A PROPOSED AIRFOIL SHAPE PARAMETRIZATION FOR AERODYNAMIC OPTIMIZATION

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Abstract. *The main objective of this paper is to present a validation methodology for airfoil parametrization. This methodology aims at wing optimization in a context of aerodynamics. The proposed method is based on an airfoil description for a set of functions stored on a numerical database. The method is implemented on a MATLAB environment, and its results are compared using an open source software. The presented results are concerned for a set of four airfoils, one of them a part of the NACA airfoil family. It is intended for the methodology the description of each airfoil using a minimum number of parameters in order to obtain the before mentioned description with minimum computational cost. The results are compared with experimentation obtained by NASA airfoils*

Keywords: *: parameterizations, airfoil, optimization*

1. INTRODUCTION

Computational methods for optimization has become more popular on aircraft design since they require less time compared to traditional methods. Also it make possible to reach improbable solutions by classical methodologies. The efficiency of optimization method is directly connected to geometrical parametrization, therefore both descriptions should be efficient to describe wing geometries with a reduced number of parameters. The optimum airfoil were included in this options. The description of an airfoil shape is complex, because there are regions, like leading edge, where geometrical derivative is high. This is the reason for the necessity of a great number of coordinates. With it, the geometry can be accurately described.

Recently diverse types of parameterizations had been developed for the use in projects of aerodynamic optimization and parametric studies. First and the less efficient one it was to the discrete description of the airfoil by means of coordinates. This method is easy of being implemented, however it requires a great number of points, particularly close to the leading edge.

The analytical functions are an other very popular method in the mathematical representation of airfoils. Polynomial representations and for splines reduce the number of necessary parameters in a large way, however, for points as the leading edge, these functions require a polynomial of high degree what increases the number of necessary parameters. Moreover, this method can be capable to represent only one part of the set of airfoil.

An evolution of this method is the parameterizations using Bezier curves which is described by Cosentino and Holst (1984). In this method exist the possibility to represent the curve using segments of functions of Bezier, which reduces the number of necessary parameters since in this representation the degree of the function can be reduced.

Descendant of this methodology the PARSEC method, that uses 11 parameters to represent an airfoil, is currently used on large scale in the industry. The PARSEC method is able to represent, with a small number of airfoil parameters, any desired set of airfoil shape. Because of this, the PARSEC method is superior when compared with the representation by coordinates.

In this direction, many studies have been performed. On this context, one can quote Sobiesky (1998) where it is used algebraic and analytical relations to generate realistic geometries. In this work is made an extensive quarrel on the methodology of method PARSEC.

In relation to the implementation of new analytical and half-analytical methods Sohn and Lee (2000) can be cited, where is implemented a based half-analytical method in Bezier curves. In the work of Wu et al (2003) three methodologies of parameterizations, PARSEC, Hicks-Henne and mesh-point are compared and validated.

The method free-form described at Lepine (2001) also called NURBS (rational Non-uniform B-splines) represents an airfoil for a series of points of control that determine a local B-spline. This point of control are generally project variables. Finally, there are non-analytical methods for the representation of airfoil using bases of orthogonal functions (Robinson and Keane, 2001) where these functions are based on airfoils or a set of airfoils. The main advantage on analytical methods is that the not analytical representation has taken in account the description of optimizations made for each airfoil in how much the analytical method exactly that simpler does not make it. The method to be described in this paper uses not orthogonal bases that exactly becoming its more complex describing algorithm obtain to represent a greater number of airfoil.

The following list represents desirable features for any geometric representation:

1. Well behaved, and produce smooth and realistic shapes;

2. Numerically-stable and efficient with consistent, accurate and fast process;

3. Require relatively few variables to represent a large enough design space to contain optimum aerodynamic shapes for a variety of design conditions and constraints;

4. Systematic and Consistent - The way of representing, creating and editing different types of geometries must be the same;

These had been described in Kulfan and Bussoletti (2006). All the described methods above consider to follow such rules, however the concept of more relevance is presented by item 3.

2. PARAMETRIZATION METHODOLOGY

The method proposed in this paper is to use base functions made from a set of preexisting airfoil shapes. In this process, the base functions are numerically obtained by a iterative method. This method uses least-square fittings with error minimization between the original and the represented airfoils.

The representation of the airfoils shapes by the based functions is made with the following expression:

$$y_j = \sum_{b=1}^{nb} \alpha_b F_{bj} \tag{1}$$

The first step of the process is applied the least-square fitting in any group of base functions, for example a set of polynomial functions. In this moment *nb* coefficients are found for each airfoil shape by the using of the least-square fitting. The second step fix the coefficients found in the previous step and the value of the functions base are determined by the least-square method. However the method is applied to find the value of the functions base in each point which minimized the error between the airfoil package and the coefficients found in the first step. The next step normalizes the functions bases found.

The convergence of the method is characterized when there are no more changes in the functions bases and the coefficients found. The method is applied successively until it converges.

