ANALYTICAL AND NUMERICAL PROCEDURES TO PROCESS VISCOELASTIC EXPERIMENTAL TIME DATA

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Abstract. *This paper considers several procedures to process experimental data for polymeric materials in time domain. The raw experimental data are, usually, of total relaxation or total creep tests. However, numerical computation requires the knowledge of deviatoric and volumetric contributions. This paper utilize the differential constitutive equation to derive several relations between the material parameters, aiming to processing experimental data obtained in total creep tests, and extract material parameters of total relaxation modulus, deviatoric and volumetric modulus, constitutive equation and rheological mechanical parameters .*

Keywords**:** *Viscoelasticity, parameter identification, Prony series.*

1. INTRODUCTION

Although not all polymers behaves viscoelasticaly, and not are linearly viscoelastic, this theory provides a usable engineering approximation for many applications in polymer and composite structural parts. Common plastics used in today's industry are required to withstand working loads for long periods of time, in some cases periods of ten to fourty years. It is important to be capable to perform time experimentations of common rigid polymers like PVC, in different conditions of temperature, humidity, age and other aggressive agents. Also, these data have to be processed to render the material information into material models, which, in turn, are to be used in finite element codes to perform the structural analysis of components.

Many different types of functions have been used to describe creep effects in plastic, in addition to Prony series, for instance, laws based on power laws and based on a kernel of the form $\mathrm{Exp}(t/\tau)^m$. This last form was proposed still in 1847 by Kohlrausch and is extensively used by Tomlins (1994, 1996) and by Struik (1987a,b, 1989), among others, to describe the creep of both, semicrystaline and amorphous polymers (which include rigid PVC, PP, HPDE, PP, among the most used in industry) over limited timescales. The power laws are observed to model adequately experimental data in the intermediate range of the time span, but it is unable to fit correctly the beginning of the curves. However, none of these families of functions posses consistent physical or mathematical grounds, as does the Prony series. Another type of functions also able to model experimental data over arbitrarily large time periods are series based on Mittag-Lefler functions, which are solution of constitutive equations based on differential equations of fractional order.

The linear viscoelastic material behavior can be represented by different formulations, of which we consider the following:

I. Differential equations usually associated with a rheological spring-dashpot model, like the generalized Kelvin-Voigt and generalized Maxwell (also denominated Wiecherst model). Therefore, the material can be characterized by the knowledge of one of the sets of constants:
   a) The set of parameters in the differential equation;
   b) The set of spring and damping "mechanical" parameters in the generalized rheological model;
II. A second type of representation for the material properties are through a function which represent the material response to one of the simple standard tests, of which the most usual are:
   a) Creep tests, where a specimen is subject to axial tensile loading (or bending, compression or torsion ones), uniform in time, and deformation is recorded.
   b) Relaxation tests, where the specimen is subjected to tensile deformation, constant in time, and the stress evolution is recorded;
   c) Dynamic tests, where a uniform oscillatory signal of stress or strains are applied to the specimen.

Additionally, one observes that a number of special tests have been designed (see Lemetre and Chaboche, 2000, for instance). The constants associated with any of the five groups above listed completely characterize the linear viscoelastic behavior of the material, within the limitations of the model itself. Each one of the three standard tests generates its particular set of material constants. Usually, only one type of test is performed to characterize the material. The need for multiple tests appears, for example, when the dynamic test is conducted to extend the time spectrum of creep or relaxation tests to low values, in the range where time tests are not practical. However, for rigid plastics, numerical structural analysis in time domain rarely requires data in times inferior of 10 seconds, such that a single type of test, creep or relaxation, is sufficient. The type of test can be chosen simply by the availability of equipment and personnel in a given laboratory. When one considers the utilization of the material properties in structural analysis of a part, one notices that each algorithm, each numerical procedure and each commercial finite element code, requires a proper set of material constants. In this situation, it is important to have available efficient procedures to recast one set of material constants into another among the standard tests.

A second type of constant conversion is between one of the sets in the group II above and one of the sets of group I. Although the parameters of the Prony series in sets II(a) and II(b) have to be non-negative, the procedures of least square errors used to process experimental data do not always produce coherent values. A conversion to the mechanical parameters in set I(b) provides an additional test for soundness of the parameter determination. Also, the knowledge of constants of any of the two sets I(a) and I(b) enables direct computation of the results in any of the standard tests in group II.

