DETERMINATION OF THE INTRINSIC PERMEABILITY OF MICROTOMOGRAPHIC IMAGES BY THE LATTICE BOLTZMANN METHOD

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Abstract*. The pore scale low Reynolds flow, in a high resolution tomographic image of a sedimentary rock, was calculated by numerical simulation of the Navier-Stokes equations using the Lattice Boltzmann Method. These results were used to investigate the influence of sample size on the determination of the intrinsic permeability. The size of the representative elementary volume as a function of the autocorrelation length was estimated based on the permeabilities obtained from simulation on several sub-domains. The permeability results of Lattice Boltzmann simulation for the tomographic image and three dimensional reconstructed media from sections of the same sample were compared with experimental results.*

Keywords: *Lattice Boltzmann Method, flow in porous media, Microtomographic Image, REV, Porous Media Reconstruction*

1. INTRODUCTION

Flow in porous media is analyzed in several scales: pore (microscopic), local (macroscopic) and field scales. Dominant process and governing equations also vary with spatial scales. The latter scale is of more practical interest, for example, for the problem of water resources and oil recovery. The macroscopic variables, related to the field scale, e. g. permeability, are commonly defined as volume average of microscopic variables over a Representative Elementary Volume (REV). The REV is well known concept and it is explained in detail by Bear (1972). The size of the REV is believed to be large when compared to the pore dimension or solid particle structures, but small compared to the dimension of the physical domain of interest in macroscopic analysis. Zhang *et al.* (2000) treated questions about what is the actual size of a REV and how a REV varies inside a medium. In this work, the medium autocorrelation function was computed and used as a basis to estimate the size of the REV.

The porous medium structure could be measured using a non-invasive technology, namely X-ray microtomography. This technology has been successfully applied to rock geometries imaging and has addressed several problems about REV and medium reconstruction. Specifically, the new technology of high resolution microtomography is able to obtain three-dimensional images of rocks with a resolution of the order of one micrometer. These images have been also successfully used in pore scale studies of flow in porous media (Zhang *et al.*, 2000).

In this research we investigate the relation between the REV and the autocorrelation length, calculating the permeability using the Lattice Boltzmann method (dos Santos *et al.* 2005). We also compare the permeability results of the microtomographic sample with those of tridimensional media, reconstructed using the Truncated Gaussian (Liang *et al.*, 1998) and Simulated Annealing (Torquato and Kim, 2000) methods.

2. REV DETERMINATION METHODOLOGY

For the REV determination we used a microtomography obtained from a Botucatu sandstone which was measured in the Petrobrás Research Facility (CENPES). This sample was scanned with a SKYSCAN 1172 X-Ray microtomograph equipment using a resolution of 3.81μm. The porosity of this sample is 31.16% and its autocorrelation length is 40 voxels, comprising 152.4 μm. The experimental permeability measured was 5.000mD. The scanned microtomography had 150^3 voxels and can be observed in Fig. 1a, with the pore space shown in gray scale. A slice image, extracted approximately at the middle of the sample, is shown in Fig. 1b.

The autocorrelation graphic and its autocorrelation length corresponding to the microtomography can be seen in the Fig 2.