Following the used algorithm:

1. By a set of any functions base determine the coefficients that minimized the error resolving the following equation:

$$\frac{dE_a}{d\alpha_{ka}} = \sum_{j=1}^{np} \sum_{b=1}^{nb} (\alpha_{ab} F_{kj} F_{bj} - y_{aj} F_{kj}) = 0 \tag{2}$$

2. Fixing the coefficients of 1 determine the value points of the functions base that minimized the error of the airfoil package:

$$\frac{dE_j}{d\alpha_{ki}} = \sum_{a=1}^{na} \sum_{b=1}^{nb} (F_{ij} \alpha_{ai} \alpha_{ai} - y_{aj} F_{kj}) = 0 \tag{3}$$

3. Normalize the functions bases found in 3 by the maximum value of the functions.

4. Repeat 1 with the new bases found in 3 until the method converge

The success of any method of parameterizations is directed related to calculations of accurate sensitivity derivatives Li and Padula (2004). The requirement that the base functions be smooth was imposed by applying Savitz-Golay smoothing in the base functions after the iterations.

3. RESULTS AND ANALYSIS

A key convergence question relative to any geometrical parameterizations is the following: How many numbers of parameters are required for a sufficient capture of a meaningful design space to contain a true optimum design? The validation method was defined in order to obtain the answer to that question with a two approaches:

1. Compare actual airfoil and represented airfoil geometries for a wide variety of airfoils.
    - Compute and compare curvature, surface slopes and between actual and approximate airfoil shapes for a variety of numbers of parameters
2. Conduct a panel method with boundary layer analysis of the actual and the represented airfoil shape.
    - A comparative study with the numbers of parameters in the convergence of the aerodynamics coefficients and pressure distributions.

More then 530 airfoils have been analyzed applying step one. These include symmetric NACA airfoils, cambered NACA airfoils, high lift airfoils, natural laminar flow airfoils, Utralight, shock-free airfoils, supercritical airfoils and transonic multipoint optimized airfoils. Results of the analysis of some of these airfoils will be shown to illustrate the evaluation processes and to demonstrate the rate of convergence of the parameterizations airfoil representation, to the corresponding specified airfoil geometry with increasing orders parameters.

The results obtainable were based in a set of approximately 400 airfoils. The figure 1 presents 5 functions based derivable from this set of airfoils :

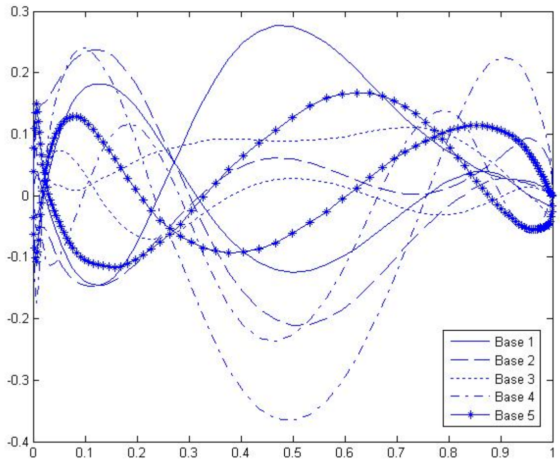


Figure 1. Set of 5 base functions derivate from the package of airfoils

In the convergence studies are included very detailed comparisons of the geometric and aerodynamic characteristics of the originals airfoils and those calculated with the base functions, for 3 to 11 bases.

3.1 NACA 0012: Symmetric Airfoil Study

The NACA0012 is a very known symmetrical airfoil which has been used to comparison in similar methodologies of parameterizations. Following will be presented the geometric comparisons (step 1) and the aerodynamic comparisons (step 2) for the study case.