In what concerns numerical structural analysis, it is important to observe that a single standard test is not enough to characterize a material, even a linear isotropic one, in the way it is usually considered in applications where the only concern is quality control or comparison between different material compositions. For the isotropic material, two independent time functions are needed, being the most usual the following:

(a) Elastic modulus and Poisson coefficient;
(b) Volumetric and deviatoric modules;
(c) In frequency domain, the storage and loss modules.

Few works are available on interchanging between different sets material properties. Flugge, (1975), Findley at alli (1976) and Christensen (1982), among others, present relations between relaxation (or creep) modulus and complex modulus. The formulations are left open for a generic function $J(t)$ or $G(t)$, etc., or are, at best, developed only to short chains of Kelvin-Voigt or Maxwell models. However, the character of the exponential basis in the Prony series makes the development of the formulation for arbitrarily long chains by no means a simple task. In this paper a brief treatment is given for relationships between some of the parameter sets listed above, with emphasis in the computation of deviatoric relaxation constants from experimental total creep test data.

The organization of the paper is the following: first, the constitutive equation for a viscoelastic linear solid is briefly derived following the classic development, according to a generalized Maxwell model, using Laplace transform. The constitutive equation in differential form and its Laplace counterpart are determined, and a relationship between the two sets of material parameters, mechanical and constitutive sets, is established. Next, the creep modulus is obtained in form of Prony series, and its parameters are related with the constitutive constants. Likewise, the Prony series of the relaxation modulus is obtained and its constants are related with mechanical parameters of the material. The Prony series for the relaxation deviatoric modulus is derived for a material with elastic bulk modulus, in terms of the creep parameters. A set of experimental creep data is processed to test all different relationships developed, performing a close loop, from creep data to creep Prony series to relaxation Prony series, to mechanical parameters, to constitutive parameters and back to creep Prony series.

2. RELATIONSHIP BETWEEN CONSTITUTIVE AND STANDARD TESTS MATERIAL CONSTANTS

Let one consider the well known generalized Maxwell model of order N, consisting of a single spring with constant k_o in parallel with N Maxwell spring-dashpots units (see standard texts as Flugge, 1975, Findley at alli, 1976 and Christensen, 1982). The constitutive relation for the model is obtained considering that, for an arbitrary unit,

$$\varepsilon(t) = \frac{\sigma}{\mu_j} + \frac{d\sigma/dt}{k_j}, \quad \text{for} \quad j = 1, \cdots, N \qquad \text{and} \qquad \varepsilon(t) = \frac{\sigma}{k_o}. \tag{1}$$

Performing Laplace transformation and adding the transformed stresses in each unit one obtains

$$L[\sigma] = \left\{ k_o + s \sum_{j=1}^{N} \frac{1}{\left(\dfrac{1}{\mu_j} + \dfrac{s}{k_j} \right)} \right\} L[\varepsilon], \tag{2}$$

Where $L[\bullet]$ denotes Laplace transform of $[\bullet]$ and s is the coordinate in the Laplace domain. Expanding the denominators and collecting terms one obtains the standard form of the constitutive relation in Laplace domain

$$P(s)L[\sigma] = Q(s)L[\varepsilon], \quad \text{with} \tag{3}$$

$$P(s) = \sum_{j=0}^{N} s^j p_j, \quad Q(s) = \sum_{j=0}^{N} s^j q_j, \qquad \text{with } p_o = 1. \tag{4}$$

The inverse Laplace transform of Eq. (3) gives the constitutive equation in differential form as

$$\sum_{j=0}^{N} p_j \frac{d^j\sigma}{dt^j} = \sum_{j=0}^{N} q_j \frac{d^j\varepsilon}{dt^j}. \tag{4a}$$

The set of $2N - 1$ constitutive parameters, p's and q's, relates to the $2N - 1$ mechanical parameters by

$$p_j = \frac{\bar{p}_j}{\bar{p}_o}, \quad q_j = \frac{q_o\bar{p}_j + \bar{q}_j}{\bar{p}_o}, \quad \text{for} \quad j = 1, \cdots, N, \quad \text{with} \quad p_j > 0, \ q_j > 0, \tag{5}$$

where

$$\bar{p}_o = \prod_{a=1}^{N} \frac{1}{\mu_a}, \qquad \bar{p}_1 = \sum_{b=1}^{N} \frac{1}{k_b} \prod_{a=1, a\neq b}^{N} \frac{1}{\mu_a}, \qquad \bar{p}_2 = \sum_{c=2}^{N} \frac{1}{k_c} \sum_{b=1}^{c-1} \frac{1}{k_b} \prod_{\substack{a=1, a\neq b \\ a\neq c}}^{N} \frac{1}{\mu_a}, \qquad \bar{p}_N = \prod_{a=1}^{N} \frac{1}{k_a}. \tag{6}$$