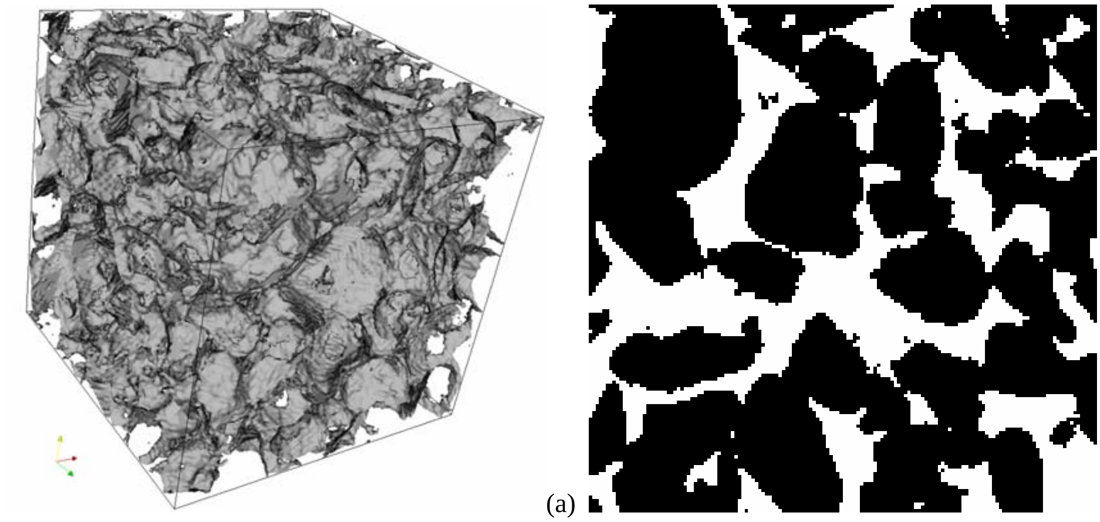


Figure 1. Botucatu sandstone microtomography with 3.81μm resolution. 3D (a) and binarized 2D views (b) with edge size of 150 voxels.

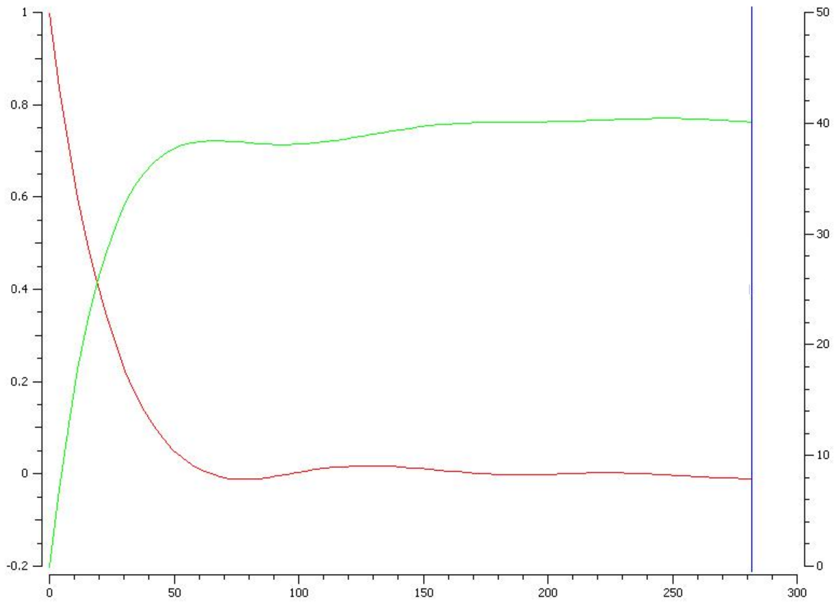


Figure 2. Autocorrelation function (R[u]) and autocorrelation length (Integral[u]) from the microtomographic sample shown in Fig 1.

The procedure started subdividing the microtomography sample in smaller media, comprising 60^3, 80^3, 100^3, 120^3 and the complete 150^3. For each one of these samples we calculated the permeability using the Lattice Boltzmann method described in dos Santos *et al.* (2005).

The Lattice Boltzmann Method (LBM) is a relatively new scheme for the simulation of the hydrodynamics of nearly incompressible fluids, based in a special discretization of the Boltzmann transport equation. In the particular implementation of this numerical method used in this work, the particle distribution function, $f_i(\vec{x},t)$, defined in specific sites of a regular lattice, evolves following the Lattice Boltzmann Equation with the Bhatnagar-Gross-Krook collision model, Eq. (1).

$$f_i(\vec{x}+\vec{c}_i\delta t, t+\delta t) = f_i(\vec{x},t) + \frac{f_i^{eq} - f_i(\vec{x},t)}{\tau}, \tag{1}$$

where $\vec{x}$ indicates vertices of a regular lattice, linked by a set of velocity vectors $\vec{c}_i$, δt denotes the time step, and the parameter τ is the relaxation time.