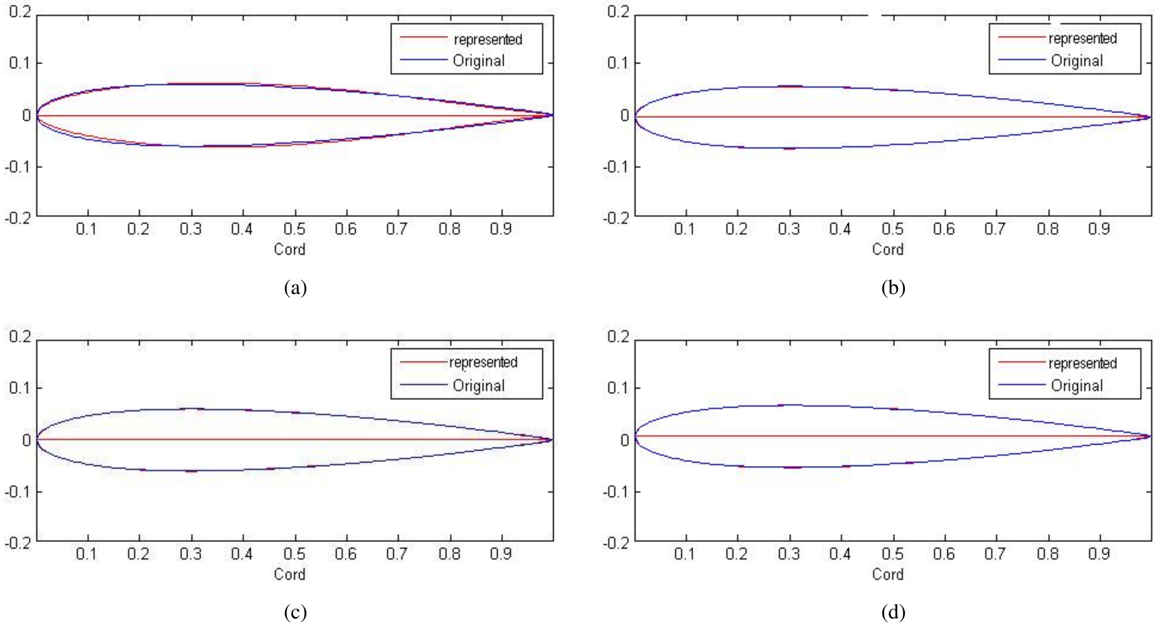


Figure 2. Geometric Comparisons for base functions a)3, b)5, c)7 e d)11

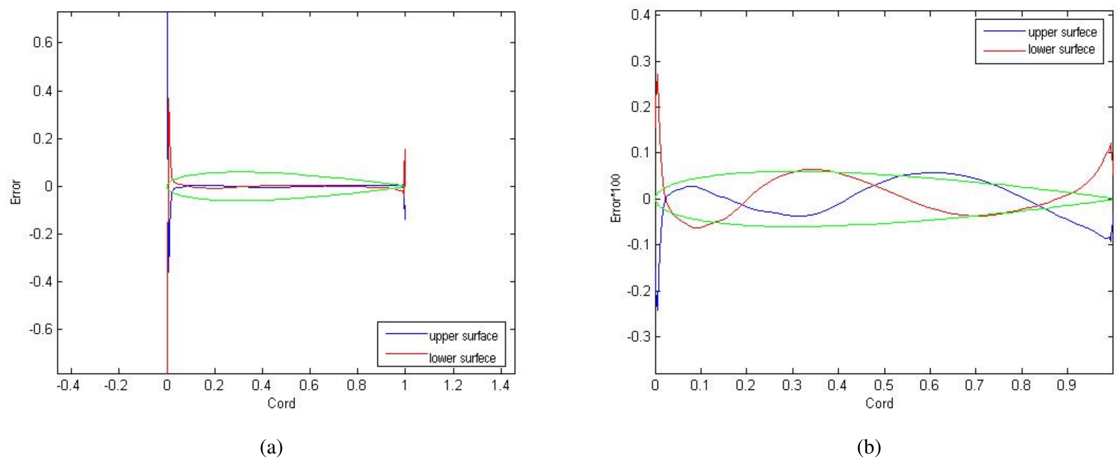


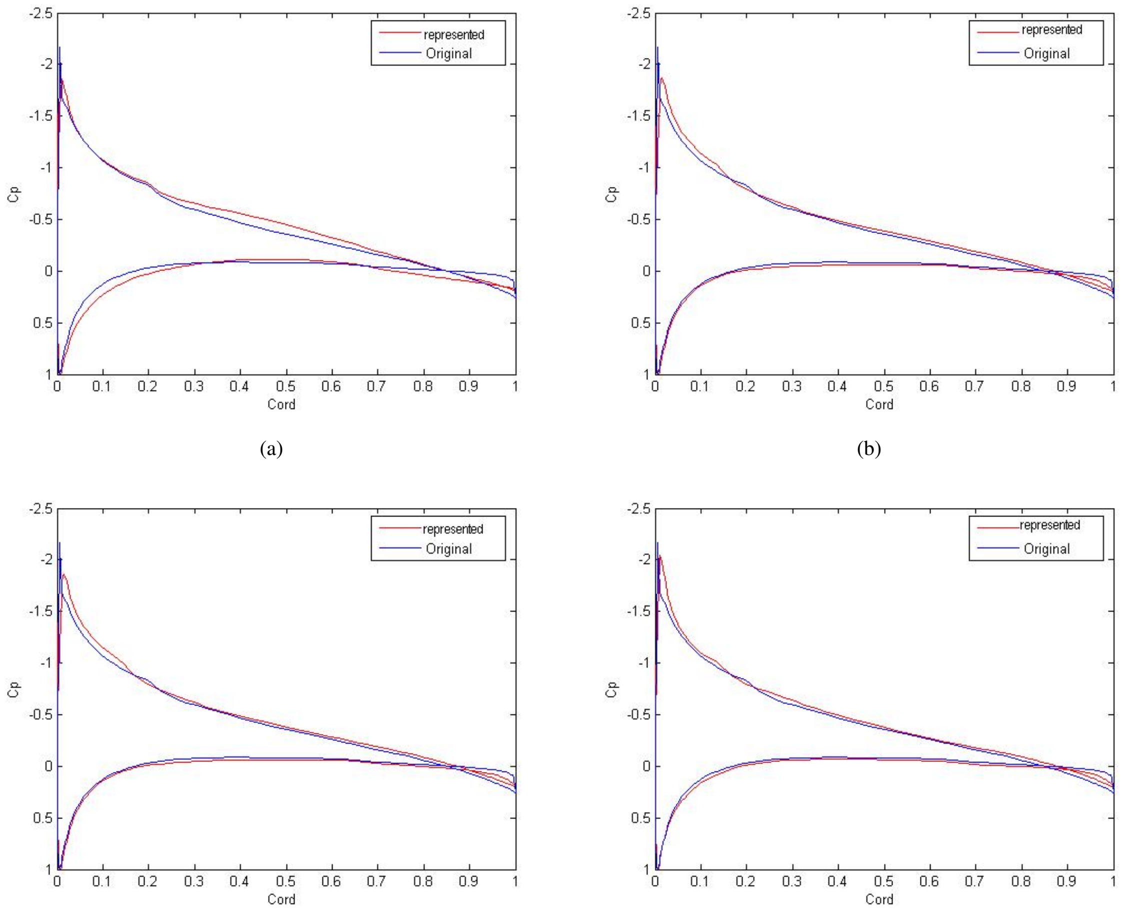
Figure 3. Geometric Errors a) 1^{st} derivative e b) Thickness percent cord with 5 BF

