PRE-PROCESSING AND CHEBYSHEV CURVE-FITTING FOR FORM ERROR ASSESSMENT

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Abstract. *The concept of quality of products and services has evolved from attending the functional performance into exceeding customer expectations. In this context, the capability to meet increasingly tighter tolerances is paramount and is being well supported by enhanced manufacturing processes and new machine-tool technologies. This poses to the Metrology area the challenge of developing faster and more accurate measurement systems and evaluating uncertainties more precisely. The accuracy of measurement systems is affected to some extent by the algorithms used to evaluate form errors. Through a computational perspective, many general-purpose optimization algorithms are commercially available and data-processing capabilities have greatly improved. Therefore, this work presents a proposal of a pre-processor that enables the use of the commercially available optimization tools for form error assessment. A Linear Programming curve-fitting model was developed to provide the mathematical basis for the pre-processor routines. The model was applied to flatness and straightness error assessment. In order to validate it and evaluate its efficiency, a measurement system with integrated software was assembled and experimental tests were carried out. The proposed system was compared to the traditional Least Squares algorithm, and the results showed its higher accuracy in the evaluation of the error.*

Keywords*: Chebyshev curve-fitting, pre-processor, Linear Programming, optimization, flatness error.*

1. INTRODUCTION

The interactions between market, society and current technology being developed generate stimuli that mould the requisites of a competitive product or company. Since these stimuli are dynamic, the concept of competitiveness has also been changing during the decades. The economic and industrial scenario around 1960 was characterized by high volume production, a low level of diversity of products and services, long product life cycles, low speed of changes and a not strong influence of globalization over the markets, that is, more emphasis was given on regional and local markets instead of on global ones. The context in this current decade is almost the opposite.

The main factor of competitiveness of those days was cost. On the next decade, a strong focus was set on quality with Deming, Juran and the Japanese engineers, among others. Statistical techniques and problem-solving tools were developed to processes control to improve quality. Attending the specifications and improving process capabilities became paramount. Later on, a demand for flexibility in the sense of diversification of products and production mix produced with lower volumes. In this current decade, short time-to-market and the ability of rapidly changing the productive structure and process to launch innovations are being pointed out as the main factors of competitiveness.

Therefore, the customer choices nowadays are not exclusively driven by quality, but this factor surely still has a relevant weight, that is, meeting the project specifications is an expected requirement, a basic pre-requisite for competitiveness. Moreover, the level of what is acceptable regarding quality is becoming higher and increasingly tighter specifications are being required. Thus, special attention must be given to the measurement systems that can accurately assess non-conformities.

Reliable data is an important input for quality tools such as Statistical Process Control and Design of Experiments, as well as for current quality methodologies such as Design for Six Sigma, which is characterized as data-driven approaches for problem solving. Accurate measuring systems may be highlighted as an important basis to support these quality programs.

The demand for tighter tolerances is being well supported by enhanced manufacturing processes and new technologies for machine-tools. Some of the manufacturing technologies that may be referred include micromachining, dry machining, and CAD/CAM interfaces. Also, machines are becoming remarkably faster, more rigid, and, specially, more accurate. As a result, evident gains in speed and capability are achieved.

The context depicted shows that challenge in Metrology consists of addressing topics such as performing faster measurement processes, evaluating measurement uncertainties more precisely and developing more accurate measurement systems. The accuracy of the algorithms used to evaluate geometrical errors may be emphasized as an important component of the general accuracy of a measurement system. Therefore, the general objective of this work is to present an algorithm for form errors evaluation as an alternative to the traditional Least Squares Method (LSM), i.e., a model that yields a more accurate evaluation and equally easy to implement. This last characteristic should be strongly taken into account, being the main reason for which the Least Squares Method has been largely used. Some applications of this curve-fitting method can be found in Di Giacomo, Magalhães and Paziani (2003), Forbes (2006), and Gao and Kyono (1997), among others. Huang, Fan and Wu (1993), however, draw attention to that LSM provides an approximate solution that does not guarantee the minimum zone value and suggest an alternative minimum zone method for evaluating flatness errors.

A considerable number of algorithms have been developed for solving optimization problems. A brief list includes the traditional Simplex method, the interior point method, genetic algorithm, Newton-Raphson method and others. Many of these computational algorithms are commercially available in optimization packages, but they are usually designed for generic applications. The question that rises and deserves attention is how to model the curve-fitting problem and correctly set the data to benefit from the use of these algorithms and attain a more accurate error assessment. The specific objective of this work is to present a data pre-processor based on a Linear Programming model to evaluate form errors. The pre-processor is responsible for converting data into a suitable form so that it becomes an appropriate input for the optimization routines. In other words, it sets up matrices and vectors of coefficients to be processed by the solver, that is, the optimization tool.

As a case study, the proposed mathematical model was applied to flatness error assessment of a surface plate and to straightness error assessment of steel artifacts. In order to validate it, measurement systems were assembled and experimental tests were carried out. A software was developed for gathering data to be used as input to the pre-processor routines, which on their turn output a set of matrices that are processed by Matlab® optimization package tools. It is also a goal of this work to develop a system that can be easily programmed, integrated to commercially available tools and thus set for industrial applications.

This work presents an application of the concepts of Linear Programming that is to some extent diverse from what is traditionally developed in the area. Topics such as logistics, transportation problems, Master Production Planning or stock portfolio balancing are more usual applications of it rather than curve fitting. In this sense, this research has a multidisciplinary perspective.