The velocity $\vec{u}$ and density ρ are defined as macroscopic moments of the distribution function,

$$\rho = \sum_i f_i \, , \tag{2}$$

$$\rho\vec{u} = \sum_i f_i \vec{c}_i . \tag{3}$$

A Chapman-Enskog multiscale analysis of the Lattice Boltzmann Equation gives the continuity and the Navier-Stokes equation for incompressible flows

$$\partial_t u_\alpha + u_\beta \partial_\beta (u_\alpha) = -\frac{1}{\rho}\partial_\alpha p + \nu \partial_\beta \partial_\beta (u_\alpha) \, , \tag{4}$$

where $p = \rho c_s^{\,2}$ and $\nu = c_s^{\,2}(\tau - 1/2)$.

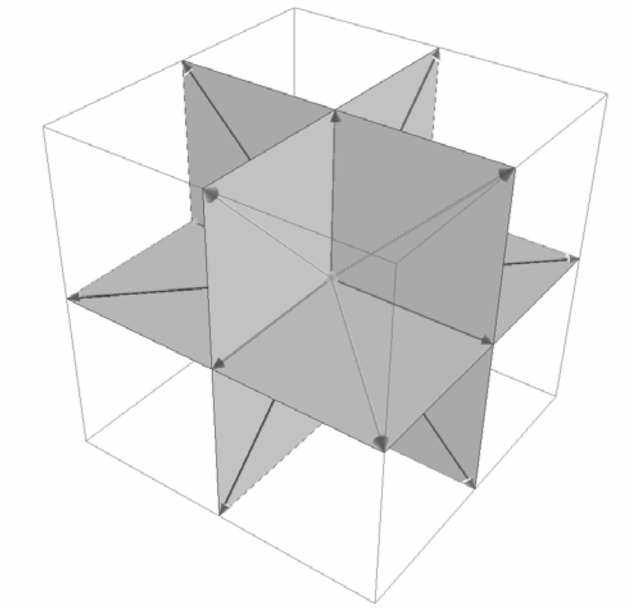


Figure 3. The three dimensional lattice with nineteen velocities used in the simulations, usually called 3DQ19 lattice.

For all cases the 3D lattice with nineteen velocities presented in Fig. 3 was used. The relaxation time was set as $\tau = 1$ and a body force field was used to drive the flow. These choices simplify the Eq. (1) and allows the use of only four real numbers, ($\rho, \vec{u}$), to represent the state of a site. The explicit and local nature of the algorithm provides its straightforward implementation in parallel computers. Using the MPI communication library, a parallel version of the lattice Boltzmann method was written, with a simple slice domain decomposition method in one direction. The permeability values were determined according to the Darcy law as

$$\langle u \rangle = \frac{k\phi}{\nu} X \, , \tag{5}$$

where ϕ is the porosity, X is the body force and the mean velocity was calculated as the average value of the momentum of all fluid sites

$$\langle u \rangle = \sum_x \rho u \Big/ \sum_x \rho . \qquad (6)$$

For every sub-sample, the intrinsic permeability k was calculated in the three orthogonal orientations and the mean value was taken as the representative value for each size. For example, for the 80^3 sample the values are 3917, 4520 and 4223 mD. Generally, the three values are closed showing the isotropy of the Botucatu sandstone. The sub-sample permeability results are given in Tab 1.

Table 1. Results from permeability calculated in the microtomography slices.

