ESTIMATION OF PARAMETERS IN THE BOUC-WEN MODEL FOR MAGNETO-RHEOLOGICAL DAMPERS USING A HYBRID METHOD

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Abstract. *Magneto-rheological (MR) dampers are semi-active control devices which have attracted the interest of several researchers in recent years mainly because they require a low power input and are considered fail-safe, i.e., in case of control hardware failure, they become passive dampers. Several models have been developed in order to predict the nonlinear and hysteretic dynamic behavior of MR dampers. Among them, the Bouc-Wen model stands out due to its relative simplicity and accurate results. Nonetheless, the inverse problem of parameter estimation of such models has demonstrated to be a difficult task. In the present work, the gradient based Levenberg-Marquardt method, the stochastic Simulated Annealing method and a combination of both methods are considered in order to estimate the parameters of the Bouc-Wen model.*

Keywords: *Magneto-rheological dampers, Bouc-Wen model, Inverse problems, Levenberg-Marquardt, Simulated Annealing.*

1. INTRODUCTION

Semi-active suspensions or isolation devices can have their damping and stiffness properties controlled in an active way (Hong et al., 2002). For instance, the control of force intensity given by a viscous damper can be done by changing a valve aperture. Magneto-rheological (MR) fluid is a kind of material with the ability to change its properties when a magnetic field is applied on it. One of its main applications is the use of this kind of fluid to fill viscous dampers. In magneto-rheological dampers, the fluid viscosity is continuously controled by a magnetic field. Different strategies for the control of semi-active MR suspensions have been published (Ahmadian and Paré, 2000, Guo et al., 2004, Du et al., 2005). Many models were formulated to characterize the dynamic behavior of MR dampers: The Bingham model (Stanway et al., 1987); the bi-viscous hysteresis model (Wereley et al., 1998); the phenomenological Bouc-Wen model (Spencer et al., 1997); and a unified model for ER (Electro-rheological) and MR vibration dampers (Sims et al., 2004), just to cite a few.

Among these models, the one that describes well not only the bi-viscous behavior but also the hysteretic behavior of MR dampers is the Bouc-Wen model, presented in the next section. This model takes a 8-parameter first-order nonlinear differential equation, which is used to analyse the nonlinear hysteretic MR behavior. The Bouc-Wen model stands out due to its relative simplicity and accurate results. However, to solve the inverse problem, that means, to obtain a reliable parameter estimation based on experimental data, it has been shown to be a difficult task, mainly due to the high number of parameters involved.

In the present work we use a hybrid identification method — based on the Levenberg-Marquardt method (LM), the Simulated Annealing method (SA) and a combination of both methods, (SA-LM) — in order to estimate the parameters of the Bouc-Wen model for a magneto-rheological damper.

2. THE BOUC-WEN MODEL

In order to model the nonlinear dynamic behavior of the MR damper, the Bouc-Wen model, depicted in Fig. 1, is considered in this work. This phenomenological model has been shown to accurately predict the behavior of a prototype MR damper over a wide range of inputs (Spencer et al., 1997).

The main idea in this model is a parallel combination of an element of hysteresis, a viscous damper, and a spring, as shown. In the Bouc-Wen model, the force $u(t)$ generated by the MR damper is given by

$$u(t) = c_0\dot{q}(t) + k_0(q(t) - x_0) + \alpha h(t), \tag{1}$$

where $q(t)$ is the displacement, c_0 is the plastic damping coefficient and k_0 and x_0 account for the effects of the accumulator in the MR damper, which, in a phenomenological perspective, acts as a spring. The internal variable $h(t)$ in Eq. (1)

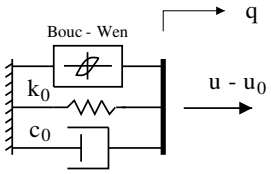

Figure 1. The Bouc-Wen model for magneto-rheological dampers.

represents the hysteretic displacement and its evolution equation is given by

$$\dot{h}(t) = -\gamma|\dot{q}(t)|\, h(t)\, |h(t)|^{n-1} - \beta\dot{q}(t)|h(t)|^n + A\dot{q}(t), \tag{2}$$

where the parameters γ, β, A and n control the shape of the hysteresis curve.