Other p_j can be obtained exchanging k and m in the expression of p_{N-j}. Parameters q_j are the following:

$$\bar{q}_1 = \sum_{b=1}^{N} \prod_{a=1, a\neq b}^{N} \frac{1}{\mu_a}, \qquad \bar{q}_2 = \sum_{c=1}^{N} \frac{1}{k_c} \sum_{b=c+1}^{N} \prod_{a=1, a\neq b,c}^{N} \frac{1}{\mu_a}, \qquad \bar{q}_3 = \sum_{d=1}^{N-1} \frac{1}{k_d} \sum_{c=d+1}^{N} \frac{1}{k_c} \sum_{\substack{b=1 \\ a\neq b,d}}^{N} \prod_{\substack{a=1 \\ a\neq b,c,d}}^{N} \frac{1}{\mu_a}, \tag{7}$$

and $q_o = k_o$. Other $\bar{q}_j$ are obtained exchanging k and m in the expression of $\bar{q}_{N-j+1}$.

The **solution for the creep test** with constant stress σ_o is obtained substituting $L[\sigma] = \sigma_o / s$ in Eq. (3), which generates the creep modulus $J(t)$, which Laplace transform is $L[J] = P(s)/sQ(s)$. The N roots δ_j of the polynomial $Q(s)/q_N$ allows its representation as $Q(s)/q_N = \prod_{j=1}^{N}(s - \delta_j)$. This operation renders the inverse transformation of $L[J]$ to be easily performed, resulting in the Prony series

$$J(t) = B_\infty - \sum_{j=1}^{N} B_j e^{-t/\rho_j}, \quad \text{with} \quad B_\infty > 0, \quad B_j > 0, \quad \rho_j > 0, \tag{8}$$

where $\rho_j = -1/\delta_j$ and B_j form the set of creep parameters. Its relationship with the set of constitutive constants, p's and q's, is the following

$$B_\infty = \frac{(-1)^N}{q_N} \prod_{a=1}^{N} \frac{1}{\delta_a}, \qquad B_j = -\frac{1}{d_j} \sum_{a=0}^{N} \delta_j^a p_j, \qquad \text{where} \qquad d_j = q_N \delta_j \prod_{\substack{a=1 \\ a\neq j}}^{N} \left(\delta_j - \delta_a\right). \tag{9}$$

An additional parameter is the initial value of $J(t)$, $J(0) = B_o = B_\infty - \sum B_j$. In case the creep parameters are approximately known from experimental data fitting, the constitutive parameter can, in principle, be estimated, as follows. The form $Q(s)/q_N = \prod_{j=1}^{N}\left(s - \delta_j\right)$ can be expanded and have the coefficients of powers j of s, i.e., s^j, collected, resulting

$$\frac{Q(s)}{q_N} = \sum_{j=0}^{N} s^j f_j, \qquad \text{where, for } N \leq 5, \tag{10}$$

$$f_1 = -\prod_{a=1}^{N} \delta_a, \qquad f_2 = \sum_{b=1}^{N} \frac{1}{\delta_b} \prod_{a=1}^{N} \delta_a, \qquad f_3 = -\sum_{c=1}^{N-1} \frac{1}{\delta_c} \sum_{b=c+1}^{N} \frac{1}{\delta_b} \prod_{a=1}^{N} \delta_a,$$

$$f_4 = \sum_{d=1}^{N-1} \frac{1}{\delta_d} \sum_{c=d+1}^{N} \frac{1}{\delta_c} \sum_{b=c+1}^{N} \frac{1}{\delta_b} \prod_{a=1}^{N} \delta_a. \tag{11}$$

The pattern is followed until $f_N = (-1)^N \sum_{a=1}^{N} \delta_a$. Next, one equates the coefficients of both polynomials in Eqs (4) and (10), resulting

$$q_N = \frac{1}{B_\infty f_1}, \qquad q_o = q_N f_1 = \frac{1}{B_\infty}, \qquad q_j = q_N f_j, \qquad \text{for } j = 1, \cdots, N. \tag{12}$$

This is an explicit algebraic system of equations for the constitutive constants *q*'s, which can be directly solved once δ's and *f*'s are known from experimental data and Eqs. (11). Determination of the remaining constants, *p*'s, is performed recognizing Eq. (9) as an algebraic system in terms of *p*'s, which can be recast in matrix form as

$$\mathbf{D\,p = g}, \quad \text{where} \quad D_{ij} = \delta_i^j, \quad g_i = B_i d_i - 1 \qquad \text{(no summation in } j\text{)} \tag{13}$$

The system can, in principle, be solved, but the matrix **D** is extremely ill conditioned. In practice, a more efficient method should be seek to generate the values of *p*'s , as the procedure introduced in Eqs. (16)-(17).