Size in voxels	**Permeability k (mD)**
60	5937
80	4220
100	4296
120	4961
150	4533

The permeability results exposed in the Tab. 1 shows a stability tendency around the 4.500mD value, which is very near to the experimental one. This would also discard the first processed slice as a valid simulation, indicating that the slice should have, at least, twice the autocorrelation length to perform a valid simulation. A larger microtomographic sample, however, is necessary for a final conclusion. Nevertheless, a conservative estimative of the REV size, suggested also by previous simulation in reconstructed media, has the order of twice the autocorrelation length.

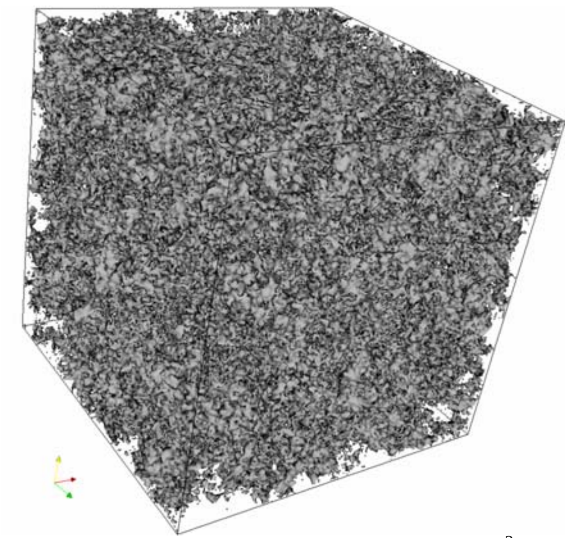


Figure 3. Truncated Gaussian simulation with 120^3 voxels.

3. COMPARISON WITH RECONSTRUCTION METHODS

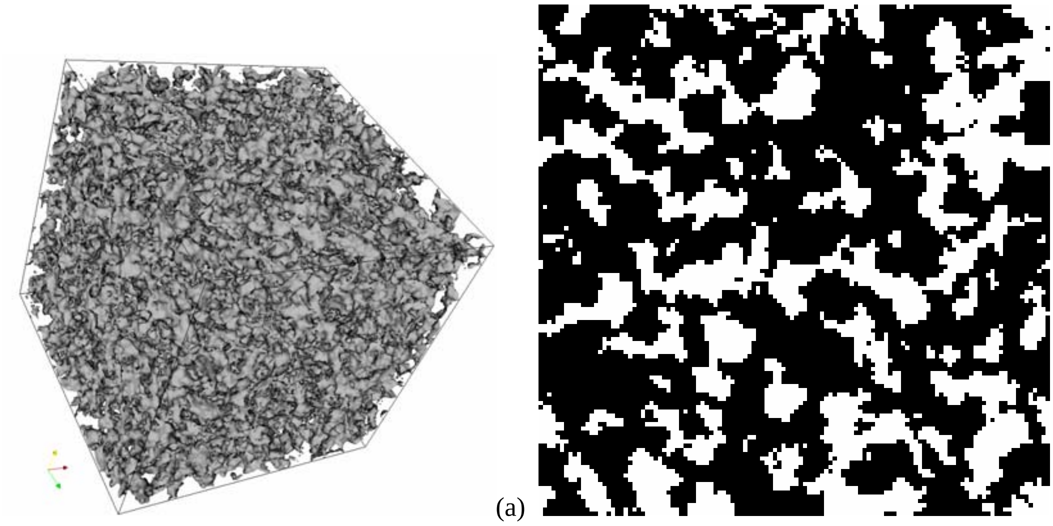
We used two methods to reconstruct tridimensional structures from slices of the original tomographic sample. The instrinsic permeability of this reconstruction, with sizes equal or greater than twice the autocorrelation length, to verify the reconstruction methods and the conclusions about the REV, as determined in the previous section.