The figure 1 shows the geometric comparisons between the approximated and the official shape. For this example, the difference between the shape functions for the actual and the approximated geometries are hardly discernible even for the 3 base functions, 3BF. The corresponding Geometric errors are indicated also in figure 2 by the 1^{st} derivative error and the thickness error or residual difference. The residuals are show as Error x 100.Typical wind tunnel model tolerance are 0.1 % of thickness(Kulfan and Bussoletti (2006).The oscillating nature of the thickness error are characteristic of any least square fit. All residuals from 5BF are less then the required to Wind tunnel.

The residuals of the 1^{st} derivatives show that numerically the airfoil slopes go to infinite near the leading edge of the airfoil and consequently it is difficult to see differences between matched geometry and actual airfoil geometry in the leading edge region. However the graphic shows a excellent behavior of the method form 5BF.

The 2^{nd} previously described step in the process to determine the numbers of base functions are necessary to adequately match the actual airfoil geometry, involved a panel method analysis of the actual and approximate airfoil geometries comparing the calculated surface pressure distributions and aerodynamic coefficients.

The aerodynamics analysis were made in the Xfoil (6.94) panel method code if bounder layer. It's a traditional free software panel code with good results compare to wind tunnel experiments. It was develop for primarily analysis of airfoil shapes. The converge of the software can be found in Derla and Giles (1987).

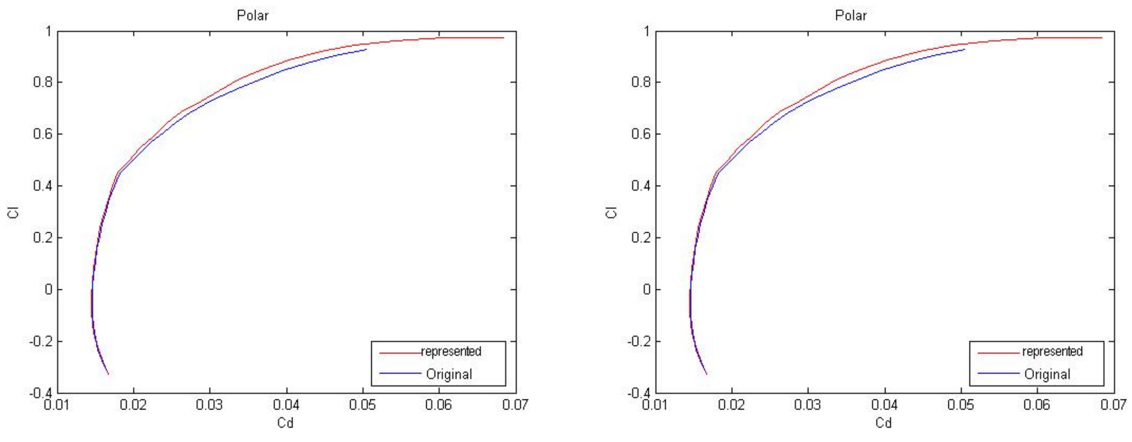


(a) (b) (c) (d)

Figure 4. NACA0012 Pressure distribution convergence for a)3BF, b)5BF, c)7BF e d)11BF ($\alpha = 5^o$, Re=200.000)

Calculations were made of the pressure distributions and aerodynamics coefficients for the original and the represented airfoil shape with 3BF to 11BF .Some results from the NACA0012 analysis are shown in figure 4. The predictions of the 5BF to 11BF analytic airfoils closely match the lower surface pressure distributions for the original airfoil. The predictions of the upper surface CPs of the represented airfoils have a slight difference from those of the original. These can be explained by a small oscillation in the 1^{st} derivative that can reproduce an oscillation in the pressure distribution prediction for the panel code.

