REAL TIME MULTISATELLITE ORBIT DETERMINATION FOR CONSTELLATION MAINTENANCE

Jairo Cavalcanti Amaral, jairo_amaral@hotmail.com
Hélio Koiti Kuga, hkk@dem.inpe.br
Marcelo Lopes de Oliveira e Souza, marcelo@dem.inpe.br
Instituto Nacional de Pesquisas Espaciais, Av. dos Astronautas 1.758, Jardim da Granja, São José dos Campos – SP, Brasil

Abstract. *In this work, we present a nonlinear Kalman filter algorithm for monitoring the orbital motion of a single satellite in real time. It is based on the linearized and extended Kalman filters. Some tests are done to show its correctness. After that, we intend to extend such algorithm to a triangular equatorial constellation of satellites to monitor their absolute positions and velocities. The tests will also be extended to show its correctness. The orbital motion will be modeled as simple as possible so as not to burden the computer load and, at the same time, to provide enough accuracy to this sort of problem. The source of measurements will be simulated to represent actual measurements such as GPS, geostationary satellite to satellite tracking, or conventional ground tracking stations. The errors in orbit determination will ultimately affect the performance of the control and maneuvering of the constellation, therefore providing the basis for future corrective and preventive constellation maintenance.*

Keywords*: extended Kalman filter, orbit determination, constellation maintenance.*

1. INTRODUCTION

The problem to be solved here is to develop a nonlinear Kalman filter algorithm which uses real-time measurements to produce estimates of a satellite position and velocity – the best possible for the information available. The algorithm will allow a preliminary study of the filter behavior in face of varying parameters such as measurement precision and measurement rate. This can later be extended to provide good preventive/corrective maintenance of a satellite constellation, where it is necessary to have precise estimations of the satellite positions to support maintenance decisions.

2. SYSTEM DESCRIPTION

The simulated satellite has an orbit in a bidimensional space, and the measurements considered are three simultaneous distances from it taken by other three satellites simulated at GPS-altitude orbits in the same plane.

The state vector **x** contains the position (x_1, x_2) and velocity (x_3, x_4) of the simulated satellite in each dimension (x,y), while the vector of observation **y** contains the distances from the simulated satellite observed by the other three GPS satellites. Below, **G** is the dynamic noise addition matrix, **ω** is the **continuous** dynamic noise vector, assumed normal with zero mean and covariance **Q**; and **v** is the **discrete** observation noise vector, assumed normal with zero mean and covariance **R**. The Earth gravitational parameter considered in the simulations is μ = 398600 km^3/s^2. So, the equations of the system are:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) + \mathbf{G}\boldsymbol{\omega}(t), \forall t \in [t_0 ; t_f] \\ \mathbf{y}(t_i) = \mathbf{h}(\mathbf{x}(t_i)) + \mathbf{v}(t_i), t_i = t_0 + i.T_s, i = 1,2,3 \ldots N_s \end{cases} \tag{1}$$

where $\boldsymbol{\omega} = N(\mathbf{0}, \mathbf{Q})$ and $\mathbf{v} = N(\mathbf{0}, \mathbf{R})$.

The functions of transition **f**(**x**) and observation **h**(**x**) are described as follows:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_3 \\ x_4 \\ -\dfrac{\mu}{r^3} x_1 \\ -\dfrac{\mu}{r^3} x_2 \end{bmatrix} \tag{2}$$

where $r = \sqrt{x_1^2 + x_2^2}$ is the orbit radius magnitude.

$$\mathbf{h}(\mathbf{x})=\begin{bmatrix}\sqrt{(x_1-X_1)^2+(x_2-Y_1)^2}\\ \sqrt{(x_1-X_2)^2+(x_2-Y_2)^2}\\ \sqrt{(x_1-X_3)^2+(x_2-Y_3)^2}\end{bmatrix} \tag{3}$$

where X_i, Y_i, are the position coordinates of the observing GPS satellite i.