The **solution for the relaxation test,** performed with constant deformation ε_o, is obtained substituting $L[\varepsilon] = \varepsilon_o / s$ in Eq. (3), which generates the relaxation modulus $L[Y] = Q(s)/sP(s)$. The *N* roots ζ_j of the polynomial $P(s)/p_N$ enables the representation $P(s)/p_N = \prod_{j=1}^{N} (s - \zeta_j)$. Therefore, the inverse Laplace transform of $L[Y]$ is obtained in the form of a Prony series

$$Y(t) = C_\infty + \sum_{j=1}^{N} C_j e^{-t/\lambda_j}, \tag{14}$$

where the characteristic times $\lambda_j = -1/\zeta_j$ and the strengths C_j form the set of relaxation parameters. Its relationship with the set of constitutive parameters, *p*'s and *q*'s, it the following:

$$C_\infty = \frac{q_o}{p_N} \prod_{a=1}^{N} \frac{1}{\zeta_a}; \qquad C_j = \frac{\sum_{a=0}^{N} q_a \zeta_j^a}{p_N \zeta_j \prod_{\substack{b=1 \\ b \neq j}}^{N} (\zeta_j - \zeta_b)}. \tag{15}$$

This procedure is analogous to that of creep modulus, Eqs. (8)-(9), but the relationship of the relaxation constants λ's and C's, with mechanical constants *k*'s and μ's, is extremely simple. In fact, one can prove that it reduces to the following:

$$C_\infty = k_o, \quad C_j = k_j \quad \text{and} \quad \lambda_j = \frac{\mu_j}{k_j} = -\frac{1}{\zeta_j}, \quad \text{for} \quad j = 1, \cdots, N, \tag{16}$$

Also, it is frequently useful defining the initial relaxation modulus $C_o \doteq Y(0) = C_\infty + \sum_{p=1}^{N} C_p$. Therefore, the mechanical constants are easily obtained from relaxation test results. Since the creep constants are not as amicable to generate other constant sets, instead of using Eqs. (12)-(13) to obtaining *p*'s and *q*'s, it is easier to convert directly creep into relaxation constants using the known relationship in Laplace domain

$$L[Y] = \frac{1}{s^2 L[J]}. \tag{17}$$

Once the relaxation set is known, Eqs. (16) produce the mechanical set and, in turn, Eqs. (6)-(7) produce the constitutive set.

3. DETERMINATION OF PRONY SERIES FOR DEVIATORIC RELAXATION MODULUS

Let one consider the bulk and deviatoric modules pair for characterization of an isotropic viscoelastic material under three-axial state of stress and strains. These modules can be obtained in time domain from one of the standard tests of creep or relaxation. For many metal and polymeric materials, the bulk modulus can be considered to be insensitive to viscous deformation, such that it can be considered purely elastic, such that

$$K(t) = K_{el} = \frac{E_{el}}{3(1-2\nu_{el})}, \tag{18}$$

where E_{el} and ν_{el} are the elastic modules and Poisson coefficient, respectively, obtained from a separate test. The deviatoric relaxation counterpart, *G(t)*, has to be derived indirectly from one of the standard tests. One alternative consists in the torsion test, which measures directly the deviatoric modulus, but it is more complex to perform than those of creep or relaxation. Therefore, in what follows, an efficient procedure is introduced to extract the Prony parameters of *G(t)* from those of *J(t)*, obtained from creep tests.

The goal consists in determining the deviatoric relaxation parameters g_∞, g_j and λ_j of the corresponding Prony series

$$G(t) = g_\infty + \sum_{j=1}^{N} g_j e^{-t/\lambda_j}, \tag{19}$$

given the constants of the Prony series of the total creep deformation *J*(*t*), i.e., B_o, B_j and ρ_j as in Eq. (8). In case $K(t) = K_{el}$, the relationship between *G(t)* and *J(t)* assumes a simple form (see Flugge, 1975, Chistensen, 1982) in Laplace transform domain:

$$\bar{G}(s) = \frac{3K_{el}}{9K_{el}s^2\bar{J} - s}, \tag{20}$$

Where, in order to compact notation, it is used $\left[\bar{\bullet}\right] \doteq L\left[\bullet\right]$ to represents the Laplace transform. The transform $\bar{J}(s)$ is obtained from Eq.(8) as

$$\bar{J}(s) = \frac{B_o}{s} + \sum_{j=1}^{N} \frac{B_j}{s\left(1+s\rho_j\right)},$$

which can be substituted in Eq. (20) to result

$$3\bar{G} = \frac{1}{sf(s)} \prod_{j=1}^{N} \hat{\rho}_j, \quad \text{with} \quad \hat{\rho}_j \doteq \left(1+s\rho_j\right), \quad \text{and}$$
$$f(s) = \bar{B}_o \prod_{b=1}^{N} \hat{\rho}_b + \sum_{a=1}^{N} \frac{B_a}{\hat{\rho}_a}, \quad \text{and} \quad \bar{B}_o = B_o - \frac{1}{9K_{el}}. \tag{21}$$

An analytic form for the inverse Laplace transform of $\bar{G}(s)$ in Eq. (21) seems to be impossible for large values of N. However, we introduce here a different approach to obtain the deviatoric relaxation function from Eq. (21). First, let one consider that the *N* roots a_j of the polynomial *f(s)* in Eq. (21) can be computed, such that the polynomial can be represented as

$$f(s) = C\sum_{j=1}^{N} \left(s - a_j\right), \quad \text{with} \quad C = \frac{(-1)^N\ f(0)}{\prod_{b=1}^{N} a_b} \quad \text{and} \quad f(0) = B_\infty - \frac{1}{9K_{el}}. \tag{22}$$

Therefore, $\bar{G}(s)$ in Eq. (21) takes the form

$$3\bar{G} = \frac{\prod_{j=1}^{N} \hat{\rho}_j}{sC\prod_{i=1}^{N}\left(s-a_i\right)}. \tag{23}$$

This form can be easily Laplace transformed back, resulting in Eq. (19), with the deviatoric relaxation parameters given by

$$\lambda_j = \frac{1}{a_j}, \quad g_\infty = \frac{1}{3f(0)}, \quad \text{and} \quad g_j = \frac{\prod_{b=1}^{N}\left(1+a_j\rho_b\right)}{3Ca_j\prod_{\substack{c=1\\c\neq j}}^{N}\left(a_j-a_c\right)}. \tag{24}$$

These expressions completely extract the deviatoric relaxation set of parameters from standard creep test data.

The roots of *f(s)* necessary in Eq. (22) are obtained by first identifying explicitly its polynomial coefficients, such that

$$f(s) = \sum_{g=0}^{N} f_g s^g, \quad \text{where} \quad f_o = \bar{B}_o + \sum_{m=0}^{N} B_m,$$

$$f_g = \sum_{j_1=1}^{N-g+1}\sum_{j_2=j_1+1}^{N-g+2}\cdots\sum_{j_g=j_{g-1}+1}^{N}\rho_{j_1}\rho_{j_2}\cdots\rho_{j_g}\left(\bar{B}_o+\sum_{\substack{m=1\\m\neq j_1,j_2,\cdots,j_g}}^{N} B_m\right), \quad \text{until} \quad f_N = \bar{B}_o\prod_{m=1}^{N} B_m. \tag{25}$$

3. NUMERICAL RESULTS

The formulation developed in the preceding sections was tested by processing a set of experimental data points obtained from a PVC blend tested in tension creep in the laboratory of Grante/EMC/UFSC (Pagliosa, 2004, Medeiros, 2006). First, the raw data is filtered to reduce local fluctuation and smooth the curve, using the simple moving average filter (linear, with all weights equal, summing the unity, see Smith, 1997). Next, a subset of points is selected, in this case, 36 points, to represent the material. The bulk modulus is computed from short duration tests as $K_{el} = 5.361\cdot 10^{12}$ Pa. Usually, it suffice choosing N equal to the number of decades spanned by the experimental data. Therefore, it is selected a constitutive model of order N = 5. The fitting of the data are performed using the procedures analyzed by Gerlach, 2005, coded by the authors in Fortran and Mathematica™.

The characteristic creep times chosen are shown in the second column of Table 1. A curve fit procedure generates the creep curve strengths shown in the first column of the table. The Prony series thus obtained and some experimental points are shown in Figure 1a.

Table 1. Strengths and characteristic times for creep modulus $J(t)$ of PVC.