2. LINEAR PROGRAMMING BASICS

A Mathematical Programming model is an analytical formulation to a problem where the objective is finding an extreme point of a function that satisfies a set of constraints. It is an optimization model that aims to minimize or maximize a function, named objective function, by systematically choosing values of real or integer variables from an allowed set. Linear Programming, LP, it is a specialization of Mathematical Programming where the objective function is a linear combination of variables and the set of feasible solutions is specified only by linear equalities or inequalities.

The standard form of a LP problem is given by Eq.(1).

$$\begin{aligned} &\text{Minimize or Maximize } \mathbf{c}^{\mathbf{T}}.\mathbf{x} \\ &\text{Subject to (s.t.)} \quad \mathbf{Ax} \le \mathbf{b} \ \text{(constraints)} \\ &\qquad\qquad\qquad\quad \mathbf{x} \ge \mathbf{0} \ \text{(sign restrictions)} \end{aligned} \tag{1}$$

where $\mathbf{c}^{\mathbf{T}}$ is a real vector of coefficients of the objective function, $\mathbf{x}$ is the vector of variables, $\mathbf{A}$ is a real matrix of coefficients of the constraints and $\mathbf{b}$ is a real vector.

One of the most traditional algorithms to solve LP problems is the Simplex method. It requires the problems to be converted to the augmented form before starting the iterations. This form introduces non-negative slack variables to replace non-equalities by equalities in the constraints, and can be obtained by defining a matrix $\mathbf{A_{eq}}$ and a vector of variables $\mathbf{x'}$ as in Eq.(2). The augmented form is presented in Eq. (3).

$\mathbf{A_{eq}} = [\mathbf{A} \ \ \mathbf{I}]$ and $\mathbf{x'} = [\mathbf{x} \ \ \mathbf{x_{sl}}]$, where $\mathbf{x_{sl}}$ are the newly introduced slack variables, $\mathbf{I}$ is the identity matrix. (2)

$$\begin{aligned} &\text{Minimize or Maximize } \mathbf{c}^{\mathbf{T}}.\mathbf{x} \\ &\text{s.t. } \mathbf{A}_{eq}\mathbf{x'} = \mathbf{b}_{eq} \\ &\quad \mathbf{x'} \ge \mathbf{lb}, \ \text{where } \mathbf{c}^{\mathbf{T}}, \mathbf{A}, \mathbf{x} \text{ and } \mathbf{b} \text{ are the ones defined in Eq. (1) and } \mathbf{lb} \text{ is the vector of lower boundary values for } \mathbf{x'} \end{aligned} \tag{3}$$

The equalities and inequalities that express the constraints define a subset of R^n space that is a closed convex polytope, which is called the feasible region. Basically, there are two situations where the problem does not have an optimum solution. One of them is if the inequalities contradict one another and do not define a feasible region, that is, the feasible region is empty. The other alternative is in the case of an unbounded problem in the direction of the gradient of the objective function, when maximization is being performed, or umbounded in the opposite direction of the gradient when the function is being minimized. Excluding these particular situations, the convexity of the feasible region and the linearity of the objective function imply that an optimal solution can only occur at a boundary point of the feasible region, unless the objective function is constant. This solution is not necessarily unique since the optimum points may lie on an edge or face of the polytope. It can be stated, however, that if an optimum solution exists, there is at least one optimum solution located in one vertex of the polytope. Moreover, it can also be stated that the set of feasible solutions is infinite, but the set of the ones located in the vertices is finite. These statements are proved by fundamental theorems in LP theory, and most of the algorithms are based on them.

According to Simplex method, an initial feasible solution is taken in the vertex of the polytope. If the objective is a maximization, at each iteration an adjacent vertex is chosen such that the value of the objective function does not decrease. As this vertex is not usually unique, a pivot rule is established to determine which one must be chosen. For the case of maximization, the optimization ends when no vertex can be found such that the function value increases.

In contrast to algorithms that remain on the boundary searching for the optimum point, there are other ones called interior point methods that try to improve the value of the function by moving across the interior of the feasible region until the vertex of the optimum solution is reached.

As it is valid for other modeling efforts, the effective application of Linear Programming requires a pertinent interpretation of the obtained analytical solutions and a good understanding of the underlying modeling assumptions. Rather than being concerned with the numerical algorithm, this work set the focus on the understanding of the modeling process.

3. THE LINEAR PROGRAMMING MODEL FOR FORM ERROR ASSESSMENT

The evaluation of form errors consists of assessing the deviations of a measured profile from the ideal geometric form that the actual profile is expected to have. Therefore, in order to assess the straightness, flatness or roundness errors of a part, a reference straight line, plane or circle must be firstly fitted to the set of data. In other words, the evaluation depends on the determination of the parameters of the best-fit geometric entities. The method of extreme points has its importance in practical situations since it has a physical sense. For instance, one practical way of assessing the straightness error of a part would be laying its two extremities on leveling devices on a reference surface plate, aligning these extreme points and then reading the total range of deviation with a dial indicator.

