ON THE IDENTIFICATION OF VORTICAL STRUCTURES

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Abstract. *The concept of vortex plays a crucial role in turbulence. In spite of this, no single definition of a vortex is universally accepted. This fact lead us to perform in this paper a pure kinematical analysis of currently popular vortex definitions based on the velocity gradient tensor. We describe the inter-relationships between the different definitions, illustrating them by means of the ABC flow. In addition, we discuss the objectivity requirement for vortex definition as well as the issue of identification of vortical structures.*

Keywords*: structural turbulence, vortex definition, turbulence, vortical structures*

1. INTRODUCTION

In turbulence, the definition of vorticity and the notion of vortex are widely used to characterize this type of flow. When we assume that the turbulence has rotational nature, is immediate to consider the kinematics and the dynamics of the vorticity as tool of aid to make possible an ampler understanding of the flow. In this context, the two terminologies are sometimes confounded and used erroneously to illustrate some concepts associated to the phenomenon. The vorticity is a mathematical operation defined as the rotational of velocity field and is a local structure whereas a vortex is characterized with aid of physical arguments, beyond of mathematical definitions, by a spatial structure. Some geometrical concepts defined in function of the vorticity field as the line, the sheet and the vortex tube has structure non-local and are useful to characterize a vortex. We understand that the term “vortical structure” can be interpreted like any structure defined by all these concepts.

2. IDENTIFICATION CRITERIA

The methods of characterization of vortices embedded in a turbulent flow can be divided in *intuitive and objective identifications*, the former suggests the definition of a vortex without any quantitative argument while the latter, unlike, employs variables of field as velocity, pressure and vorticity to formulate the criteria.

An example of definition intuitive can coarsely be taken as *a swirl motion of a multitude of material particles around a common center* (Lugt, 1995). This type of definition suggests the idea of a visualization of trajectories (pathlines) of particles with aid of the tracer.

There are many forms of objective characterization of vortices which uses arguments based on: *i*) the magnitude of vorticity; *ii*) the minimum pressure; *iii*) the topology of trajectories and streamlines of fluid particles; *iv*) kinematics relations; and *v*) combinations these methodologies.

The methods based on magnitude of vorticity identify vortices as regions of domain occupied by vectors vorticity that satisfy an arbitrary threshold adopted by analyst. The structures composed by vectors of magnitude smaller than this threshold are not considered in the analysis like vortices. The criteria of minimum pressure follow a similar procedure using a threshold to identify the vortices as the regions where the pressure is below of a certain value, also adopted by analyst.

The criteria that use the topology of trajectories and streamlines identify as vortices the structures that obey a definition like that of Lugt (1995), but now with quantitative arguments. The notions of vortex based on topology of closed or spiraled pathlines and streamlines can be viewed as criteria which combine the two ideas if we observe that the system that governs the trajectories of particles is an expansion until first order in Taylor's series of local velocity around of origin. Identification criteria based on kinematics relations (only) consider relations involving the velocity gradient tensor or its decomposition additive given by tensors rate-of-strain and rate-of-rotation.

We point out that the methods based on the trajectories and streamlines also can be considered as kinematics criteria since the variation of the position (velocity) is approached as function of the velocity gradient.

The scope of this paper is to analyze three methodologies which identify the vortices in incompressible flow considering only kinematics relations: the criteria Q (Hunt *et al*., 1988), Δ (Chong *et al*., 1990) and λ_2 (Jeong and Hussain, 1995). The regions occupied by vortices according to these criteria are shown with aid of a field of velocity known as ABC flow (Dombre *et al*., 1986). At this point is important to emphasize that the three criteria identify the structures as vortex tubes, only. So that, additionally, we consider in this paper two classification methods that identification the structures as vortex sheets or vortex tubes, namely, the classification proposed by Tanaka and Kida (1993) and Horiuti (2001), developed based on criteria proposed by Hunt *et al*. (1988) and Jeong and Hussein (1995) respectively. Others methods of vortex identification (like tubes) that we found in literature are the criteria proposed by Zhou *et al*. (1999), Chakraborty *et al*. (2005), Haller (2005) and Cucitore *et al*. (1999).

