# ROBUST MULTIOBJECTIVE OPTIMIZATION OF VISCOELASTIC STRUCTURES

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***Abstract.*** *In the context of passive control of noise and vibration the use of viscoelastic materials is an interesting means of achieving effective mitigation in various types of industrial applications, where these materials can be applied either as discrete devices or surface treatments at a relatively low cost. A natural extension of modelling is the optimization of the viscoelastic devices aiming the reduction of cost and/or maximization of performance. In the field of optimization strategies much interest has been devoted to multiobjective evolutionary algorithms (MOEA) since they provide the possibility of addressing tradeoffs between multiple objective functions by sampling a number of Pareto solutions. In general, optimization methods can be classified in two main categories: deterministic and stochastic. Deterministic approaches neglect the effects of uncertainties in the design space, and the solutions may violate the physical reality when the design is actually realized. It is then said that the solutions lack robustness. As an alternative, the method implemented in this paper consists in finding the robust optimal solutions by introducing additional cost functions (known as robustness functions) for each original objective function, defined on the basis of the dispersions of the original cost functions. During the optimization process, the robustness and the original cost functions are evaluated simultaneously, in order to find the robust optimum. To take into account the uncertainties on the design variables, one proposes to use Monte Carlo sampling combined with a parametric approach which directly uses the variation on the design parameters. The interest in reducing the computational burden involved in multiobjective optimization based on large finite element models motivates the use of condensation method adapted for the viscoelastic structures. For the purpose of illustration, the suggested methodology is applied to a stiffened panel treated with constraining damping layer.*

***Keywords****: robust multiobjective optimization, Viscoelastic damping, finite element*

## 1. INTRODUCTION

The use of viscoelastic materials has been regarded as a convenient strategy in many types of industrial applications, where these materials can be applied either as discrete devices, such as translational and/or rotational mounts, or surface treatments (free or constrained layers) at a low cost of application (Nashif et al., 1985; Samali and Kwok, 1995; Rao, 2001). Another interesting strategy consists in incorporating viscoelastic materials as a means of adding damping to vibration neutralizers (Espíndola et al., 2005). In the last decades, much effort has been devoted to the development of finite element models capable of accounting for the typical dependence of the viscoelastic behavior with respect to frequency and temperature (Balmès and Germès, 2002). As a result, it is currently possible to perform finite element modeling of complex real-world engineering structures such as automobiles, airplanes, communication satellites, tall buildings and space structures.

A natural consequence of the modeling capability is the optimization of the viscoelastic devices aiming at the reduction of cost and/or maximization of performance. In the quest for optimization, the engineers are frequently faced with conflicting objectives. Such situations are conveniently dealt with by the so-called multi-objective or multicriteria optimization approach (Eschenauer et al., 1990). In general, optimization methods can be classified in two main categories: deterministic approaches that neglect the effects of uncertainties in the design space, and the solutions may violate the physical reality when the design is actually realized. It is then said that the solutions lack robustness; the second class is the one of stochastic approaches, in which the uncertainties and variations of design variables are considered. The uncertainties are normally associated to variability in materials properties (Young modulus, density, etc.) and tolerances in the manufacturing processes (thickness, other geometrical variables, etc.). Some authors (Lee and Park, 2001) proposed a method of optimization in which an additional objective function is defined having the same mean and standard deviation as the original objective function. The principal disadvantage of this method is the use of a weighting method of these functions. The weakness of the weighting method is that not all of the Pareto optimal solutions can be found unless the problem is convex. In reference (Lee and Park, 1996), the authors completed similar work by using the Taguchi method, where the robustness is evaluated only at the end of the optimization process.

The method proposed in this paper consists in finding the robust solutions by introducing additional cost functions (known as robustness functions) for each original objective function, defined on the basis of the dispersions of the original cost functions (Ait Brik et al., 2003). During the optimization process, the robustness functions and the original cost functions are evaluated simultaneously by using an evolutionary multiobjective algorithm with sharing technique, in order to find all the optimal and robust Pareto solutions. To take into account the uncertainties on the design variables of the viscoelastic damping treatment, one proposes to use Monte Carlo sampling combined with a parametric approach which directly uses the variation of the design parameters. The interest in reducing the computational burden involved in multiobjective optimization based on large finite element model composed by a free-free stiffened panel treated with passive constraining damping layer motivates the use of a robust condensation methodology adapted for the viscoelastic structures (Lima et al., 2006).

## 2. STOCHASTIC MULTIOBJECTIVE OPTIMIZATION

A general multiobjective optimization problem (MOP) involve simultaneous optimization of multiple objective functions, which may be in conflict with each other, and the goal is to find the best design solutions, which lead to the minimum or maximum values of the objective functions. In general, in a multiobjective optimization problem there is no single optimal solution and the interaction among different objectives gives rise to a set of compromised solutions, known as the Pareto optimal solutions (Ait Brik et al., 2003). Since none of these Pareto optimal solutions can be identified as better than the others without any further consideration, the interest is to find as many Pareto optimal solutions as possible. A deterministic multiobjective problem includes a set of $k$ parameters (decision variables) and a set of $n$ objective functions ( $n \geq 2$ ), and can be summarized as follows:

$$\begin{cases} \min\limits_{\boldsymbol{x}} \; \boldsymbol{F}(\boldsymbol{x}) = \left(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x})\right) \\ \quad g_j(\boldsymbol{x}) \leq 0 \quad j = 1, \ldots, m \\ \quad \boldsymbol{x}_L \leq \boldsymbol{x} \leq \boldsymbol{x}_U \quad \boldsymbol{x} \in C \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_k]^T$ is a the vector of design variables and $C \in R^k$ is the decision space. For a practical design problem, $\boldsymbol{F}(x)$ is non-linear, multi-modal and not necessary analytical.