3. FILTER DESCRIPTION

The Kalman filter (Kalman, 1960) works by trying to minimize the mean squared estimation error of the state of linear systems with noises in the dynamics and in the measurements. According to references such as Kuga (2005), Maybeck (1979), Brown e Hwang (1996), it has the advantage that it can be used recursively, instead of being obliged to collect and process all data at once as other minimum square methods. Besides that, it can easily include the dynamic noise in the formulation. It consists of two phases: 1) propagation in time, where it takes initial estimates of the mean state vector $\bar{\mathbf{x}}(t)$ and the state covariance matrix $\mathbf{P}(t)$ and predicts their behavior until the next measurements; and 2) updating in measure, where it uses the available measurements to obtain new estimates by comparing them with propagated ones.

The mean state vector $\bar{\mathbf{x}}(t)$ will follow the (ideal) physical model without noise, but the uncertainty (given by the covariance matrix $\mathbf{P}(t)$) will be updated taking into account the uncertainties attributed to the model. The Kalman filter theory assumes normal distributions of deviations; actually they are not always normal, but this is a fair approximation in most cases.

As most things in Nature, satellites in orbit do not exhibit linear behavior, so we need to use linear approximations in a variant called **the extended Kalman filter**. At the initial instant t_0, it starts the first nominal state trajectory $\mathbf{x}_n(t)$ in the initial state estimate $\mathbf{x}_0$, i.e., $\mathbf{x}_n(t_0)=\bar{\mathbf{x}}(t_0)=\mathbf{x}_0$. Between updating instants (t_i; t_{i+1}) it propagates the nominal state trajectory $\mathbf{x}_n(t)$, as being the mean trajectory $\bar{\mathbf{x}}(t)$, i.e., $\mathbf{x}_n(t)=\bar{\mathbf{x}}(t)$ from that point on, until the new updating. At each updating instant t_i, it starts a new $\mathbf{x}_n(t)$ in the updated state estimate $\hat{\mathbf{x}}(t_i)$, i.e., $\mathbf{x}_n(t_i)=\hat{\mathbf{x}}(t_i)$. While more computationally costly, it is more adequate to situations of poor models or initial conditions and it is easier to implement than its counterpart called **the linearized Kalman filter**, which works only with the linear deviations from a nominal trajectory $\mathbf{x}_n(t)$ that is not altered along the cycles. In this paper, we will use the extended Kalman filter.

3.1. Propagation phase

Considering a continuous system and a discrete measurement, the propagation phase of $\bar{\mathbf{x}}(t)$ and $\mathbf{P}(t)$ during $t\in$ (t_i; t_{i+1}) is defined by the following differential equation system:

$$\begin{cases}\dot{\bar{\mathbf{x}}}=\mathbf{f}(\bar{\mathbf{x}}),\bar{\mathbf{x}}(t_0)=\mathbf{x}_0\\ \dot{\bar{\mathbf{P}}}=\mathbf{F}\bar{\mathbf{P}}+\bar{\mathbf{P}}\mathbf{F}^T+\mathbf{GQG}^T,\bar{\mathbf{P}}(t_0)=\mathbf{P}_0\end{cases} \tag{4}$$

where $\mathbf{F}$ is the jacobian matrix of $\mathbf{f}$:

$$\mathbf{F}=\left.\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_n(t)}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 3\mu r^{-5}x_1^2-\mu r^{-3} & 3\mu r^{-5}x_1x_2 & 0 & 0\\ 3\mu r^{-5}x_2x_1 & 3\mu r^{-5}x_2^2-\mu r^{-3} & 0 & 0\end{bmatrix} \tag{5}$$

with $r=\sqrt{x_1^2+x_2^2}$.