We used the Truncated Gaussian method (Liang et al., 1998) and the Simulated Annealing method (Torquato and Kim, 2000), optimizing the autocorrelation and the linear chord functions. Figure 3 exposes a Truncated Gaussian reconstruction result. The microstructure size is 120^3 and the resolution is the same as the microtomography. Note that the method does not retain the pore sizes from the original microtomography. This problem is circumvented when the spatial resolution is reduced during the reconstruction process. A reconstruction result with 7.62μm resolution is shown in Fig 4. This medium does not retain the autocorrelation function, as its autocorrelation length is greater than the original one, but it represents better the pore sizes as can be noted when the slice Fig. 4b is compared to Fig. 1b.

The Fig. 5 shows the reconstruction result for the Simulated Annealing method. Although the optimized functions were better fitted than the Truncated Gaussian method, we can see some non connected porosity scattered around the reconstructed medium.

The autocorrelation functions for the three media are shown in the Fig. 6, showing that the reconstructed medium with the Truncated Gaussian does not retain the autocorrelation function.

The Lattice Boltzmann simulation results for permeability are shown in Tab. 2. As was predicted by the qualitative criteria of pore size comparison, the reconstructed media by the Truncated Gaussian method underestimate considerably the permeability values. The Truncated Gaussian with half resolution and Simulated Annealing reconstructions also has lower values of the permeability. The Truncated Gaussian method with decreased resolution provided the best results. This method is also remarkably faster than the Simulated Annealing method. The latter method, however, is more flexible, as it can optimize more parameters and represent better the original medium. This feature will be exploited in future works.

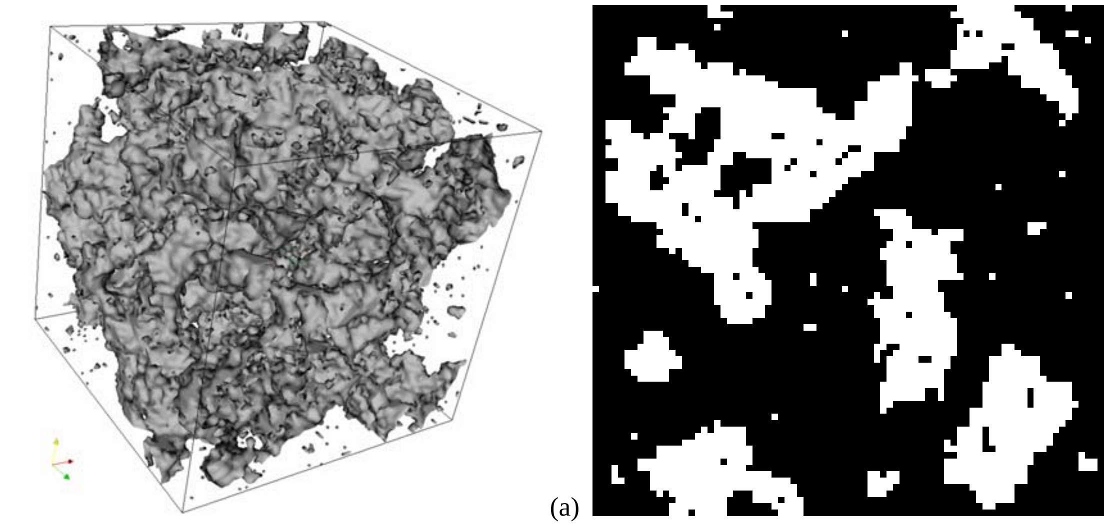


(a) (b)

Figure 4. Reconstructed medium with 120^3 voxels using Truncated Gaussian method with a half resolution of the original image. (a) 3D view (b) Transversal section. The black pixels are solid.

Table 2. Permeability results for the reconstructed media.

Reconstruction Method	**Size in voxels**	**Permeability k (mD)**
Truncated Gaussian	80	676
Truncated Gaussian	120	722
Truncated Gaussian	150	677
Simulated Annealing	80	2.291
Truncated Gaussian with decreased resolution	120	3.512



(a) (b)

Figure 5. Simulated Annealing reconstruction result with 80^3 voxels optimizing autocorrelation and the linear chord functions. (a) 3D view (b) Transversal section. The black pixels are solid.