2.1. Kinematics criteria

The Q criterion proposed by Hunt *et al.* (1988) defines a vortex as a spatial region where the second scalar invariant Q of velocity gradient $\boldsymbol{L}$ (L_{ij} in indicial notation) is positive. An additional constraint imposed by the criterion is that the pressure is lower than the ambient value inside of the vortex. The Q criterion for incompressible flows is defined as:

$$Q(\boldsymbol{L}) := \frac{1}{2}(|\boldsymbol{W}|^2 - |\boldsymbol{S}|^2) := -\frac{1}{2}(S_{ij}S_{ji} + W_{ij}W_{ji}) > 0 \quad (1)$$

where S_{ij} and W_{ij} are respectively the rate-of-strain and rotation tensors defined by additive decomposition of velocity gradient L_{ij}. Equation (1) also defines the Laplacian of pressure which is obtained by taking the divergent of Navier-Stokes equations. This method identifies the structure as vortex tube because the rate-of-rotation magnitude is larger than that of rate-of-strain. The structures visualized with the results obtained of several analysis by DNS of isotropic turbulence suggests tubelike structures when $\boldsymbol{W} > \boldsymbol{S}$.

The Δ criterion proposed by Chong *et al.* (1990) defines a vortex as the region where the discriminant Δ of characteristic polynomial of velocity gradient $\boldsymbol{L}$ is positive. In this case, the polynomial supplies (conjugated) complex eigenvalues to the velocity gradient. The Δ criterion for incompressible flows is defined as follows:

$$\Delta(\boldsymbol{L}) := (Q/3)^3 + (R/2)^2 > 0 \quad (2)$$

where Q is defined by Eq. (1) and R is the third invariant of velocity gradient $\boldsymbol{L}$ given by determinant of velocity gradient. This method also identifies the structure as a vortex tube, but now due to topology of streamlines. The conjugated complex eigenvalues of gradient velocity define a pattern of closed or spiraled trajectories for the group of fluid particles, namely, a vortex tube.

The λ_2 criterion proposed by Jeong and Hussein (1995) defines a vortex as the region where the symmetric tensor $\boldsymbol{S}^2+\boldsymbol{W}^2$ has two eigenvalues negative. An equivalent definition requires only that the intermediate eigenvalue λ_2 of the same tensor be negative:

$$\lambda_2(\boldsymbol{S}^2 + \boldsymbol{W}^2) < 0 \quad (3)$$

since any symmetric tensor has only real eigenvalues which can be ordered as $\lambda_1 \geq \lambda_2 \geq \lambda_3$. Jeong and Hussein (1995) take the gradient of Navier-Stokes equations to obtain, after some algebra and some assumptions, the tensor $\boldsymbol{S}^2+\boldsymbol{W}^2$ which express the Hessian of pressure by kinematics quantities. While the Q criterion measures the relative magnitude of the excess of the rate-of-rotation in relation to the rate-of-deformation in all of the directions, the criterion λ_2 valuates this excess in a specific plan, defined by eigenvectors related to the negative eigenvalues. This plan is normal to the axial vector (of a tubelike structure) defined by related eigenvector to the eigenvalue λ_1, which is positive due to own criterion.

The method proposed by Tanaka and Kida (1993) defines a vortex as described by the Q criterion of Hunt *et al.* (1988) with this vortex characterized as a tube or a sheet. Tanaka and Kida (1993) argue, based on numeric simulations and experiments, which a *strong* vortex tube can be characterized as a region of low pressure or as a region where the vorticity $\boldsymbol{\omega}$ (or rate-of-rotation $\boldsymbol{W}$) is *large* and the rate-of-strain $\boldsymbol{S}$ is *small*. A *strong* vortex layer (sheet vortex), according to Tanaka and Kida (1993), is observed where the rate-of-rotation $\boldsymbol{W}$ and rate-of-strain $\boldsymbol{S}$ are *large* and of equivalent magnitudes. Tanaka and Kida (1993) showed some of their results using arbitrary thresholds as $N^2>2$ to extract vortex tubes and $1/2<N^2<4/3$ to extract the vortex layers, where $N^2=|\boldsymbol{W}|^2/|\boldsymbol{S}|^2$. By using $4/3<N^2<2$ they affirm that there are typically vortical structures around the cores of tubes, or between the tubes and sheets.

Horiuti (2001) proposes a criterion of structural identification based on the method λ_2 of Jeong and Hussain (1995) and in the solutions of Burgers [12] of Navier-Stokes equations. This criterion identifies the structures vorticais as tubes and vortex (plane and curves) sheets, being, therefore a generalization of the method λ_2, that identifies the vortices only as tubes. The methodology of Horiuti (2001), that uses the approach of Jeong and Hussain (1995) of the Hessian pressure , it is compared with the solution (exact) of Burgers.According to Horiuti (2001) the Burgers' structures that happens in the regions dominated by the rate-of-rotation are characterized by tubes and those that happen where the rate-of-rotation and deformation are equivalent are characterized by flat sheets, with the curved sheets (around the core of tube), occupying intermediate regions, where the rate-of-strain is dominant.