The comparisons of the lift and drag coefficients predictions are shown in figure 5.The predictions of the coefficients of the original and the represented airfoil have a convergence interval between 0.2 to 0.6 lift coefficients.



(a) (b)

Figure 5. Polar convergence of NACA0012 for number of parameters a) 5BF; b) 7BF(Re = 200.000)

Table 1 resumes the NACA0012 study presenting the aerodynamics coefficients errors and the thickness and 1^{st} derivative maximum errors for 3BF to 11BF. The converge of the method can be seen from the 5BF where the errors seem to stabilized so the number of base functions capable of relativity represented the NACA 0012 is that.

Table 1. NACA0012 Aerodynamics and Geometrics convergence with Re $= 200.000$ e $\alpha = 5^o$.

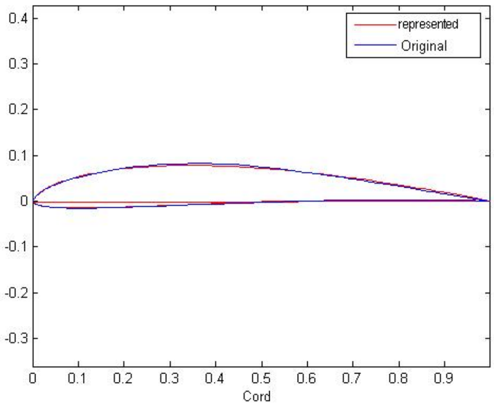
Numbers of base functions	cl error	cd error	Maximum error of thickness	Maximum error of derivate
3	0.0784 (16,5%)	0.00024 (1,3%)	0.004439	0.599446
5	0.0528 (11,6%)	0.00077 (4,2%)	0.002717	0.732319
7	0.0539 (11,8%)	0.00074 (4,1%)	0.00283	0.728065
9	0.0491 (10,8%)	0.00088 (4,8%)	0.002141	0.628515
11	0.0545 (11,9%)	0.00070 (3,9%)	0.00121	0.446805

The results of the pressure distributions and aerodynamics coefficients have been shown, along the previously discussed geometric comparisons, imply that a relative low number of base function can represent a symmetric type airfoil.

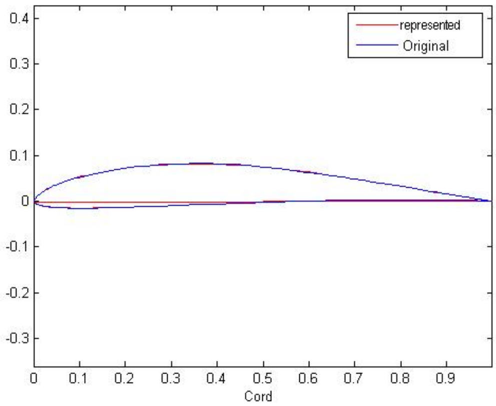
3.2 Eppler 387: low Reynolds airfoil study

The Eppler 387 airfoil is intended for model sailplanes having Reynolds numbers greater than 200.000. Its thickness is 9,1 percent chord. It has been tested in several wind tunnels making it a low Reynolds number calibration standard. Following the methodology of the previous case, the geometric and aerodynamic analysis are presented.

The results of the pressure distributions and aerodynamics coefficients have been shown, along the previously discussed geometric comparisons, imply that a relative low number of base function can represent a symmetric type airfoil.

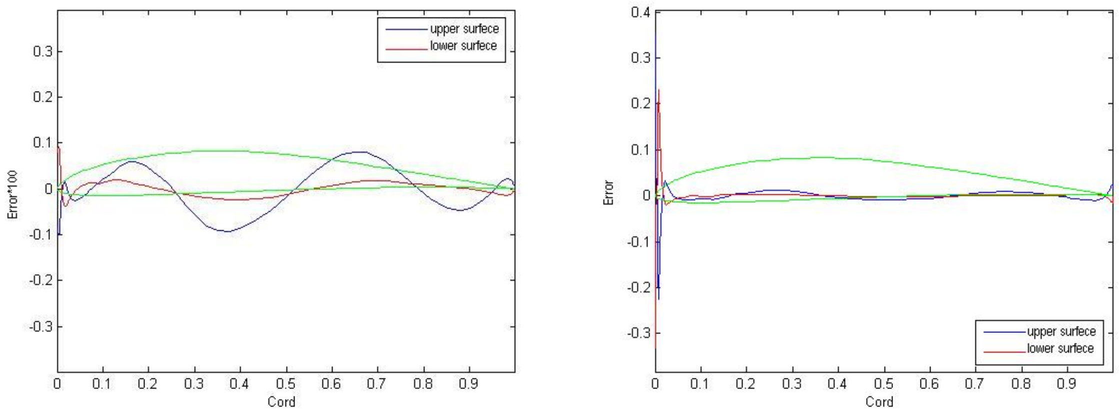


(a)