3. MATHEMATICAL FORMULATION OF THE INVERSE PROBLEM

In the original Bouc-Wen model, the dynamics is governed by eight parameters, as can be seen from Eqs. (1) and (2). Aiming at reducing the number of parameters to be estimated, the following hysteretic force is defined as

$$z(t) = \alpha h(t). \tag{3}$$

Considering Eqs. (1), (2) and (3), the dynamics of the model may be described by the set of equations:

$$\begin{aligned} u(t) &= c_0\dot{q}(t) + k_0(q(t) - x_0) + z(t); \\ \dot{z}(t) &= -\bar{\gamma}|\dot{q}(t)|\, z(t)\, |z(t)|^{n-1} - \bar{\beta}\dot{q}(t)|z(t)|^n + \bar{A}\dot{q}(t), \end{aligned} \tag{4}$$

where the parameters $\bar{\gamma}$, $\bar{\beta}$ and $\bar{A}$ are given by:

$$\bar{\gamma} = \frac{\gamma}{\alpha^{n-1}}; \qquad \bar{\beta} = \frac{\beta}{\alpha^{n-1}}; \qquad \bar{A} = \alpha A. \tag{5}$$

Therefore, the number of parameters to be estimated in the MR Bouc-Wen model is reduced from eight to seven and the vector of unknown parameters is given by

$$\mathbf{x} = [c_0, k_0, x_0, \bar{\gamma}, \bar{\beta}, \bar{A}, n]^T. \tag{6}$$

The present inverse problem of parameter estimation can then be stated as the minimization problem

$$\min_{\mathbf{x}} Q(\mathbf{x}) = \min_{\mathbf{x}} \frac{1}{2}\, \mathbf{F}(\mathbf{x})^T\mathbf{F}(\mathbf{x}), \tag{7}$$

where $\mathbf{F}(\mathbf{x})$ is the residual vector, whose components are defined as

$$F_i(\mathbf{x}) = u_i(\mathbf{x}) - u_{exp,i}, \qquad i = 1, 2, ..., N_t, \tag{8}$$

where N_t is the total number of time samples considered in the estimation process and u_i and $u_{exp,i}$ stand, respectively, for the predicted and experimental damper force at the time instant t_i.

According to Eq. (4), for computing the predicted force, the displacement $q(t)$ and velocity $\dot{q}(t)$ at the end of the MR damper must be available from the experiments, besides the damper force, which is required for computing the residual vector in Eq. (8).

3.1 The Levenberg-Marquardt (LM) Method

The optimality condition associated with the estimation problem in Eq. (7) is given by

$$\frac{\partial Q(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{J}^T\mathbf{F}(\mathbf{x}) = \mathbf{0}, \tag{9}$$

where $\mathbf{J}$ is the Jacobian matrix, whose elements are defined as

$$J_{ij} = \frac{\partial u_i(\mathbf{x})}{\partial x_j}, \quad i = 1, 2, ..., N_t, \quad j = 1, 2, ..., 7. \tag{10}$$

Using a Taylor's expansion for $\mathbf{F}(\mathbf{x})$ about $\mathbf{x}^k$ and keeping only the terms up to the first order, one has

$$\mathbf{F}(\mathbf{x}^{k+1}) = \mathbf{F}(\mathbf{x}^k) + \mathbf{J}^k \Delta\mathbf{x}^k, \tag{11}$$

where $\mathbf{J}^k = \mathbf{J}|_{\mathbf{x}=\mathbf{x}^k}$ and $\Delta\mathbf{x}^k = \mathbf{x}^{k+1} - \mathbf{x}^k$.

Considering Eqs. (9) and (11), the iterative procedure of the Levenberg-Marquardt method is given by

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \left[\left(\mathbf{J}^k\right)^T \mathbf{J}^k + \lambda^k \mathbf{I}\right]^{-1} \left(\mathbf{J}^k\right)^T \mathbf{F}(\mathbf{x}^k), \tag{12}$$

where k is the iteration counter, $\mathbf{I}$ is the identity matrix, and λ is a damping parameter, which is added with the aim of improving the stability of the iterative procedure (Silva Neto and Moura Neto, 2005).