Strengths [1/Pa]	Times [s]
$B_\infty = 7.760\cdot 10^{-10}$	
$B_o = 3.897\cdot 10^{-10}$	
$B_1 = 1.650\cdot 10^{-11}$	$\rho_1 = 5\cdot 10^2$
$B_2 = 2.870\cdot 10^{-11}$	$\rho_2 = 5\cdot 10^3$
$B_3 = 9.074\cdot 10^{-11}$	$\rho_3 = 5\cdot 10^4$
$B_4 = 1.641\cdot 10^{-10}$	$\rho_4 = 5\cdot 10^5$
$B_5 = 8.627\cdot 10^{-11}$	$\rho_5 = 2\cdot 10^6$

Next, the deviatoric relaxation parameters are estimated using the Prony series of *J(t)*. The function *f(s)* in Eq. (21) is identified and its roots are determined as

$$a_1 = -2.086\cdot 10^{-3},\qquad a_3 = -2.430\cdot 10^{-5},\qquad a_5 = -5.571\cdot 10^{-7},$$
$$a_2 = -2.143\cdot 10^{-4},\qquad a_4 = -2.632\cdot 10^{-6}. \tag{28}$$

Deviatoric parameters are directly computed using Eqs. (24), resulting the values in Table 1. The corresponding curve obtained from Eq. (19) is shown in figure 1b.

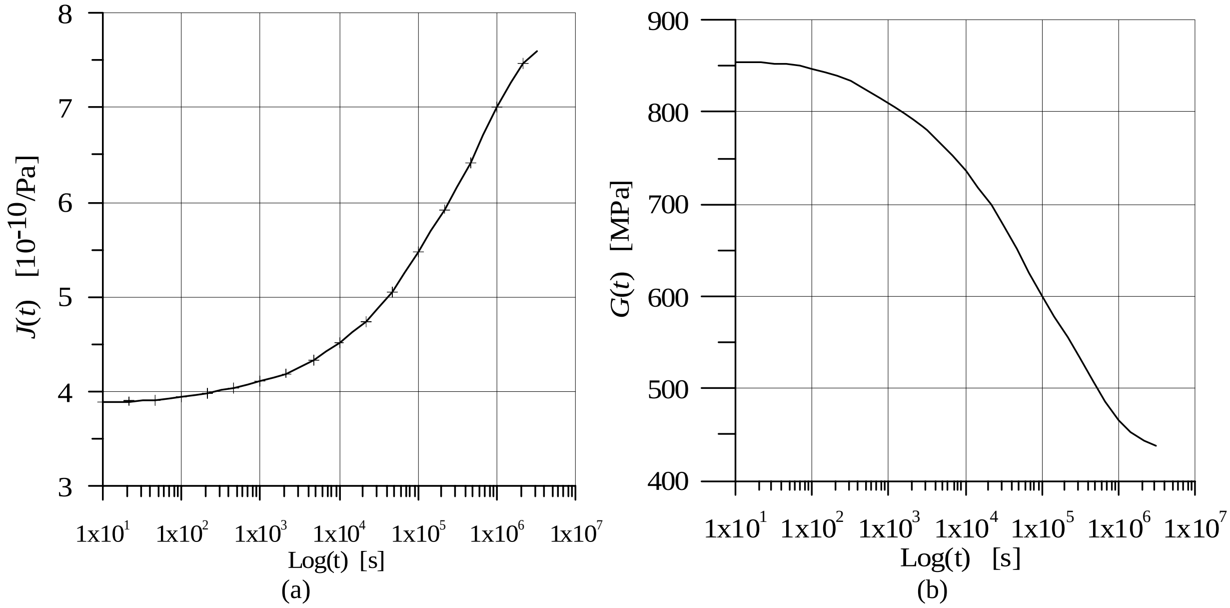

Figure 1. Diagrams for total creep modulus in (a) and for deviatoric relaxation modulus in (b).

Table 2. Strengths and characteristic times for deviatoric relaxation modulus $G(t)$ for PVC.