3.1. Updating phase

The updating phase at $t = t_i$ is done by first defining the Kalman gain $\mathbf{K}$, and then using it to find the updated $\bar{\mathbf{x}}(t)$ and $\mathbf{P}(t)$:

$$\mathbf{K} = \overline{\mathbf{P}}\mathbf{H}^T\left(\mathbf{H}\overline{\mathbf{P}}\mathbf{H}^T + \mathbf{R}\right)^{-1} \tag{6}$$

$$\hat{\mathbf{P}} = (\mathbf{I} - \mathbf{K}\mathbf{H})\overline{\mathbf{P}} \tag{7}$$

$$\hat{\mathbf{x}} = \overline{\mathbf{x}} + \mathbf{K}[\mathbf{y} - \mathbf{h}(\overline{\mathbf{x}})] \tag{8}$$

$$\text{with } \mathbf{H} = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{x=x_n(t_k)} = \begin{bmatrix} \frac{(x_{n1}-X_1)}{C_1} & \frac{(x_{n2}-Y_1)}{C_1} & 0 & 0 \\ \frac{(x_{n1}-X_2)}{C_2} & \frac{(x_{n2}-Y_2)}{C_2} & 0 & 0 \\ \frac{(x_{n1}-X_3)}{C_3} & \frac{(x_{n2}-Y_3)}{C_3} & 0 & 0 \end{bmatrix} \tag{9}$$

$$\text{where } \begin{cases} C_1 = \sqrt{(x_{n1} - X_1)^2 + (x_{n2} - Y_1)^2} \\ C_2 = \sqrt{(x_{n1} - X_2)^2 + (x_{n2} - Y_2)^2} \\ C_3 = \sqrt{(x_{n1} - X_3)^2 + (x_{n2} - Y_3)^2} \end{cases} \tag{10}$$

The new $\hat{\mathbf{x}}$ and $\hat{\mathbf{P}}$ are then used for initializing the next propagation phase.

4. DESCRIPTION OF THE SIMULATED CASES

The positions of each satellite were produced by a numerical simulation with MatLab (versions 6.1) using the two body dynamics, as described in Kuga and Rao (1995). For the observed satellite, a dynamic noise was introduced as an acceleration in each dimension to account for the imprecision of the model. Eclipses in observation caused by Earth were neglected.

The simulation parameters (absolute tol. = 10^{-6}, relative tol. = 10^{-8}) were chosen so that, with dynamic noise $\sigma_D = 0$, the order of the numerical error in position after one revolution be smaller than 1 m, using the variable step integration methods ODE1-13 of Matlab 6.1. We plotted the results for each satellite in N equally distributed instants along the time span $[t_0;t_f]$. In the same time interval the measurements were done at a rate T_s. The observed satellite initial conditions $\mathbf{x}_0 = \mathbf{x}(t_0)$ are assumed to be known with a standard deviation σ_P at each dimension of position and σ_V at each dimension of velocity; the dynamic noise has a standard deviation σ_D in each dimension, and the measurement noise has a standard deviation σ_M in each sensor.

The tests were conducted according to Table 1, by applying deviations in the initial conditions $\mathbf{x}_0 = \mathbf{x}(t_0)$ that were equal to the uncertainty attributed to them, and then letting the Kalman filter try to approach the actual positions by using the measurements affected by noise.

Table 1: Parameters of the Graphics Shown in Figures 1-8.

Tests	σ_M	σ_D	T_s	Initial State x(t_0)	Initial Estimate $\overline{\mathbf{x}}_0$	Initial Covariance $\mathbf{P}_0$ (diagonal)
Figure 1	10 m	1e-3 m/s^2	60 s	[7000, 0, 0, 7.5]	[7010, 10, 1, 8.5]	[10^2, 10^2, 1^2, 1^2]
Figure 2	1 km	1e-3 m/s^2	60 s	[7000, 0, 0, 7.5]	[7010, 10, 1, 8.5]	[10^2, 10^2, 1^2, 1^2]
Figure 3	10 m	1e-5 m/s^2	60 s	[7000, 0, 0, 7.5]	[7010, 10, 1, 8.5]	[10^2, 10^2, 1^2, 1^2]
Figure 4	10 m	1e-3 m/s^2	10 s	[7000, 0, 0, 7.5]	[7010, 10, 1, 8.5]	[10^2, 10^2, 1^2, 1^2]
Figure 5	10 m	1e-3 m/s^2	60 s	[7000, 0, 0, 8.5]	[7010, 10, 1, 9.5]	[10^2, 10^2, 1^2, 1^2]
Figure 6	10 m	1e-3 m/s^2	60 s	[7000, 0, 0, 7.5]	[7100, 100, 2, 9.5]	[100^2, 100^2, 2^2, 2^2]
Figure 7	10 m	1e-3 m/s^2	60 s	[7000, 0, 0, 7.5]	[7010, 10, 1, 8.5]	[10^2, 10^2, 1^2, 1^2]
Figure 8	10 m	1e-3 m/s^2	60 s	[7000, 0, 0, 7.5]	[7010, 10, 1, 8.5]	[10^2, 10^2, 1^2, 1^2]