(b)

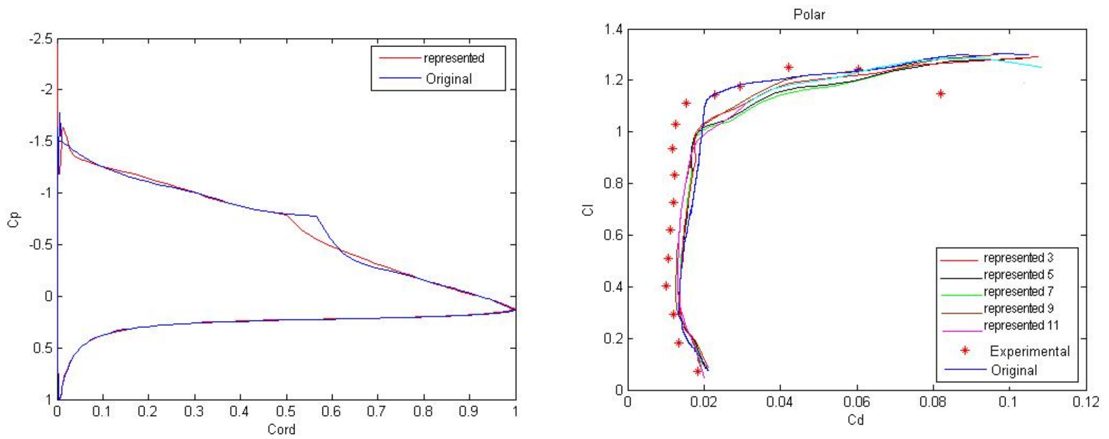
Figure 6. Geometric Comparisons for base functions a)3, b)5



(a) (b)

Figure 7. Geometric Errors a) 1^{st} derivative e b) Thickness percent cord with 7 BF

The figure 6 shows the geometric comparisons between the approximated and the official shape for the eppler 387.In example are barely visible the difference the actual and the approximated geometries for 5BF. Like the previously airfoil study the corresponding Geometric errors are indicated also in figure 7 by the 1^{st} derivative error and the thickness error or residual difference. For the eppler study the residuals from 7BF are less then the required to wind tunnel (0,1%). The behavior of the 1^{st} derivative as shown in the graphic is acceptable.



(a) (b)

Figure 8. a)Eppler 387 Pressure distribution convergence with 7 BF (Re $= 203.800$ e $\alpha = 5^o$) b) Polar convergence of Eppler 387 (Re= 203.800)

Some results of the 2^{nd} step proposed in the validation are shown in figure 8. The predictions of pressure distributions from 7BF are equivalent the lower surface for the original airfoil and the upper surface have a slight difference but is still acceptable.

In the 2^{nd} graphic are presented the polar convergence of the aerodynamics coefficients from 3BF to 11BF and comparer with the original and experimental data for the same number of Reynolds. The aerodynamics coefficients convergence is visible since the 3BF in a interval between 0,2 to 1,0 of lift coefficient. There is a slight difference between the original and experimental data that can be explained by the precision capability of the Xfoil code.

Tables 2 resumes the Eppler 387 study presenting the aerodynamics coefficients errors and the thickness and 1^{st} derivate maximum errors for 3BF to 11BF. The number of base functions capable of represent the Eppler 387 is 5BF.

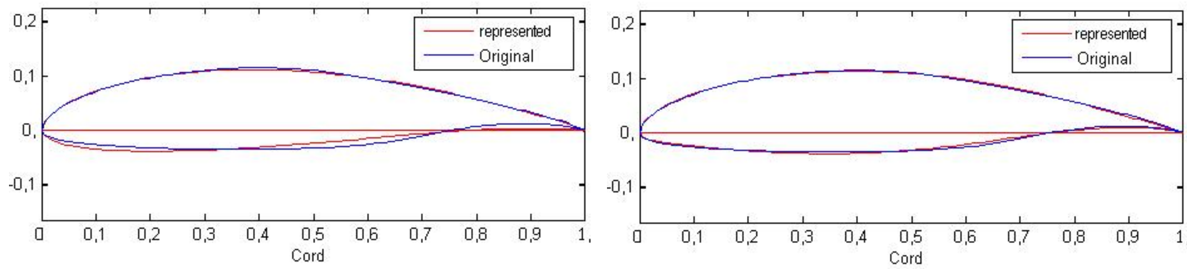
Table 2. Eppler 387 Aerodynamics and Geometrics convergence with Re $= 203.800$ e $\alpha = 5^o$.