The criterion proposed by Horiuti (2001) it considers the eigenvalues of the symmetric tensor $\boldsymbol{S}^2+\boldsymbol{W}^2$ defined as λ_z, λ_+ and λ_-, where λ_z is that eigenvalue related to the most aligned eigenvector with the vorticity $\boldsymbol{\omega}$, and the remaining two are characterized by the inequality. $\lambda_+ > \lambda_-$. This criterion establishes that vortex tubes happen where $0 > \lambda_+ \geq \lambda_-$

when ***W*** is dominant; that curved sheet happen where $\lambda_+ \geq \lambda_- > 0$ when ***S*** is dominant; and that flat sheet happen where $\lambda_+ > 0 > \lambda_-$ when ***S*** and ***W*** are equivalent.

2.2. Vortices and ABC flow

The field of velocity given by the ABC flow is an (steady) exact solution of Euler equations, exhibiting a pattern of chaotic streamlines, according to Dombre *et al*. (1986). When turbulent flows are analyzed in the context of dynamical systems the behavior of streamlines obtained as (steady) solutions of these systems is chaotic. Haller (2005) adds that the ABC flow shows elliptic regions with certain features (KAM-type) that are arguably called vortices, as regions that display patterns of swirling motion and low dispersion of particles. The swirling motion is an argument used to define vortices according with criteria proposed by Chong *et al*., 1990, Zhou *et al*. (1999), Chakraborty *et al*. (2005) if the velocity gradient ***L*** has (conjugated) complex eigenvalues. The low dispersion of two particles is the basis of the criteria proposed by Cucitore *et al*. (1999).

The ABC flow used to display the regions occupied by vortices is defined as follows:

$$u(x) = (A\sin z + C\cos y, B\sin x + A\cos z, C\sin y + B\cos x) \qquad (4)$$

with the parameters configuration given by $A=\sqrt{3}$, $B=\sqrt{2}$ and $C=1$ so that Eq. (4) generates chaotic streamlines as described in Dombre *et al*. (1986). With the ABC flow, we write a simple computational code in Matlab to generates the illustrations that show the regions occupied by vortices according to criteria Q (Hunt *et al*., 1988), Δ (Chong *et al*., 1990) and λ_2 (Jeong and Hussein, 1995).

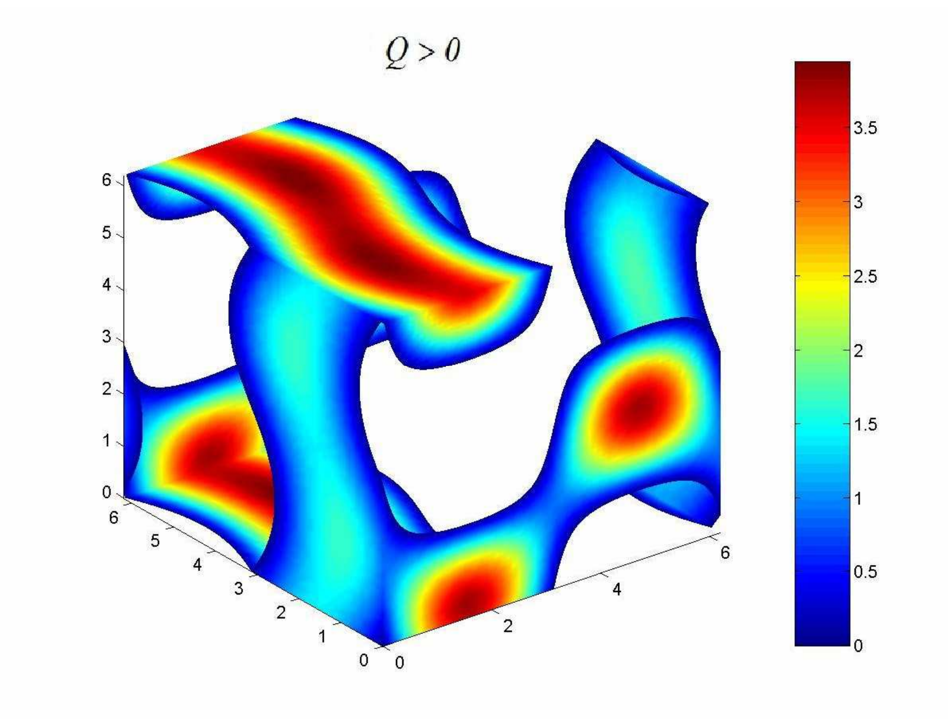

Figure 1. Plot of regions occupied according to Q criterion (Hunt *et al*., 1988).