The accuracy required for some applications and the evolution of computers justifies the use of mathematical curve-fitting methods. The basic concept of discrete curve fitting theory is simultaneously minimizing the set of deviations of each data point in relation to each corresponding value yielded by the function chosen for approximation. The deviation can be stated as in Eq. (4):

$$d_i(a, x_i) = f(x_i) - \sum_j \left[a_j g_j(x_i)\right] \tag{4}$$

where d_i represents the deviation of the i-th point, **a** is a p-dimensional real vector **a** = (a_j), $f(xi)$ is the value of the real-valued function f at point x_i and $g_j(x)$ is a set of real-valued functions ($0 \le j \le p$) used for the approximation.

For the intended application, the combination of coefficients a_j and functions $g_j(x)$ should represent the equations of geometric entities such as planes, straight lines or circles. Taking Eq. (4) as a basis, the LSM principle consists of minimizing the sum of the squares of the deviations. The Chebyshev Norm, $L\infty$, seeks to minimize the maximum deviation of the set of deviations. In this case, as the determination of the maximum deviation depends on the parameters of best fit form and both of these things are unknowns, an iterative process is required to solve this problem. The curve-fitting method, also known as Minimax, can be modeled as a Linear Programming problem and iterative optimization tools can be applied. Equation (5) below shows the objective function of the problem.

$$\text{Minimize } (\max|d_i|; i = 1,2,...m*n) \tag{5}$$

The relationship between the deviations and the parameters of the best-fit geometric form, or the limitations for the values that variables d_i can assume are stated in Eq. (4), which constitutes the first set of constraints of the problem. Depending on the relative position of the experimental data points and the points of the curve at a given coordinate, the sign of the deviations might be positive or negative. It is a requirement of Simplex and other optimization algorithms to work in non-negative algebraic spaces. In order to fulfill this condition, the deviations must be written as a difference of two positive real numbers u_i and v_i which enables rewriting Eq. (4) into Eq. (6).

$$\sum_j \left[a_j g_j(x_i)\right] + u_i - v_i = f(x_i) \tag{6}$$

The variable change shown in Eq. (7) must be applied to eliminate the modulus operator in the objective function statement. This procedure leads to another set of constraints that directly result from the definition of the new variable h as the maximum of deviations. These constraints are presented in Eq. (8).

$$\max|d_i| = h; \qquad i = 1,2,....,m \tag{7}$$

$$h \geq |d_i| = |u_i - v_i|, \quad i = 1,2,...,m \tag{8}$$

The modulus operator in Eq. (8) may be eliminated with the use of a derived form from the triangle inequality for normed vector spaces, shown in Eq. (9). After some careful mathematical analysis, it is possible to show that Eq. (8) may be replaced by Eq. (10).

$$|u_i - v_i| \leq |u_i| + |v_i| \tag{9}$$

$$h \geq u_i + v_i \tag{10}$$

The sign of the parameters a_j is unrestricted. To turn them into positive variables is enough to use the same kind of variable change previously described for the deviations d_i, which results in Eq. (11).

$a_j = (a_j^+ - a_j^-), j = 0,1,...p$, where a_j^+ and a_j^- are variables that may assume just positive values. (11)

The proposed LP problem for general form error assessment is a combination of Eq.(5), (6), (10) and (11), that are summarized in Eq. (12).

Minimize h

Subject to:

$$\begin{cases} -h + u_i + v_i \leq 0 \\ \sum_j \left[(a_j^+ - a_j^-)g_j(x_i)\right] + u_i - v_i = f(x_i), \quad \text{with } h, u_i, v_i, a_j^+, a_j^- \geq 0 \end{cases} \tag{12}$$

As a case study, the proposed model was applied to assess the straightness error of two different artifacts and the flatness error of a surface plate. It was considered relevant to present the specific equations of the model applied to flatness error assessment since the equations for straightness error evaluation can be easily derived from the first ones. For the selected case, the combination of functions g_j should represent an Euclidean plane. These functions are dependent on two variables and $p = 2$, that is, j varies from 0 to 2. The function $f(x_i)$ that appears in Eq. (6), in this case, also depends on two variables and represents the measured data. All these functions are defined in Eq. (13).

$$g_0(x,y) = 1\,;\; g_1(x,y) = x\,;\; g_2(x,y) = y\,; f(x_s, y_t) = z_i \tag{13}$$

where z_i is the measured height at a point of coordinates (x_s,y_t) of the surface. The variation of the indices s, t and i are shown in Eq. (14).

$$s = 0,1,2,...,m-1\,;\; t = 0,1,2,...,n-1\,;\; i = 0,1,2,...,m*n \tag{14}$$

After these considerations, the final form of the LP model for flatness error assessment is presented in Eq. (15).

Minimize h

Subject to:

$$\begin{cases} -h + u_i + v_i \leq 0 \\ a_0^+ - a_0^- + a_1^+ x_s - a_1^- x_s + a_2^+ y_t - a_2^- y_t + (u_i - v_i) = z_i \\ u_i, v_i, a_j^+, a_j^- \geq 0 \end{cases} \tag{15}$$

The optimization of this model provides the parameters a_j of the best plane and the maximum deviation h. This variable represents the symmetrical distance from the best-fit plane to the farthest measured point above it and below it. Therefore, the flatness error is, by definition, twice the distance h.