Numbers of base functions	cl Error	cd Error	Maximum error of thickness	Maximum error of derivative
3	0.0125 (1,4%)	0.0010 (6,2%)	0.003772	0.328951
5	0.0082 (0,9%)	0.0010 (6,0%)	0.001784	0.201632
7	0.0079 (0,9%)	0.0012 (7,3%)	0.001104	0.354094
9	0.0057 (0,6%)	0.0012 (7,3%)	0.001028	0.344213
11	0.0012 (0,1%)	0.0008 (5,0%)	0.000773	0.262436

The results imply, as in the symmetric airfoil study, that low number of base function can represent a low Reynolds airfoil.

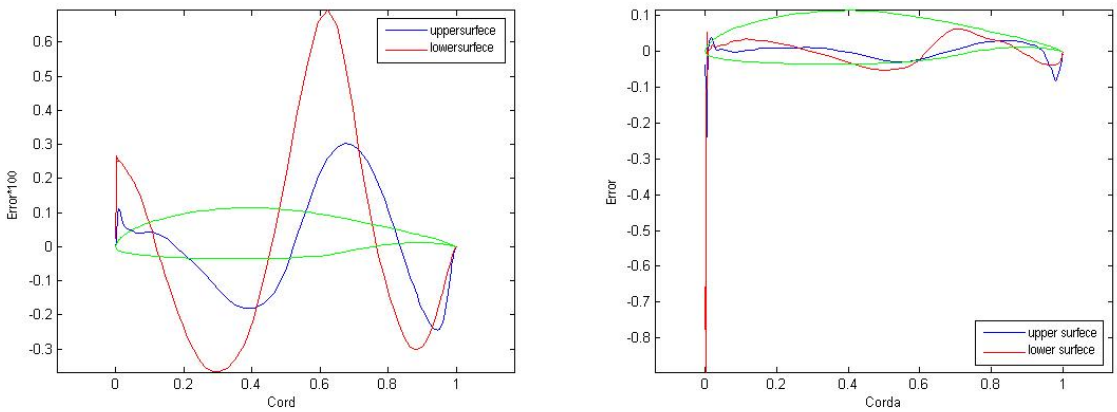
3.3 Nasa/ Lanagley NFL (1)-0215f: Natural laminar flow Airfoil Study

The NFL (1) 0215f airfoil was develop by Airfoils Incorporated founded by Dan M. Somers, which is a firm specialized in airfoil design, analysis and testing. It has been used in several aircrafts as Lancair 360 (Neico Aviation). The term natural-laminar-flow airfoil refers to an airfoil that can achieve significant extents of laminar flow(30% of cord) on both the upper and lower surfaces simultaneously, solely through favorable pressure gradients (no boundary suction or cooling).



(a) (b)

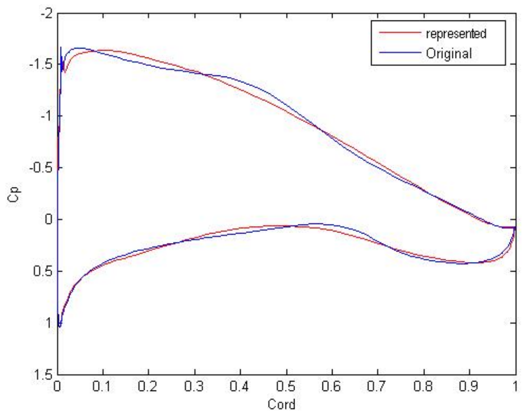
Figure 9. Geometric Comparisons for base functions a)3, b)7



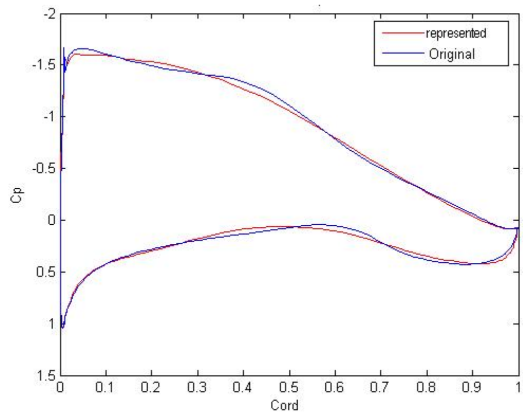
(a) (b)

Figure 10. Geometric Errors a) 1^{st} derivative e b) Thickness percent cord with 7 BF

Figure 9 shows that the geometrical comparison (the difference between the actual and the approximated geometries) are barely seen only from 7BF. The level of complexity of the original geometry especially in the lower surface leads to a later converge of the method compared to the previous study cases. This can also been seen in figure 10 where maximum thickness error are located in this area. The small fluctuations that can be observed in the 1st derivative errors can also be explained by the complexity of the geometry of the original airfoil shape. Nevertheless the numeric errors of the thickness and the 1^{st} derivative are acceptable for the approximation of the NFL (1) 0215f.