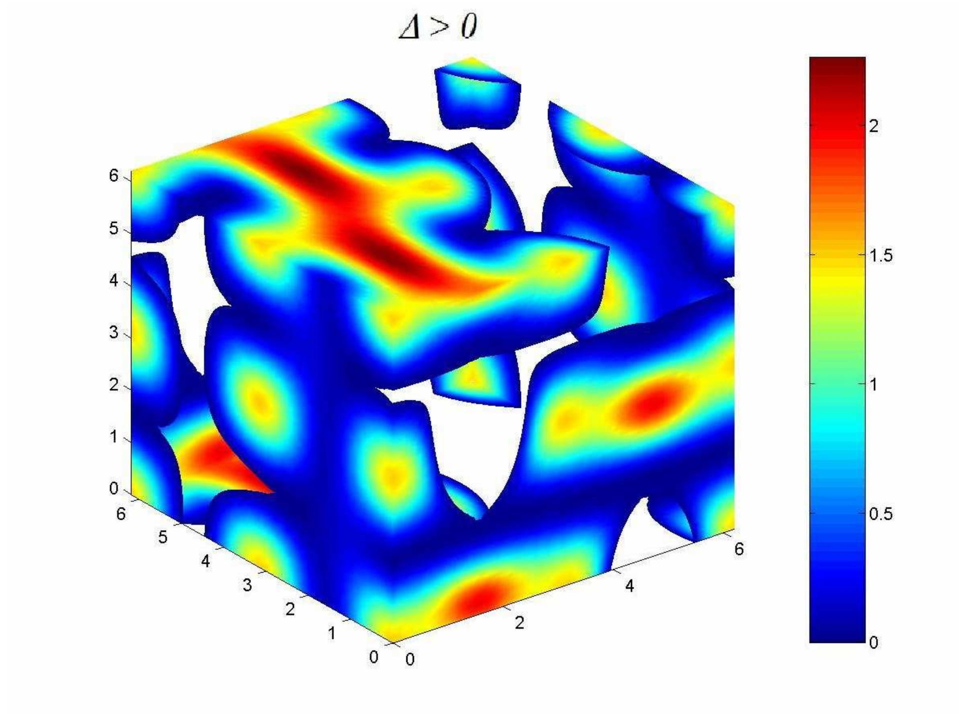

Figure 2. Plot of regions occupied according to Δ criterion (Chong *et al*., 1990).

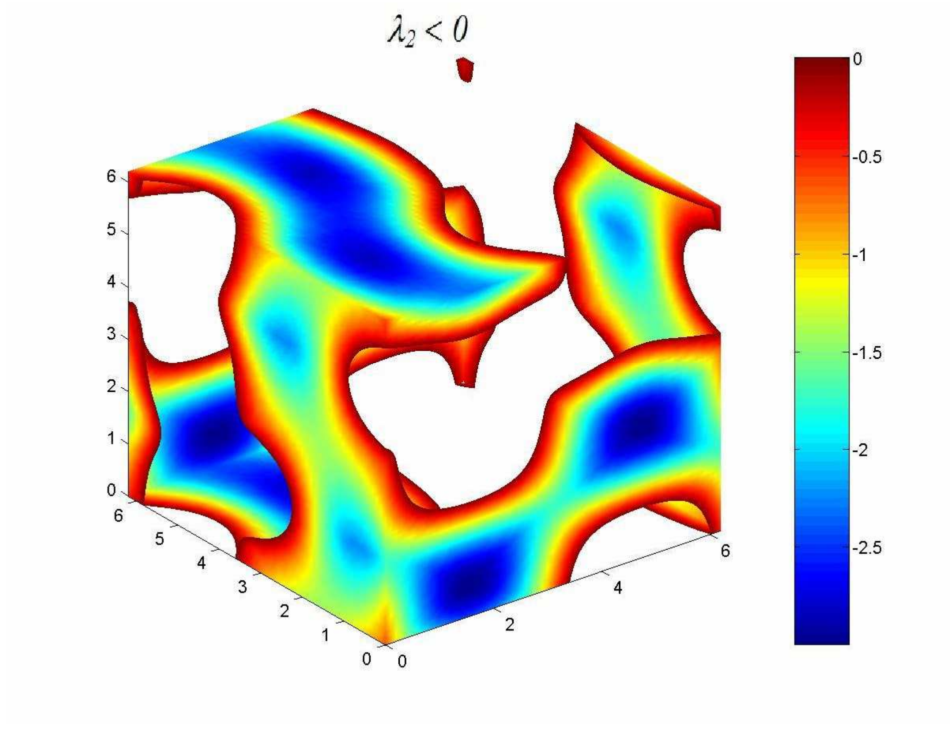

Figure 3. Plot of regions occupied according to Δ criterion (Chong *et al*., 1990).

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