Therefore, the parameter estimation built on the Levenberg-Marquardt method starts with an initial guess $\mathbf{x}^0$ and new estimates are obtained according to Eq. (12). The damping parameter λ is reduced along the iterative procedure, which is continued until some convergence criterion is satisfied.

3.2 The Simulated-Annealing (SA) Method

At the core of the Simulated-Annealing method is an analogy with the way that metals cool and anneal. At relatively high temperatures, the molecules of a liquid metal have a great mobility, and they can move freely with respect to one another. If the metal is cooled slowly, this thermal mobility is lost and the system has enough time for redistribution of its atoms and it eventually achieves the minimum energy state. If the system is cooled quickly, it does not reach this minimum state, but, rather, it reaches a state having somewhat higher energy. Therefore, the essence of the process is slow cooling for ensuring that a low energy state will be achieved.

Based on statistical mechanics reasoning, Metropolis et al. (1953) introduced a simple algorithm that can be used to accomplish an efficient simulation of a system of atoms in equilibrium at a given temperature T. In each step of the algorithm, a small random displacement of an atom is performed and the variation of the energy ΔE is calculated. If $\Delta E < 0$, the displacement is accepted, and the configuration with the displaced atom is used as the starting point for the next step. In the case of $\Delta E > 0$, the new configuration may be accepted according to Boltzmann probability

$$P(\Delta E) = \exp(-\Delta E / k_b T) \tag{13}$$

A uniformly distributed random number $p \in [0, 1]$ is calculated and compared with $P(\Delta E)$. Metropolis criterion establishes that the new configuration is accepted if $p < P(\Delta E)$, otherwise it is rejected and the previous configuration is used again as a starting point.

In order to apply the SA method in the present parameter estimation problem, the objective function $Q(\mathbf{x})$, defined in Eq. (7), takes the place of the energy and some configurations are defined by a set of vectors $\mathbf{x}$. Then, the Metropolis procedure generates a collection of configurations for the optimization problem at some "temperature" T, which is simply a control parameter of the method. Hence, the SA procedure consists of first "melting" the system being optimized at a high effective temperature, then lowering the temperature until the system "freezes" and no further change occurs. The main control parameters of the algorithm implemented ("cooling procedure") are the initial temperature T_0, the cooling rate r_t, the number of steps N_s performed through all elements of vector $\mathbf{x}$, the number of times N the procedure is repeated before the temperature is reduced, and the number of minimum points N_ϵ (one for each temperature) that are compared and used as the stopping criterion, if they all agree within a tolerance ϵ.

3.3 The Simulated-Annealing and Levenberg-Marquardt (SA-LM) Hybrid Method

A parameter estimation procedure built on a gradient based method, as the LM method, may yield a solution which, in fact, represents a local minimum of the function to be minimized. Therefore, in order to achieve a global minimum, a global optimization method, as the stochastic SA method, is required. However, the relatively high number of function evaluations may lead a global method prohibitive from a computational point of view.

Aiming at keeping the best features of each method, a Simulated Annealing and Levenberg-Marquardt (SA-LM) hybrid method was considered in the present work as follows. The SA stochastic method was allowed to run for a while, yielding a solution which was further used as initial guess in the gradient based LM method. Within a few iterations, the LM method is supposed to reach a minimum of the function to be minimized. However, it is not guaranteed that this minimum is the searched global one. Hence, in order to verify it, the SA is allowed to run again. If the same solution is reached, it is likely that the global minimum of the function was reached, and the iterative procedure is interrupted. If, in the other hand, a different solution is obtained, it means that the previous one was a local minimum. In that case, the LM and SA methods are allowed to run until the global minimum is obtained.

4. NUMERICAL RESULTS

In the following, some numerical results for the inverse problem of parameter estimation in the the Bouc-Wen model are presented. In this step of the research, the identification is done based strictly on the theoretical model, by giving a initial guess with a deviation of 10% to 30% from the exact values of the parameters adopted in the model. It is worth noting that the experiment itself gives some reliable information about the neighborhood where the parameters values must stay, so that the proposed deviation is a realistic one.