(a)



(b)

Figure 11. NFL (1) 0215f Pressure distribution convergence with a)9BF b)11BF (Re $= 3.10^6$, $\alpha = 0^o$ e M $= 0,4.$)

The pressure distribution convergence with 9BF and 11BF are presented in figure 11. The small differences of the actual and the represented pressure distributions predictions can be explained by the complexity of the airfoil shape and consequently the enlarged error of thickness and the 1^{st} derivative.

In sum, the NFL (1) 0215f study, as in the previous studies, is presented in table 3 with the the aerodynamics coefficients errors and the thickness and 1st derivative maximum errors for 3BF to 11BF. The number of base functions capable of represent the NFL (1) 0251f is 7BF.

Table 3. NFL (1)0215f aerodynamic and geometric convergence with Re $= 3.10^6$, $\alpha = 0^o$ e M $= 0,4.$

Numbers of base functions	cl Error	cd Error	Maximum error of thickness	Maximum error of derivative
3	0.0125 (1,4%)	0.0010 (6,2%)	0.003772	0.328951
5	0.0082 (0,9%)	0.0010 (6,0%)	0.001784	0.201632
7	0.0079 (0,9%)	0.0012 (7,3%)	0.001104	0.354094
9	0.0057 (0,6%)	0.0012 (7,3%)	0.001028	0.344213
11	0.0012 (0,1%)	0.0008 (5,0%)	0.000773	0.262436

The results imply as in the symmetric airfoil study and the low Reynolds airfoil study a low number of base functions can represent a Natural laminar flow type airfoil

4. CONCLUSION

As the first work using the presented method of parameterizations it can be concluded that the obtained results had been acceptable. Visually the method can represent the airfoil with a small number of parameters than others methods. And the results for the errors of 1st derivative and the other aerodynamic parameters also are adequate. For the three studied cases it was observed the necessity of a maximum 7 parameters to describe the airfoils which is less then 4 parameters for the commercial method used today for airfoil parameterizations (PARCEC 11 parameters).

With this work was possible to obtain some important conclusions. The method depends strongly on the initial set of airfoils used to generate the describing bases. This conclusion is not a surprise, since the existing information in the database is in certain form carried to the derived bases. Thus the importance is standard out here to have a set of airfoils that is large and representative of what have been done until today in airfoil optimization methods, to take this information of as to construct an airfoil in an efficient way for the parameterizations method.

Some problems had been found in the attempt to construct the described database above, being the problem most immediate the lack of standardization of the airfoils what made the pre-processing more difficult. Other problems of standardization was that both leading and trailing edge were out of unitary chord in the most of cases, beyond airfoils with very small number of points that generated bad interpolations.

The set of airfoil used currently, with 530 airfoils, already is a filtered version of another package, where airfoils with wrong formatting had been removed. However, it can be concluded that is possible to make a considerable improvement in the time process with a better airfoil package. Another solution would be the use of filters that remove the existing errors in some airfoils, preventing these errors propagate.

5. REFERENCES

Cosentino, G. B. and Holst, T. L.,1984," Numerical Optimisation Design of Advanced Transonic Wing Configurations"

Drela, M. and Giles, M.B.,1987," Viscous-Inviscid Analysis of Transonic and Low Reynolds Number Airfoils",AIAA Journal, 25(10), pp.1347-1355

Lepine, J., Guibault, F., Trepanier, J.-Y., and Pepin, F., 2001," Optimized Nonuniform Rational B- spline Geometrical Representation", for Aerodynamic Design of Wings, AIAA Journal, Vol. 39, No. 11.

Li, Wu and Padula Sharon, 2004 "Using Resolution Design Spaces for Aerodynamic Shape Optimization Under Ucertainty",NASA /TB-2004-213003.

Kulfan, Brenda M. and Bussoletti, John E.,2006, ""Fundamental" Parametric Geometry Representations for Aircraft Component Shapes", AIAA 2006-6948-972

Sobieczky, H., 1998," Parametric Airfoils and Wings, Notes on Numerical Fluid Mechanics", edited by K. Fujii and G. S. Dulikravich,Vol. 68, Vieweg Verlag, 1998, pp. 71-88.

Sobieczky,H. and Klein,M.2001," Sensitivity of Aerodynamic Optimization to Parameterized Target Functions", H182, Proc. Int. Symp. on Inverse Problems in Engineering Mechanics (ISIP 2001), 06-09 Feb 2001, Nagano, Japan

Sohn, Myong H. and lee, Kyu J., 2000,"Bezier Curve Application in the Shape Optimization of Transonic Afoils", AIAA-2000-4523

Song, Wenbin and Keane, Andrew J.,2004,"A Study of Shape Parameterisation Methods for Airfoil Optimisation", AIAA paper 04-20441

Wu, Hsiao-Yuan,Yang, Shuchi and Liu, Feng ,2003,"Comparison of Three Geometric Representations of Airfoils for Aerodynamic Optimization", AIAA-2003-4095

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