The inputs for the optimization solvers are matrices of coefficients and vectors of independent terms that correspond to the presented mathematical model. A preprocessor was developed to set these matrices. The information about the dimensions of the matrices and about the variation range of the indices s, t and i must be taken from the experimental data set. For flatness error assessment, data is collected on an orthogonal grid. Hence, the indices s and t vary according to Eq. (14), where m is the number of horizontal lines that form the grid and n is the number of vertical ones. Moreover, the total number of intersections of the grid is given by the product $m*n$, which is equivalent to the total number of coordinate pairs (x_s,y_t) on the grid. For each one of these pairs there is an associated height z_i. This explains the variation range of index i.

A vector V containing all variables used in the optimization must be established. The order of variables might be arbitrarily defined provided that the matrices of the coefficients are set respecting the chosen order. One of the possibilities for the definition of V is presented in Eq.(16).

$$\mathbf{V^T} = \begin{bmatrix} h & a_o^+ & a_o^- & a_1^+ & a_1^- & a_2^+ & a_2^- & u_1 & v_1 & u_2 & v_2 & \dots & u_{m*n} & v_{m*n} \end{bmatrix} \qquad (16)$$

The values of the coordinates x_s and y_t that appear in the matrices of coefficients correspond to the coordinates of a Cartesian plane multiplied by the measurement step, named b here as it coincides to the base of the measuring instrument. Matrix $\mathbf{A_{eq}}$ of constraints in the form of equalities stated in Eq. (15) is presented as an example. The remaining matrices and vectors used in the optimization are defined in a similar way.

$$\mathbf{Aeq} = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & -1 & 1.b & -1.b & 0 & 0 & 0 & 0 & 1 & -1 & \dots & 0 & 0 & 0 \\ \dots & \dots & & & & & & & & & & \dots & \dots & \dots & \dots \\ 0 & 1 & -1 & s.b & -s.b & t.b & -t.b & 0 & 0 & \dots & & \dots & 0 & 0 & 0 \\ \dots & \dots & & & & & & & & & & \dots & 0 & 0 & 0 \\ 0 & 1 & -1 & (m-1)b & -(m-1)b & (n-1)b & -(n-1).b & 0 & 0 & 0 & \dots & \dots & 0 & 1 & -1 \end{bmatrix} \qquad (17)$$

4. MEASURING SYSTEMS FOR THE CASE STUDY

In order to validate the algorithms, prospective tests were firstly performed, using data and results already published in other works. After that, a series of experimental tests were carried out using two distinct measuring systems. The first one was assembled for straightness error assessment and basically included a six-degree-of-freedom industrial robot, three LVDT type probes, a probing device, a PC and interface components such as an analogue to digital conversion board. Two steel artifacts were measured using the Sequential Three Points Method. Error separation techniques were applied to the collected raw data and two data sets of interest for this work were provided: straightness profile data of the measured artifacts and vertical straightness profile data of the robot translational movement. The algorithm to perform the error separation was developed by Di Giacomo *et al*. (2005). It does not constitute the scope of this work to specifically describe this measurement system and the error separation model. The works just cited, as well as the work of Paziani (2005), focus on these topics in details and bring valuable discussions such as the strategies used to overcome the issue of the zero adjustment errors of the sensors.

A second measurement system was assembled exclusively to enable the application of the Linear Programming model for flatness error assessment. It basically consisted of an electronic Level interfaced to a PC, a surface plate and data acquisition routines. The electronic Level was employed to perform the measurements due to its advantage of allowing a much faster measuring process in comparison to the laser interferometer. The instrument was connected to a PC by means of a 12-bit resolution Analog to Digital (A/D) data acquisition board to convert into digital the analogue signals to the computer. Before the conversion, these signals were amplified so that the output voltage of the instrument matches the input range of the board, thus making a better use of the available range of A/D numbers.

Special attention was given to the interface between user and measurement system, so a software with graphical user interface (GUI) was developed. Mainly, it requires the user to inform the dimensions of the measured surface and the length of the base of the Level. The software also provides a map and a sequence of measurement. Additionally, the user may provide an input value to the routine that represents the difference between consecutive readings of the Level and is used to verify its stability before data is collected. In other words, the software will not record a data point until a certain level of stability of the instrument is reached. If this input value is not provided by the user, a default value is used.

An area of 500 mm x 300 mm of a granite plate was measured. First, an orthogonal grid was mapped on the surface and one corner was taken to be the reference of the system, where the zero of the instrument was set. One direction of the grid parallel to the horizontal or vertical lines must be chosen as the main direction. Each line in this direction may be referred to as a generator. The angles between consecutive points along each generator were measured at steps corresponding to the base of the Level. The instrument provides relative measurements, i.e., it outputs the difference of height between two points in terms of angle. Thus, the height of the t-th point measured along each generator corresponds to a sum of all the previous values of height from the origin of the generator to the considered position. This is presented in Eq.(18).

$$z'_{st} = \sum_0^t b.\sin\alpha_{st} \text{ , for } t \neq 0, \text{ and } z'_{st} = 0 \text{, for } t = 0 \qquad (18)$$

where s and t vary according to Eq.(14), z'_{st} is the height of the t-th point measured along the s-th generator, b is the length of the base of the Level and α_{st} is the measured angle between the points ($s,t-1$) and (s,t). The points where t is zero are denoted here as the origins of each generator.