Though the estimation process does not require this, we also recovered the positions pointed by the noisy measurements for comparison with the real and estimated positions. The method chosen was a Newton-Raphson iteration using the first two of the three distances measured. As the observed satellite is assumed to be always in a lower orbit than the GPS', the initial radius is equaled to zero and then used in the following recursion:

$$\mathbf{r}_{k+1} = \mathbf{r}_k + \left[\frac{\partial \mathbf{h}(\mathbf{r}_k)}{\partial \mathbf{r}} \right]^{-1} \left[\boldsymbol{\rho} - \mathbf{h}(\mathbf{r}_k) \right] \tag{11}$$

until $\left\| \mathbf{r}_{k+1} - \mathbf{r}_k \right\| \leq \varepsilon$, where $\varepsilon = 0.0001$ is the desired precision.

In a real time application, the algorithm would start propagating the initial state vector and covariance matrix and stop at the scheduled time of the next measure until it comes; then repeat the procedure with the updated state.

5. RESULTS

Figure 1 shows that the Kalman filter was able to estimate the position and velocity beyond the measurement error, even with measures related only to position. The algorithm was able to drop the position standard deviation from 10 km to 5 meters in 10 to 20 measurements of 10 m precision, and the velocity standard deviation from 1 km/s to less than 10 cm/s in an equivalent time interval.

With the standard parameters of the first example, position and velocity deviations ultimately settled around 5 m and 3 cm/s respectively. Besides that:

Figures 1 and 2 show that decreasing the measure precision by two orders resulted in a more oscillating precision $\sigma_{xx}(t)$, worse by a factor of 40.

Figures 1 and 3 show that reducing the dynamic noise by two orders also resulted in a more oscillating precision $\sigma_{xx}(t)$, but roughly 2 times better.

Figures 1 and 4 show that increasing the measure rate by 6 resulted in an increase factor of 2 in precision.

Figures 1 and 5 show that using an eccentric orbit altered the oscillation of the precision, but no other significant difference.

Figures 1 and 6 show that increasing the initial deviation and standard deviation by 10 and 2 in position and velocity, respectively, resulted in a more unstable transient, but also no other significant difference.

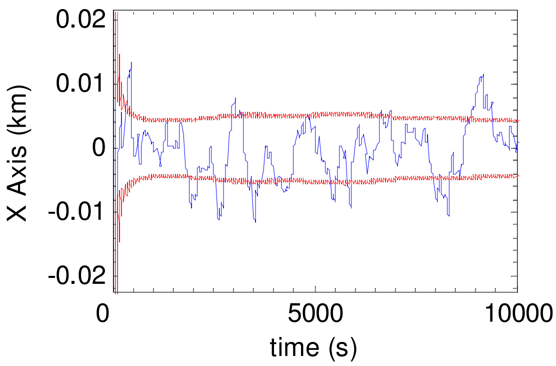

Figure 1.

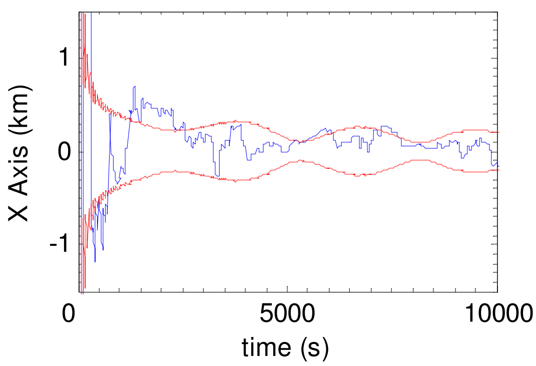

Figure 2.