As real experimental data were not available, a synthetic experimental data was generated as follows

$$u_{exp,i} = u_i(\mathbf{x})(1 + \sigma_e r), \tag{14}$$

where r is a randon number in the range $[-1, 1]$ and σ_e simulates the standard deviation of measurement errors.

The adopted values for the set of unknown parameters were (with $\alpha = 880$ N/cm):

$$k_0 = 25 \text{ N/cm},\ c_0 = 50 \text{ Ns/cm},\ x_0 = 3.8 \text{ cm},\ \gamma = 100 \text{ cm}^{-2},\ \beta = 100 \text{ cm}^{-2},\ A = 120, \text{ and } n = 2.0 \tag{15}$$

Two experiments were considered. In the first one, the excitation frequency of the MR damper was 2.5 Hz. By performing a sensitivity analysis (Stutz et al., 2007) it was observed that using a lower frequency, such as 0.5 Hz, the sensitivity of the force $u(t)$ with regard to the parameters is higher. Therefore, a second experiment with the excitation frequency of 0.5 Hz was also considered.

For each frequency it was considered four different sets of initial guesses for the unknown parameters. They correspond to 70%, 90%, 110%, and 130% of the exact values. Therefore, we have a total of eight test cases.

The results obtained for experiment 1, with 2.5 Hz are shown in Tables 1–4; the results for experiment 2, with 0.5 Hz, are shown in Tables 5–8. In all test cases synthetic experimental data with up to 5% error was used.

Three different approaches were considered for the parameters estimation. In the first one, the inverse problem was solved using only the LM method, as described in Section 3.1. In the second one, the SA method, as described in Section 3.2, was adopted with only 80 cycles. In the third approach, the hybrid SA-LM, only 3 cycles of SA were run and the obtained result was then used as the initial guess for LM, which takes just a few seconds to converge.

Each SA cycle corresponds to 700 objective function evaluations and it is related to a temperature in the cooling process. The control parameters for SA in the present paper were: $T_0 = 5.0$; $r_t = 0.75$; $N_s = 20$; $N = 5$, and $N_\epsilon = 4$. The CPU time required to run 80 cycles of SA was about 12 min on a 2.8 GHz Pentium IV processor.

All the results are expressed in terms of the rate p/p_0, where p is the identified parameter value and p_0 is the exact parameter value. In Table 1, the results of the estimation obtained by SA with 3 cycles are also shown. It is worth noting from Table 4 that the LM method alone did not yield satisfactory estimates for parameters $\bar{\gamma}$ and n, while the proposed hybrid SA-LM method got good ones.

Table 1. Identification results for drive frequency $= 2.5$ Hz and initial guess of 70% (0.7) for all parameters values.

Variables	LM	SA (80)	SA (3)	SA-LM
c_0	1.0004	1.0026	0.9914	1.0020
k_0	1.0092	1.0015	1.2132	0.9996
$\bar{\gamma}$	1.0051	1.0680	1.0268	1.0058
n	1.0050	1.0832	1.0936	1.0045
$\bar{A}$	0.9981	0.9801	0.9065	0.9876
$\bar{\beta}$	1.0028	0.9821	0.8475	0.9922
x_0	0.9868	1.0012	0.8552	1.0026

The tables 5 to 8 are related to the experiment with the drive frequency of 0.5 Hz. Here also the three approaches provided good estimates for the unknowns. Some difficulties are observed when LM is used alone at the worst case of $p/p_0 = 1.3$.