In fact, the procedure described so far provides straightness profile data of each generator. In order to obtain flatness profile data, the relative height between generators must be determined. In other words, each cumulated height along the line that is orthogonal to the generators and pass through their origins has to be added to the straightness profile of each respective generator, according to Eq.(19).

$$z_{st} = \sum_{0}^{s} b.\sin\beta_s + z'_{st}\,, \qquad (19)$$

where $z_{00} = 0$, β_s is the angle measured between the points s-1 and s along the straight line passing through the origin of the generators ($t=0$), and $z_{st} = z'_{st}$ for $s = 0$.

As a result of this procedure, all measured data refer to the two initial points where the Level was set to zero, and a matrix of flatness profile data is obtained. This matrix will be firstly read by the preprocessor, providing information for the optimization matrices set-up, and will eventually be an input for the optimization solver. Equation (18) and Eq. (19) will be relevant for calculating the uncertainty associated to the flatness error.

5. PRE-PROCESSING ROUTINES

Pre-processors are well-known tools used to improve the data flow in a data processing system, to prepare data for the application of a certain algorithm and, in some cases, to set starting conditions that reduce the number of iterations required by an algorithm. The work of Lee and Kim (1996) illustrates the use of pre-processors in Engineering Optimization. Benzley et al (1995) also provide an example where pre-processing techniques were used to increase the functionality of finite element method.

A preprocessor was developed to generate matrices and vectors that correspond to the developed mathematical model. It constitutes a key element of the measuring system as it integrates the collected raw data to the solvers of LP problems by turning this data into suitable inputs for these tools. The main procedures it executes are summarized in the block diagram of Fig. 1.

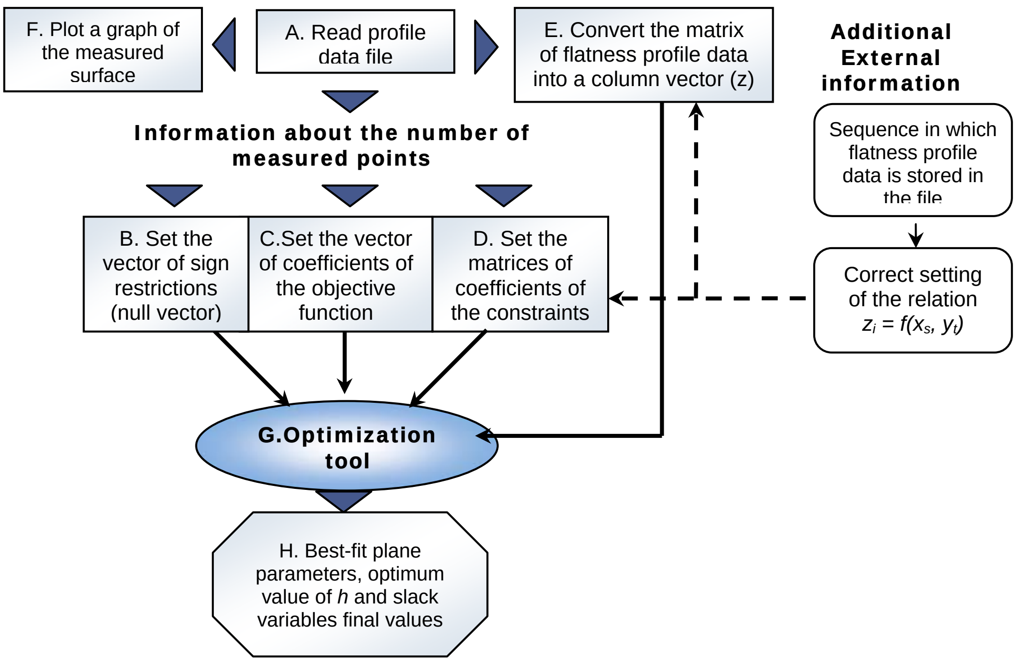

Figure 1. Block diagram of the preprocessor routines

The output of the developed acquisition software described in topic 4 is a data file with straightness or flatness profile data. The pre-processor routines start by reading this file, as shown in block A of Fig. 1 shows. The dimensions of the matrices and vectors used in the optimization derive from the dimensions of this matrix of raw data. The vector $\mathbf{c}^T$ of coefficients of the objective-function is an array with1 in the first position, which corresponds to the position of variable *h*, and zero in all the remaining positions. In block D, some subroutines set the matrices of coefficients of the constraints following the formats shown in Eq. (15). Also, according to the model, the flatness data matrix must be converted into a vector. This conversion is performed in block E. Once all the inputs are set, the pre-processor starts the optimization tool that comes up with the error value. The pre-processing subroutines were developed in Matlab's environment since the subroutine used to solve the LP problem, *linprog*, belongs to Matlab's optimization package. This tool requires, as arguments, the coefficients vector for the objective-function; the matrix **A** of coefficients of the constraints in the form $\mathbf{A.x} \leq \mathbf{b}$ as presented in Eq.(1); the real vector **b**; the matrix $\mathbf{A_{eq}}$ of coefficients of the constraints in the equality form $\mathbf{A_{eq}.x} = \mathbf{b_{eq}}$ shown in Eq.(3); the real vector $\mathbf{b_{eq}}$; and the real vector **lb** of sign restrictions for the variables, i.e., $\mathbf{x} \geq \mathbf{lb}$, also presented in Eq.(3).