Strengths [Pa]	Times [s]
$g_\infty = 4.296\cdot 10^8$	
$g_1 = 3.547\cdot 10^7$	$\lambda_1 = 4.795\cdot 10^2$
$g_2 = 5.645\cdot 10^7$	$\lambda_2 = 4.663\cdot 10^3$
$g_3 = 1.382\cdot 10^8$	$\lambda_3 = 4.116\cdot 10^4$
$g_4 = 1.511\cdot 10^8$	$\lambda_4 = 3.800\cdot 10^5$
$g_5 = 4.464\cdot 10^7$	$\lambda_5 = 1.795\cdot 10^6$

3.1. Identification of total relaxation modulus $Y(t)$ and mechanical parameters

The Prony series of the total relaxation modulus $Y(t)$ was estimated in two steps:

(a) The creep experimental data points were used to perform a numerical inversion of the Laplace transform of the relaxation modulus, using the function given in Eq. (17). The procedure to approximate inversion is that of Schapery (Christensen, 2000, Schapery, 1962), which consists in computing

$$Y(t_k) = s\bar{Y}(s)\Big|_{s=0.56/t_k}, \qquad \text{for each data point } k. \tag{29}$$

(b) The table thus obtained is used in a least squares procedure (Gerlach and Matzenmiller, 2005) to obtain the Prony series of the total relaxation modulus. The parameters of the series, according to Eq.(14), are given in the first two columns of Table 3, and its plot is shown in Figure 2.

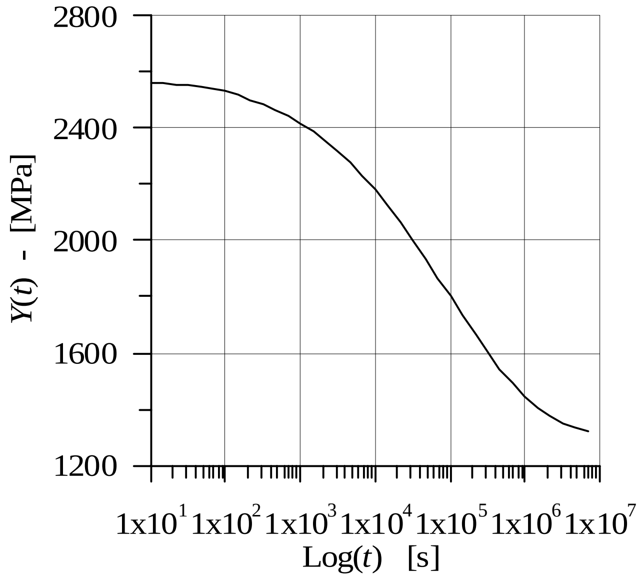

Figure 2. Diagram for total relaxation modulus of PVC, extracted from creep test.

Table 3. Strengths and characteristic times for total relaxation modulus $Y(t)$ of PVC and mechanical parameters.

Relaxation parameters		Mechanical parameters	
Strengths [Pa]	Times [s]	k [Pa]	μ [Pa·s]
$Y_\infty = 1.328 \cdot 10^9$			
$Y_o = 2.562 \cdot 10^9$		$k_o = 1.328 \cdot 10^9$	
$Y_1 = 1.000 \cdot 10^8$	$\lambda_1 = 3.5 \cdot 10^2$	$k_1 = 1.000 \cdot 10^8$	$\mu_1 = 3.500 \cdot 10^{10}$
$Y_2 = 1.944 \cdot 10^8$	$\lambda_2 = 4.0 \cdot 10^3$	$k_2 = 1.944 \cdot 10^8$	$\mu_2 = 7.780 \cdot 10^{12}$
$Y_3 = 4.247 \cdot 10^8$	$\lambda_3 = 4.0 \cdot 10^4$	$k_3 = 4.247 \cdot 10^8$	$\mu_3 = 1.699 \cdot 10^{13}$
$Y_4 = 3.639 \cdot 10^8$	$\lambda_4 = 4.0 \cdot 10^5$	$k_4 = 3.639 \cdot 10^8$	$\mu_4 = 1.456 \cdot 10^{14}$
$Y_5 = 1.509 \cdot 10^8$	$\lambda_5 = 1.6 \cdot 10^6$	$k_5 = 1.509 \cdot 10^8$	$\mu_5 = 2.414 \cdot 10^{14}$

The total relaxation parameters enables a quick estimation for the mechancial parameters, using Eqs. (16). The values obtained are shown in columns three and four of Table 3.

Table 4. Constitutive parameters of PVC.

