \frac{X_{\delta m}}{I_y} = -0.101019 \\ \frac{Z_{\delta m}}{m} = 0.0986338 & \frac{M_{\delta m}}{I_y} = 0.0906449 & \Delta\dot{u}_{bias} = -1.15626 & \Delta\dot{w}_{bias} = -0.0290062 & \Delta\dot{q}_{bias} = -0.131297 \\ \Delta\dot{\theta}_{bias} = 0.274248 & \Delta\alpha g_{ref} = -4.32966e-006 & \Delta q_{ref} = 0.00181687 & \Delta ax_{ref} = 1.16502 & \Delta az_{ref} = -0.0553747 \\ \Delta\theta_{ref} = -0.0098184 & K_\alpha = 0.0720277 & \tau = 0.284084 & & \end{cases} \quad (28)$$

The second flight test condition was accomplished at 40 kt and 5,000 ft, using again the special sequence 3-2-1-1 for exciting both the short-period and long-period longitudinal modes. Figure 5 shows the longitudinal cyclic inputs, the measured outputs (full lines) and the estimated responses (dotted lines) of the angle of attack at nose boom α_g, pitch rate q, longitudinal acceleration a_x, vertical acceleration a_z and longitudinal attitude θ.

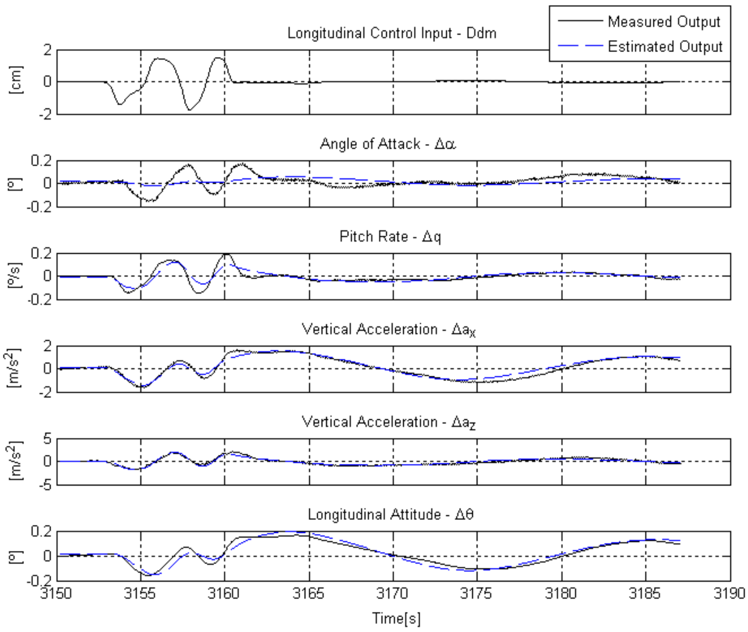

Figure 5. Results for 40 kt and 5,000 ft

The correlation coefficients, as given by Eq. (27) for each measured response and maneuver, are presented in Figure 6, with the same considerations given for Figure 4.

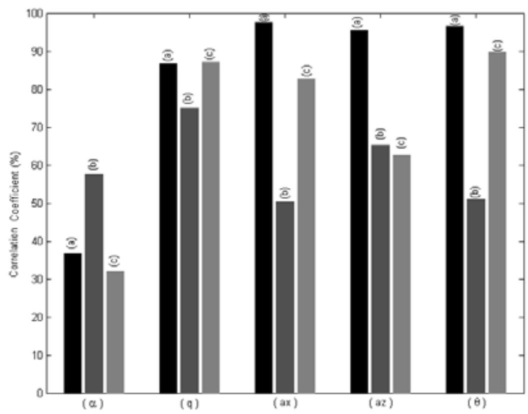

Figure 6. Correlation coefficients for each estimated response at 40 kt and 5,000 ft

As the former case, obtained at 80 kt, the calculated correlation coefficients, except for the aerodynamic angle of attack (α_g), were satisfactory. However, with regard to the validation data, the 3-2-1-1 and 2-1-1-1 maneuvers show a considerable degradation in the accelerometers and longitudinal attitude estimated responses. The estimated parameters are:

$$\begin{cases} \dfrac{X_u}{m} = -0.0309716 & \dfrac{X_w}{m} = 0.188233 & \dfrac{X_q}{m} = 5.83968 & \dfrac{Z_u}{m} = 0.097454 & \dfrac{Z_w}{m} = -0.0178824 \\ \dfrac{Z_q}{m} = 15.2636 & \dfrac{M_u}{I_y} = 0.0355739 & \dfrac{M_w}{I_y} = -0.00866944 & \dfrac{M_q}{I_y} = -0.839498 & \dfrac{X_{\delta m}}{I_y} = 0.0995183 \\ \dfrac{Z_{\delta m}}{m} = 0.103052 & \dfrac{M_{\delta m}}{I_y} = 0.115321 & \Delta\dot{u}_{bias} = -0.244209 & \Delta\dot{w}_{bias} = 0.262797 & \Delta\dot{q}_{bias} = -0.118925 \\ \Delta\dot{\theta}_{bias} = 0.0589556 & \Delta\alpha g_{ref} = 0.0150446 & \Delta q_{ref} = -0.00368096 & \Delta ax_{ref} = 0.423329 & \Delta az_{ref} = -0.430494 \\ \Delta\theta_{ref} = 0.0150779 & K_\alpha = 0.137809 & \tau = 0.298218 & & \end{cases} \quad (29)$$

From these results, obtained at 80 and 40 kt, it can be observed that the 3-2-1-1 maneuver was not enough suitable to identify the longitudinal stability and control derivatives, especially at 40 kt, since the calculated correlation coefficients are inferior to 80% for most of the validation data. This shows the need for a correct specification of the experiments so that the maximum of information of the system can be obtained. Therefore, others flight test techniques, such as longitudinal cyclic or collective pulses, should be tested to maximize the level of information contained in the flight test data and, consequently, to identify precisely the system.

Another possible error source is related with the model itself. As presented by Tischler and Remple (2006), a quasi steady model with six degrees-of-freedom is necessary to estimate the dynamic modes observed in helicopters with small effective flap stiffness as the UH-1H helicopter and a hybrid fully coupled model with thirteen degrees-of-freedom is applicable to helicopters with high flap stiffness as the Squirrel AS 355 F2. Therefore, considering the assumptions and limitations of the theoretical helicopter dynamic model, the calculated correlation coefficients, except for the aerodynamic angle of attack (α_g), were satisfactory and the estimated parameters for each airspeed are, at least, a reasonable approximation.

Another contribution of this work is related with the adopted methodology. The forward longitudinal derivatives were initially estimate using the genetic optimization algorithm and then, when the cost function tends to converge, the parameters are estimated using Levenberg-Marquardt algorithm. This process joins the mean advantages of each algorithm:

- genetic optimization algorithm: guides the parameters along the right path, despite the lack of a priori knowledge about the stability and control derivatives of the vehicle, searching a global minimum and also avoiding divergences and results without physical meaning, since GA facilitates the introduction of constraints;
- Levenberg-Marquardt optimization algorithm: guide the parameters to a precise local minimum and also requires less computational time.

7. CONCLUDING REMARKS

From the methodology M^4V, it was possible to obtain a reasonable approximation for the longitudinal stability and control derivatives in forward flight of the AS 355 F2 helicopter at 5,000 ft with 80 and 40 kts.

The results showed that the genetic and Levenberg-Marquardt optimization algorithms are powerful tools for parameter estimation problems by the output-error method, especially, when the level of information regarding the studied system is reduced. However, the helicopter dynamics has non-linear terms, with parameters that may vary abruptly with the modification of the flight conditions, or with helicopter configurations (large flap hinge-offsets), which require a larger number of degrees-of-freedom in the model to properly introduce the rotorcraft dynamics. Thus, in spite of satisfactory results obtained with linear models for the longitudinal dynamic mode, excluding the aerodynamic angle of attack, more complex models will be required to improve the curve fitting procedures. This is especially relevant to obtain level D flight simulator models and to develop control system strategies that depend on a comprehensive knowledge of the rotor and fuselage dynamic interaction in the entire flight envelope (weight, CG, altitude and airspeed) of the rotorcraft.

Concerning flight test maneuvers, it is also evident the need for an appropriate specification of the inputs, taking in accounting the a priori model knowledge is necessary to maximize the level of information contained in the flight test data, minimizing the uncertainties associated with the identification process. In this respect, the 3-2-1-1 maneuver was not suitable to identify the longitudinal stability and control derivatives, especially at 40 kts.

For the next studies, the following points will be taken in consideration:

- A more complex model will be proposed to include both longitudinal and lateral-directional dynamics modes;

- Application of different optimization techniques, such as Extended Kalman Filter (EKF), in conjunction with the maximum likelihood estimation problem;
- Application of others flight test techniques, such as longitudinal cyclic or collective pulses, to identify the AS 355 F2 longitudinal dynamic mode; and
- Application of optimization techniques to generate maneuvers in order to maximize the level of information contained in the flight test data and, consequently, to make the stability and control derivatives estimation more robust and precise.

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9. RESPONSIBILITY NOTICE

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