6. RESULTS AND DISCUSSIONS

The curve-fitting algorithms were applied to sets of straightness data profile of each artifact. The straightness profile of the robot movement can be obtained from each measurement of the artifacts, after error separation is performed. In other words, for each data set of an artifact there is an associated data set of the profile of the movement of the robot. Table 1 to 4 present the values of the errors obtained using the proposed Chebyshev method and the LSM.

Table 1. Straightness errors values for the artifact no. 1

	Straightness errors (micrometers)		Differences	
Test	Proposed Method	LSM	Micrometers	Percentage
1.1	37.3	43.8	6.5	17.3%
1.2	31.8	36.9	5.0	15.8%
1.3	36.3	42.0	5.7	15.7%
Average	35.2	40.9	5.7	16.3%

Table 2. Straightness error values of the robot motion measured with artifact no. 1

	Straightness errors (micrometers)		Differences	
Test	Proposed Method	LSM	Micrometers	Percentage
1.1	104.4	113.4	9.0	8.61%
1.2	99.4	109.4	10.0	10.01%
1.3	105.8	116.8	11.0	10.40%
Average	103.2	113.2	10.0	9.67%

Table 3. Straightness errors for the artifact no. 2

	Straightness error value (micrometers)		Differences	
Test	Proposed Method	LSM	Micrometers	Percentage
2.1	112.3	120.8	8.5	7.6%
2.2	105.8	114.2	8.4	7.9%
2.3	110.1	118.4	8.3	7.5%
2.4	108.9	116.8	7.9	7.3%
Average	109.3	117.5	8.2	7.6%

Table 4. Straightness error values of the robot motion measured with artifact no. 2

	Straightness error value (micrometers)		Differences	
Test	Proposed Method	LSM	Micrometers	Percentage
2.1	90.4	93.7	3.3	3.6%
2.2	87.7	88.6	0.9	1.0%
2.3	92.1	93.3	1.2	1.3%
2.4	91.1	93.6	2.6	2.8%
Average	90.3	92.3	2.0	2.2%

The observed differences for the error of the robot movement, presented in Tab. 2 and Tab. 4 are due to the fact that, as the artifacts are different, the robot moves along different action lines while measuring each artifact.

The straightness profile of artifact number one is shown in Fig. 2. The measurement step for the Sequential Three Points Method, by definition, coincides with the distance between sensors which is, in this case, 18mm. Figure 2 also shows the straight line adjusted by the proposed method and the straight line yielded by the Least Squares Method. Similarly, Fig. 3 show the straightness profile of the robot motion associated to the measurement of artifact no. 2 in the test 2.3. and the best-fit straight lines.

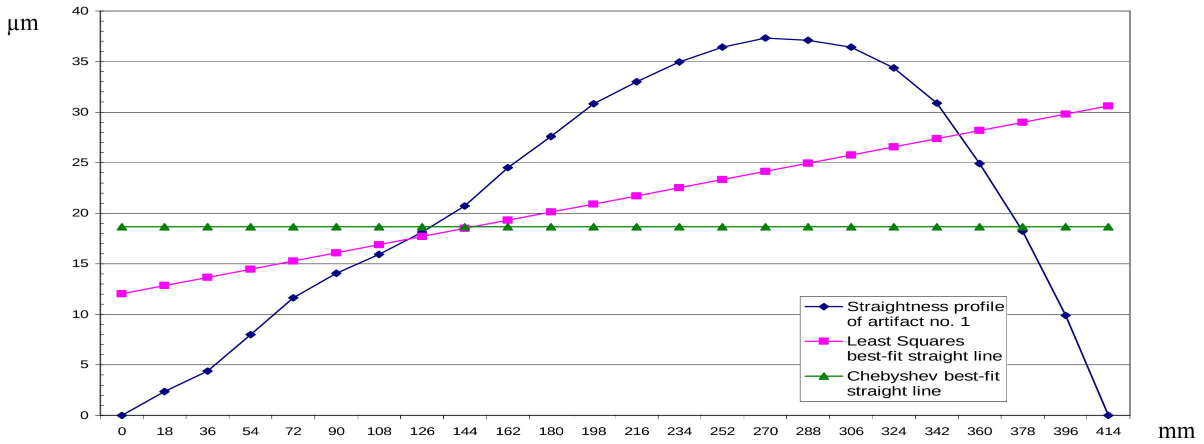

Figure 2. Straightness profile of the artifact no. 1 measured in test 1.1 and best-fit straight lines adjusted by the proposed method and by LSM