p's	q's	p's	q's
$p_o = 1$	$q_o = 1.328 \cdot 10^9\,\text{Pa}$	$k_o = 1.328 \cdot 10^9$	
$p_1 = 2.080 \cdot 10^6\,\text{s}$	$q_1 = 3.175 \cdot 10^{15}\,\text{Pa} \cdot \text{s}$	$k_1 = 1.000 \cdot 10^8$	$\mu_1 = 3.500 \cdot 10^{10}$
$p_2 = 8.023 \cdot 10^{11}\,\text{s}^2$	$q_2 = 1.477 \cdot 10^{21}\,\text{Pa} \cdot \text{s}^2$	$k_2 = 1.944 \cdot 10^8$	$\mu_2 = 7.780 \cdot 10^{12}$
$p_3 = 5.468 \cdot 10^{16}\,\text{s}^3$	$q_3 = 1.176 \cdot 10^{26}\,\text{Pa} \cdot \text{s}^3$	$k_3 = 4.247 \cdot 10^8$	$\mu_3 = 1.699 \cdot 10^{13}$
$p_4 = 1.043 \cdot 10^{21}\,\text{s}^4$	$q_4 = 2.564 \cdot 10^{30}\,\text{Pa} \cdot \text{s}^4$	$k_4 = 3.639 \cdot 10^8$	$\mu_4 = 1.456 \cdot 10^{14}$
$p_5 = 3.584 \cdot 10^{23}\,\text{s}^5$	$q_5 = 9.183 \cdot 10^{32}\,\text{Pa} \cdot \text{s}^5$	$k_5 = 1.509 \cdot 10^8$	$\mu_5 = 2.414 \cdot 10^{14}$

3.2. Constitutive parameters

Once the mechanical parameters are estimated, one proceed to compute the constitutive ones, which is done using Eqs. (3)-(7). The values obtained are shown in columns one and two of Table 4.

The quality of all estimated sets of parameters is tested by utilizing the constitutive parameters to estimate the creep parameters. Good adjustments would imply obtaining the same constants, and the same graph of the experimental data which originated the chain of computations. In the present case, the estimated values are computed from Eqs.(9) and are shown in Table 5. Comparison with the original creep parameters in Table 1 one observes a good approximation for the characteristic times. The strengths are also similar, but B_2 appears as zero. It means the pair of parameters corresponding to characteristic time $\rho_2 \approx 4000$ s is missing. It shows in the plot, as depicted in Figure 3. For ease of comparison the original curve obtained directly from experimental data is also shown in continuous line. As expected, the dashed line shows steep ascent in the vicinity of 1000 s.

Table 5. Reconstructed strengths and characteristic times for creep modulus $J(t)$ of PVC.

Strengths [1/Pa]	Times [s]
$B_\infty = 7.529 \cdot 10^{-10}$	
$B_o = 3.903 \cdot 10^{-10}$	
$B_1 = 1.577 \cdot 10^{-11}$	$\rho_1 = 3.64 \cdot 10^2$
$B_2 = -1.525 \cdot 10^{-25}$	$\rho_2 = 4.00 \cdot 10^3$
$B_3 = 1.278 \cdot 10^{-10}$	$\rho_3 = 5.30 \cdot 10^4$
$B_4 = 1.289 \cdot 10^{-10}$	$\rho_4 = 4.98 \cdot 10^5$
$B_5 = 9.038 \cdot 10^{-11}$	$\rho_5 = 1.80 \cdot 10^6$

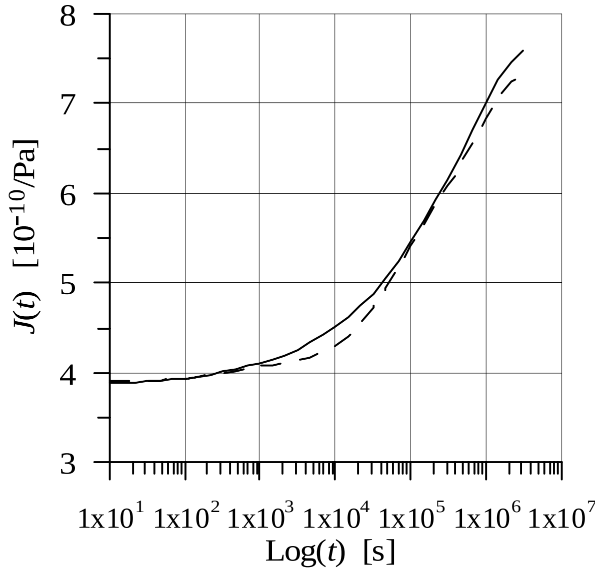

Figure 3. Diagrams for total creep modulus: continuous line obtained from experimental data and dashed line from recoverd parameters of Table 5.

4. CONCLUSIONS

This paper considers the problem of identification of material parameters in linear viscoelastic materials. Two new procedures are introduced to process experimental creep data into total creep module, total relaxation module and deviatoric relaxation module. Also, determination of the mechanical constants associated to the rheological model and determination of the coefficients in the differential constitutive equation is addressed. Numerical results obtained indicate the applicability and numerical efficiency of the procedures presented.

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