Table 2. Identification results for drive frequency $= 2.5$ Hz and initial guess of 90% (0.9) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	1.0010	1.0025	0.9986
k_0	1.0184	1.0000	0.9840
$\bar{\gamma}$	0.9958	1.0398	0.9916
n	1.0000	1.0517	0.9900
$\bar{A}$	1.0006	0.9866	1.0074
$\bar{\beta}$	1.0156	0.9908	0.9989
x_0	0.9842	1.0027	0.9978

Table 3. Identification results for drive frequency $= 2.5$ Hz and initial guess of 110% (1.1) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	1.0016	1.0026	0.9996
k_0	1.0216	0.9998	1.0320
$\bar{\gamma}$	1.0065	1.0361	1.0390
n	1.0050	1.0478	1.0450
$\bar{A}$	0.9995	0.9875	0.9948
$\bar{\beta}$	1.0045	0.9921	0.9976
x_0	0.9789	1.0030	0.9731

Table 4. Identification results for drive frequency $= 2.5$ Hz and initial guess of 130% (1.3) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	0.9986	1.0026	1.0001
k_0	0.9916	1.0002	1.0176
$\bar{\gamma}$	0.6688	1.0425	0.9846
n	0.3345	1.0547	0.9851
$\bar{A}$	1.0253	0.9861	1.0032
$\bar{\beta}$	0.7694	0.9898	1.0095
x_0	1.0815	1.0026	1.0157

Table 5. Identification results for drive frequency $= 0.5$ Hz and initial guess of 70% (0.7) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	0.9964	0.9968	0.9971
k_0	1.0228	1.0024	1.0101
$\bar{\gamma}$	0.9886	0.9923	1.0131
n	0.9801	0.9799	1.0151
$\bar{A}$	1.0028	1.0014	1.0151
$\bar{\beta}$	0.9955	0.9865	1.0012
x_0	09815	0.9958	0.9895

Table 6. Identification results for drive frequency $= 0.5$ Hz and initial guess of 90% (0.9) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	0.9976	0.9992	0.9998
k_0	0.9996	1.0092	0.9816
$\bar{\gamma}$	0.9891	1.0002	1.0087
n	0.9751	0.9951	1.0001
$\bar{A}$	1.0024	0.9983	0.9985
$\bar{\beta}$	0.9901	0.9885	0.9871
x_0	0.9973	1.0001	1.0013

Table 7. Identification results for drive frequency $= 0.5$ Hz and initial guess of 110% (1.1) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	0.9998	0.9968	0.9994
k_0	1.0192	1.0021	0.9604
$\bar{\gamma}$	0.9827	0.9892	0.9941
n	0.9401	0.9767	0.9901
$\bar{A}$	0.9985	1.0023	1.0001
$\bar{\beta}$	0.9664	0.9883	0.9988
x_0	0.9921	0.9961	1.0263

Table 8. Identification results for drive frequency $= 0.5$ Hz and initial guess of 130% (1.3) for all parameters values.

Variables	LM	SA (80)	SA-LM
c_0	1.0081	0.9968	0.9992
k_0	1.0316	1.0025	0.9928
$\bar{\gamma}$	0.6920	0.9936	0.9925
n	0.4071	0.9812	0.9901
$\bar{A}$	1.0262	1.0011	1.0011
$\bar{\beta}$	0.8492	0.9857	0.9982
x_0	1.0394	0.9957	1.0052

5. CONCLUDING REMARKS

The identification of the parameters of the Bouc-Wen model for a magneto-rheological semi-active damper was performed using three different techniques: the gradient based Levenberg-Marquard (LM) method; the stochastic Simulated Annealing (SA) technique, and a hybrid method (LM-SA), were the SA method is used to obtain an initial guess to the LM technique.

The hybrid method showed to be more economical than the SA in terms of CPU time consuming, even if not showing too much advantage in the parameters identification accuracy.

The hybrid method showed to be much better than the LM in identifying the parameters in the cases of plus 30% of error, as can be seen in Tables 4 and 8.

It is worth noting that each simulation was performed with the same initial error for all parameters. In practice, however, some of them permit a better initial guess, extracted from the experimental data, while others do not. Accurate initial guess for parameters c_0, k_0 and x_0 may be withdrawn directly from the experiments. Besides, based on the results presented in the literature, the value of parameter n is likely to be in the interval (1,2). Hence, in the numerical tests presented here a random initial guess was not considered.

In the studied situations, the LM method yielded satisfactory results, except in the 1.3 initial guess cases. However, a better comparative performance of the hybrid SA-LM method is expected when dealing with true experimental data.

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