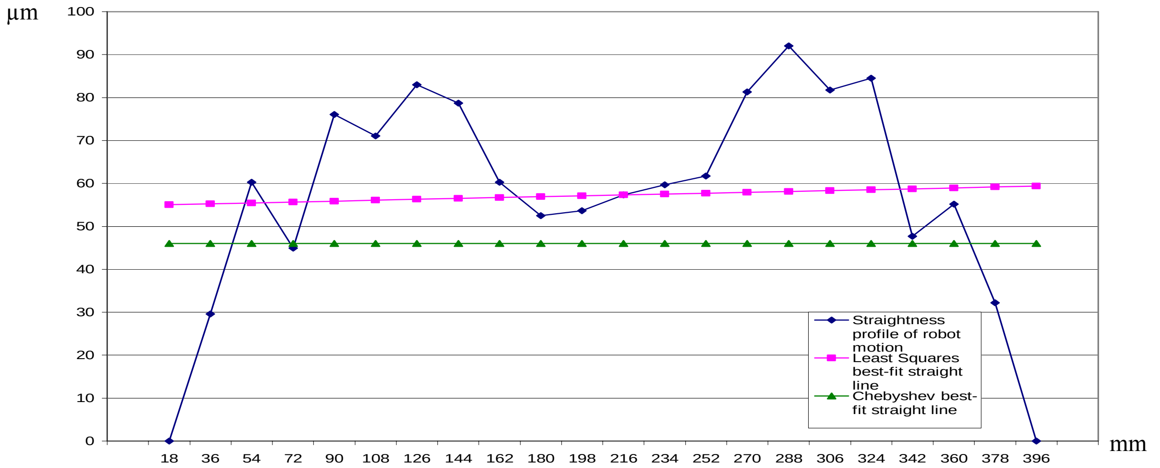

Figure 3. Straightness profile of the robot motion measured in test 2.3 and best-fit straight lines adjusted by the proposed method and by LSM

The results for the flatness error assessment of a 500x300 mm^2 granite surface are shown in Tab. 5, while Fig. 4 presents a graph of the measured surface.

Table 5. Error values of a granite flat surface

	Flatness error value (µm)		**Differences**		**Uncertainty values at 95% (µm)**		
Test	Proposed Method	LSM	Micrometers	Percentage	Proposed Method(u_s)	LSM (u_{LS})	u_{LS}/u_s
1.1	10.1	10.3	0.2	1.98%	1.28	2.16	1.69
1.2	10.1	10.3	0.2	1.98%	1.28	2.16	1.69
1.3	10.2	10.4	0.2	1.96%	1.26	2.16	1.71
Average	10.1	10.3	0.2	1.97%	1.28	2.16	1.69

In this case, there are two possible choices of directions for the generators. Two preliminary tests were carried out, one for each direction, to check out which evaluation would yield the higher value of error. This direction was taken for the subsequent tests because otherwise, the value of the error would be underestimated. However, once the set of data is defined, special attention should be given to the curve fitting models that come up with smaller values of error.

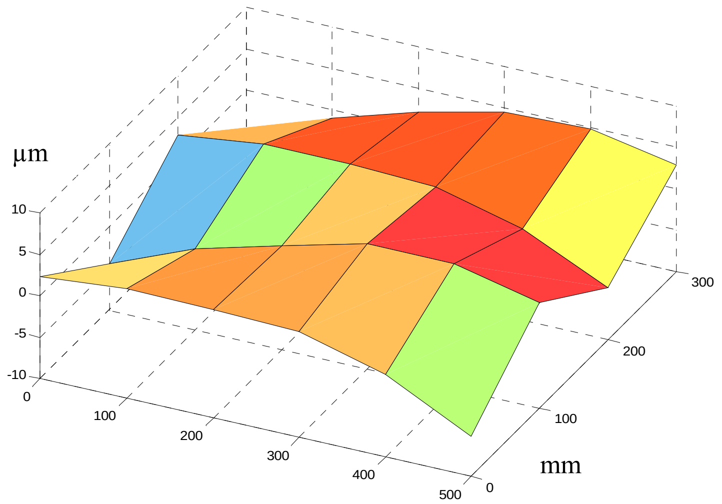

Figure 4. Graph of the measured surface

It can be observed that, for the same set of data, the errors values that were obtained using the proposed algorithm are always smaller than the ones evaluated by LSM. In some cases, this difference is of up to 17%. Moreover, for straightness error evaluation, the differences in percentage between the evaluations varied within a significant range. The first consideration to be taken about this fact is that the differences are higher as long as the angular differences between the best-fit straight lines are more significant. In other words, the higher the difference between the angular coefficients of the straight lines, the bigger is the difference between the values of error that are obtained using each of the two methods. A significant angular difference between the straight lines can be seen in Fig. 2, for which the difference of error values is 17.3%, whilst in Fig. 3, the best-fit lines are almost parallel and the difference is 1.3%. Second, in the Least Squares Method, all the points are used by the algorithm in the computation of the error, i.e, each one of the points contribute to adjust the best-fit curve. On the contrary, when Chebyshev Norm is applied, just the points that present maximum deviations are considered in the curve fitting, that is, the best-fit curve is defined just by these points. This explains why the LS straight line in Fig. 2 has a significant positive inclination, reflecting the predominant positive inclination of the profile, whereas the inclination of the Chebyshev straight line is almost null. It can be seen that the increasing portion of the profile corresponds to two thirds of the data points and this increasing trend influences the Least Squares algorithm. It could be asked whether the horizontal straight line yielded by Chebyshev algorithm, in this case, is representative of the profile. The answer is affirmative as far as this best-fit line leaded to a smaller value of error.

A parenthesis that may be stated here about the proposed method is that, for straightness error assessment, the maximum value of deviation h where verified at three points, which defined the best-fit line, whereas for flatness error assessment there were found four points in the referred conditions. These results are in accordance to the theorem attributed to de la Valle Poussin, presented by Kelley (1957) and Goldstein, Levine and Hereshoff (1957), what contributes to validate the developed mathematical model.

The fact that Chebyshev curve-fitting method always presented smaller values of error for a given set of data corroborates its higher accuracy in the assessment. Additionally, the capacity of today's computers justifies and stimulates the use of more complex algorithms, in terms of computational processing, than the traditional ones.