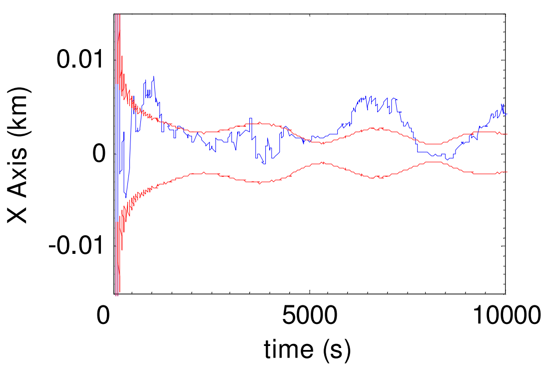

Figure 3.

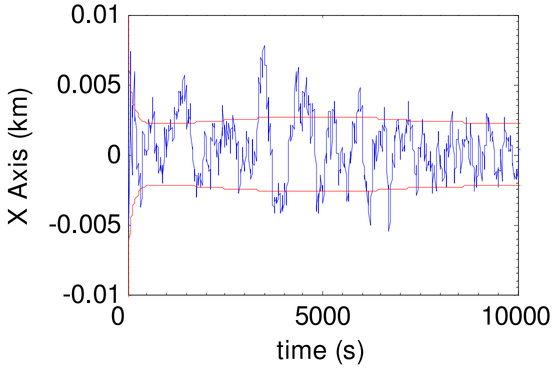

Figure 4.

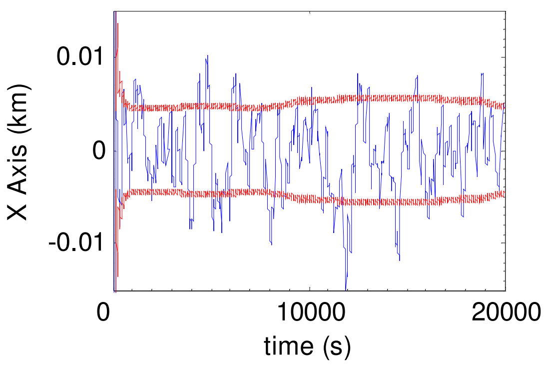

Figure 5.

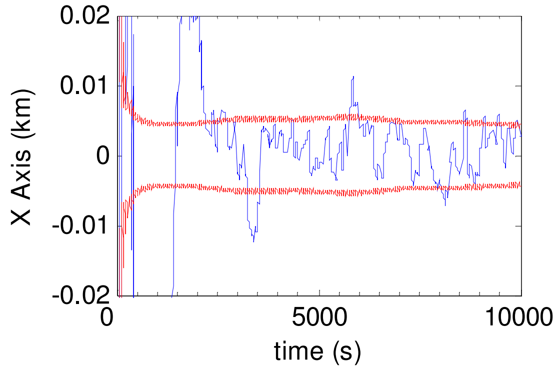

Figure 6.

For didactic purposes, we also tested two less adequate variations in the algorithm, all of them with the same parameters shown in Fig 1.

Figure 7 shows the extended Kalman filter applied to a 3D space, i.e., **x** now includes *z* and v_z; but still with only 3 observers. As in tests 1-6 the three observing satellites were in the same plane of the observed satellite, and could only measure scalar distances, they had no means to observe the signal of the displacement in the *z* axis neither its speed. So, the linearization prevented any variation in z axis from being computed from , since the first order derivatives $\frac{\partial \mathbf{h}}{\partial z}, \frac{\partial \mathbf{h}}{\partial v_z}$ are null at *x-y* plane. So, even with initial states zeroed in the *z* axis, the filter always diverged in a matter of time .

In Fig. 8, it was used the linearized Kalman filter, with an initial state of the nominal trajectory $\bar{\mathbf{x}}(t_0)$ deviated in 1 km along the *x* axis from the initial state of the real trajectory $\mathbf{x}(t_0)$. Differently from the extended Kalman filter, the nominal trajectory was not updated. It was able to work properly for one orbit; but, after that, the difference of position between the real and nominal references became too big, making the filter to diverge. As the standard deviation relies on the nominal trajectory, it did not accused the divergence, which is even worse. This outweighs the milder computational load of this filter, and shows that it would be better only in situations where there is control to keep it close to the nominal trajectory.