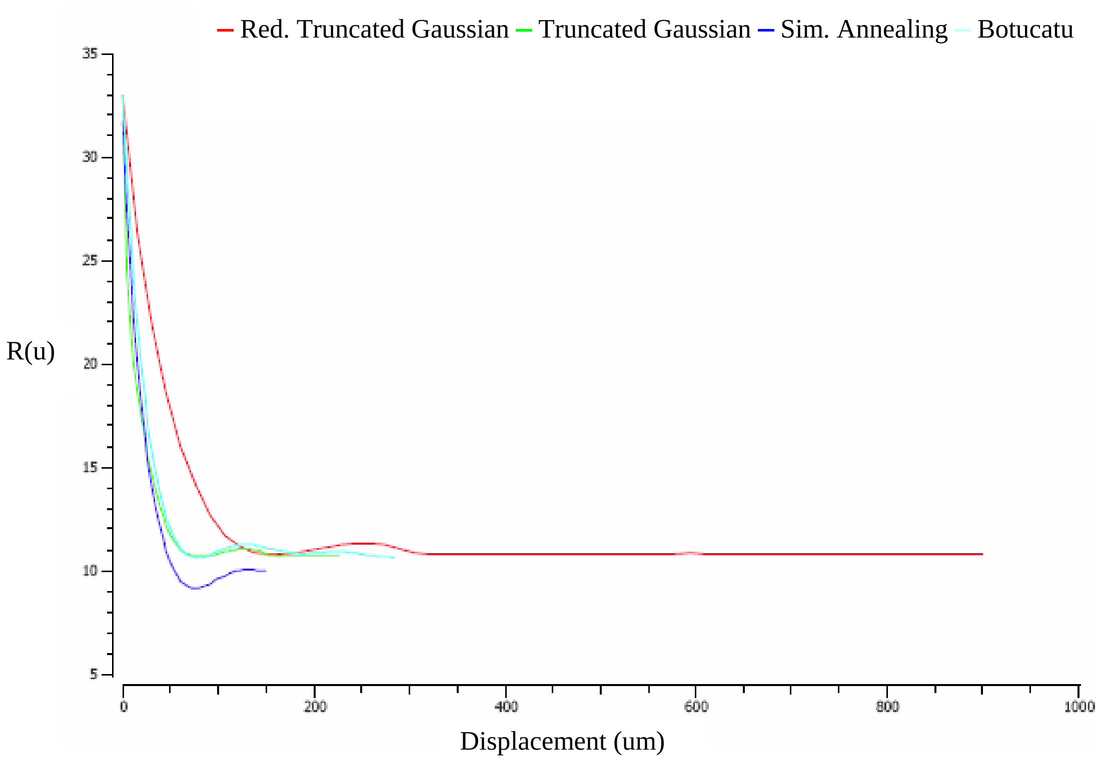


Figure 6. Autocorrelation functions from the microtomography and all the reconstructed media used in this research.

4. CONCLUSION AND DISCUSSION

As was strongly suggested by the lattice Boltzmann simulations of nearly incompressible fluid flow in microtomographic reconstructed media from Botucatu sandstone, a conservative estimative of the size of the representative elementary volume is twice the autocorrelation length.

The two reconstruction techniques used here, Truncated Gaussian and Simulated Annealing methods, do not represent the porous structure of the sandstone properly, particularly the pore size distribution. Because the strong dependency between the porous media structure details and the intrinsic permeability, these method was unable to furnish reliable values of the intrinsic permeability in this kind of porous media independently of the size of the sample.

Results for more detailed microtomographic images and different sandstones as well as others three-dimensional reconstruction methods, are in progress.

5. ACKNOWLEDGEMENTS

Authors are greatly indebted with ANP (Agencia Nacional do Petroleo), CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) and Finep (Fundação Nacional de Estudos e Pesquisas) for the financial support and to the CENPES-Petrobrás for providing the microtomographic samples used in this research.

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