Although the proposed algorithm is iterative and should, therefore, require more time to be solved, its computational processing time for each test was of the order of 10^0 seconds, which is comparable to the efficiency of the Least Squares Method. Efficiency is also a strong feature of the measuring system assembled for straightness error assessment since it is dedicated and designed for high-volume inspection. On its turn, the electronic level presents advantages over other instruments such as the laser interferometer due to the fact it requires much smaller setup times. Regarding costs, electronic levels are cheaper than interferometers and certainly cheaper than Coordinate Measuring Machines.

Besides time and economic factors, some other aspects to be analyzed in the evaluation of the success of a project or application include human factors. Considering these latter ones, the ability of the user could affect, to some extent, the accuracy in positioning the instrument. Some error-proof tools were included in the programs, such as a measurement map and a check for the instrument stability, with the purpose of reducing this influence. In addition, the analysis of the uncertainty of the flatness error showed that positioning errors of up to 1 mm in the directions x and y contributed at least 1000 times less to the combined uncertainty than the uncertainty associated to the determination of the heights in the coordinate z.

Finally, although the differences in Table 5 are of less than 2%, the smaller value of uncertainty provided by the proposed method must be highlighted as another important advantage of it. The explanation for this result comes from the fact that in the Chebyshev-Simplex method, the parameters of the best-fit plane and the maximum deviation h are determined in the same level, and all these values are yielded by the model simultaneously. Thus, the calculation of the value of the error, i.e, the calculation of the value of h, just depends on the experimental data. In the Least Squares method, on the contrary, first the parameters of the best-fit plane are determined based on experimental data and then the value of the error is computed based on these parameters. In this case, an extra level of uncertainty is introduced since part of the uncertainty in the determination of the error comes from the uncertainty in the determination of the best-fit plane parameters. Therefore, the value of the uncertainty of the measurement, in this case, is dependent on the algorithm used for the evaluation of the error.

7. CONCLUSIONS

For non-ordinary applications, where tighter tolerances and manufacturing processes with high capabilities are required, it is essential to provide measuring systems with higher accuracy. These cases justify the use of more complex algorithms such as the Least Squares Method or the Minimax instead of conventional GD&T procedures to assess form errors. Attention should be taken when collecting discrete data, to guarantee that a given set is representative of the profile of the part and that the error is not being underestimated. However, for a given set of data, a model that provides smaller values of errors is certainly more accurate and should be preferred. The results proved that the difference between the evaluations provided by the developed method and the LS Method may be significant. Moreover, an analysis showed the advantage of the proposed model regarding the associated measurement uncertainties.

A considerable set of general-purpose optimization tools is commercially available and computational capabilities do not constitute a limitation at all. The proposed pre-processing techniques have a major role in enabling the use of these tools for form error assessment, which, otherwise, would not be possible. The developed model of preprocessor may be used with other optimization solvers, as long as it is adapted to set the correct format of inputs required by these solvers.

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9. REFERENCES

Benzley, S.E., Merkley, K., Blacker, T.D. and Schoof, L., 1995, "Pre- and post-processing for the finite element method", Finite Elements in Analysis and Design, Vol. 19, pp. 243-260.

Di Giacomo, B., Magalhães, R.C.A. and Paziani, F.T., 2003, "Error separation methods applied to form measurement", Proceedings of COBEM 2003, São Paulo, Brazil, paper number 1229 (CD-ROM).

Di Giacomo, B., Tsunaki, R. and Paziani, F. T., 2005, "Robot-based dedicated measuring system with data redundancy for profile inspection", IEEE/RSJ IROS Int. Conf., Edmonton, Alberta, Canada, Aug. 2-6, 2005, pp. 1646-1649.

Forbes, A.B. 2006, "Uncertainty evaluation associated with fitting geometric surfaces to coordinate data", Metrologia, Vol. 43, pp. S282-S290.

Gao, W. and Kiyono, S., 1997, "On-Machine Profile Measurement of Machined Surface Using the Combined Three-Point Mehtod", JSME International Journal, Series C, Vol. 40, No. 2, pp. 253-259.

Goldstein, A.A., Levine, N. and Hereshoff, J.B., 1957, "On the 'best' and 'least Qth' approximation of an overdetermined system of linear equations", Journal of ACM, Vol. 4, Issue 3 (July 1957), pp. 341-347.

Huang, S.T., Fan, K.C. and Wu, J.H., 1993, "A new minimum zone method for evaluating flatness errors", Precision Engineering, Vol. 15, No. 1 (Jan. 1993), pp 25-32.

Jiang, Q., Ouyang, D., Feng, H. and Desta, M., 2004, "Roundness evaluation based on profile confidence level", Transactions of NAMRI/SME, Vol. 32., pp. 127-134.

Kelley Jr., J.E., 1958, "An Application of Linear Programming to Curve Fitting", Journal of the Society for Industrial and Applied Mathematics, Vol. 6, No. 1, pp. 15-22.

Lee, D. and Kim, S., 1996, "A Knowledge-Based Expert System as a Pre-Post Processor in Engineering Optimization", Expert Systems With Applications, Vol. 11, No. 1, pp. 79-87.

Paziani, F.T. "Development of na automated and dedicated measuring system" (in Portuguese), Ph.D. Thesis, Escola de Engenharia de São Carlos: Universidade de São Paulo, São Carlos – SP, Brazil, 2005.

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