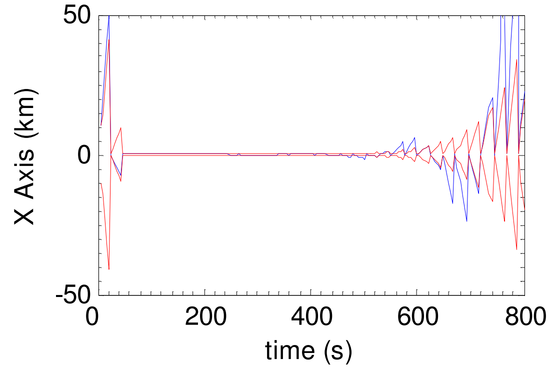

Figure 7.

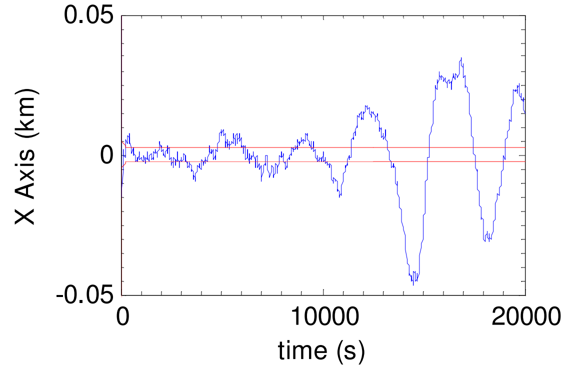

Figure 8.

6. INTENDED EXTENSIONS

Until now the movements and deviations were kept in two dimensions, we intend to increase the number of observation sources, so that we can also work in the *z* axis.

With the algorithm for a single satellite completed, the next step is to adapt it to a constellation of satellites and eventually include the modeling and detection of other GPS error sources, such as the clock synch and atmospheric interference, as treated in Raimundo (2007).

7. CONCLUSIONS

In this work, we presented a nonlinear Kalman filter algorithm for monitoring the orbital motion of a single satellite in real time. It was based on the extended Kalman filter mainly. Some tests were done to show its correctness. The tests helped to show how variations in the estimation parameters influence the quality of the resulting data. This is useful in determining what is necessary and what is superfluous, and is the key to find what parameters should be emphasized in a multi-satellite estimation system. After that, we intend to extend such algorithm to a triangular equatorial constellation of satellites to monitor their absolute positions and velocities.

8. ACKNOWLEDGEMENTS

The first author thanks the National Council for Scientific and Technological Development-CNPq of Brazil for supporting him during this research. The second and third authors thank the National Institute for Space Research-INPE/DMC of Brazil for supporting them and the Laboratory of Computational Environments for Simulation, Identification, and Modeling -LABSIM2 of AOCS where this work is being done.

9. REFERENCES

Brown, R. G. and Hwang, P. Y. C., 1996, “Introduction to Random Signals and Applied Kalman Filtering”, John Wiley & Sons, New York.

Kalman, R.E., 1960, “A New Approach to Linear Filtering and Prediction Problems”, Transactions of the ASME–Journal of Basic Engineering, 82 (Series D): 35-45.

Kuga, H.K., 2005, “Noções Práticas de Técnicas de Estimação”, INPE, São José dos Campos.

Kuga, H.K., Kondapali, R.R., 1995, “Introdução à Mecânica Orbital”, INPE-5615-PUD/064, INPE, São José dos Campos.

Maybeck, P. S., 1979, “Stochastic Models, Estimation and Control”, vol.1, Academic Press, New York.

Raimundo, P.C.P., 2007, “Determinação de Órbita via GPS Considerando Modelo de Pressão de Radiação Solar para o Satélite Topex/Poseidon”, INPE, São José dos Campos, Dissertação de Mestrado.

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.