In a classical deterministic optimization process, the uncertainties in the design variables are not considered, and the Pareto optimal solutions are not *insensitive* to the variations on the design variables. In the question of evaluating the robustness of the optimal solutions, the problem posed is not to find only the optimum but the robust optimum by taking into account the uncertainties during the optimization process. In mechanical structures, uncertainties result from several sources, including the errors associated to the modeling procedure of physical phenomena, the materials characteristics adopted such as Young modulus, mass density, Poisson ratio, etc., the finite element assembling process, the tolerances in the manufacturing processes (thickness, stiffness of junctions…), etc. Classically, the robustness of the optimal solution is evaluated starting from the deterministic optimization process, supposing that the deterministic design space contains the robust solutions obtained by the introduction of a stochastic criterion.

To mitigate the difficulty in evaluation the robustness optimal solutions, Lee and Park (1996; 2001) proposed an optimization methodology in which an additional objective function is defined having the same *mean* and *standard deviation* of the original objective function, by using a weighting method. The main disadvantage of the proposed methodology is that not all of the Pareto optimal solutions can be found unless the problem is convex. Other authors proposed similar work by using the Taguchi method, in which the robustness of the optimal solutions is evaluated only at the end of the optimization process. In this paper, one proposes a different methodology to evaluate the robustness of the optimal solutions, by supposing that an optimal and robust solution is not necessarily a solution of the deterministic design space. The principal idea is that the robustness is introduced as an additional objective function, defined to have the dispersion of each original cost function, and during the optimization process, both robustness and original cost functions are evaluated simultaneously. To introduce the uncertainties on the design variables, one uses Monte Carlo (MC) sampling. Since the computational effort required to perform a structural uncertainty analysis with MC simulation is normally very high, the so-called Latin-Hypercube (LHC) method is used instead, with the aim of reducing the number of samples required to evaluate the dispersion on the design variables.

The stochastic optimization is performed by using an evolutionary multiobjective algorithm with sharing technique [9] in order to find the Pareto optimal solutions, known as Non-dominated Sorting Genetic Algorithm (NSGA), originally developed by Srinivas and Deb (1993). The NSGA differs from the classical multiobjective genetic algorithms in the way that a selection operator is used to classify the nondominated individuals for each iteration process. The robustness of the objective functions is defined as follows:

$$f^r = \left(\sigma_f / \mu_f\right) = \left(f^v\right)^{-1} \tag{2}$$

where $f^v$ is the dispersion (vulnerability) of the cost function $f$ ; $\mu_f = E[f]$ and $\sigma_f = E\left[\left(E(f) - f\right)^2\right]$ are, respectively, the mean and the standard deviation, computed by taking into account $n$ samples $(f_i)_{1\leq i\leq n}$ of the cost function $f$ by performing a Monte Carlo simulation.

In the stochastic multiobjective optimization problem, the robustness function $f^r$ associated with a cost function $f$ is introduced as an additional cost function that must be maximized simultaneously with the original cost functions. In this case, the initial multiobjective optimization problem composed by $N$ cost functions is transformed into a stochastic optimization problem with $2N$ objective functions. To evaluate the uncertainties on a design variable $x$, one considers the parametric approach, by assuming that all design variables have normal distribution as illustrated on Fig. 1.

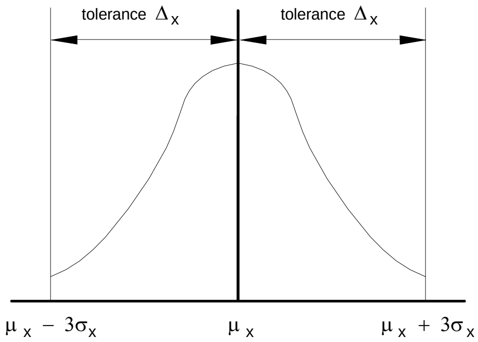



Figure 1. Mean standard deviation and the tolerance of design variables (Lee and Park, 2001).

The probability density function of the normal distribution is expressed as follows:

$$p(x) = \frac{1}{\sigma_x\sqrt{2\pi}} exp\left[\frac{-(x-\mu_x)^2}{2\sigma_x^2}\right] \tag{3}$$

where $\mu_x$ is the mean or nominal value of $x$, and $\sigma_x$ is the standard deviation, in such a way that 99.73% is the probability of having $x$ located in the interval $[\mu_x - 3\sigma, \mu_x + 3\sigma]$. The tolerance $\Delta_x$ is equal to $3\sigma_x$. The proposed methodology is applied with the aim of obtaining the robust optimal solutions of objective functions according to the variations on the design variables expressed in Fig. 1

## 3. SURFACE VISCOELASTIC TREATMENT

One considers in this paper the surface viscoelastic treatment modelled by a three-layer sandwich plate FE, according to the original development made by Khatua and Cheung (1973). Figure 2 illustrates a rectangular plate FE formed by an elastic base-plate (1), a viscoelastic core (2) and an elastic constraining layer (3), whose dimensions are denoted by $a$ and $b$, respectively. The FE model is composed by 4 nodes and 7 degrees-of-freedom per node, representing the nodal longitudinal displacements of the base-plate middle plane in directions $x$ and $y$ (denoted by $u_1$ and $v_1$), the nodal longitudinal displacements of the constraining layer middle plane in directions $x$ and $y$ (denoted by $u_3$ and $v_3$), the transverse displacement, $w$, and the cross-section rotations about axes $x$ and $y$, denoted by $\theta_x$ and $\theta_y$.

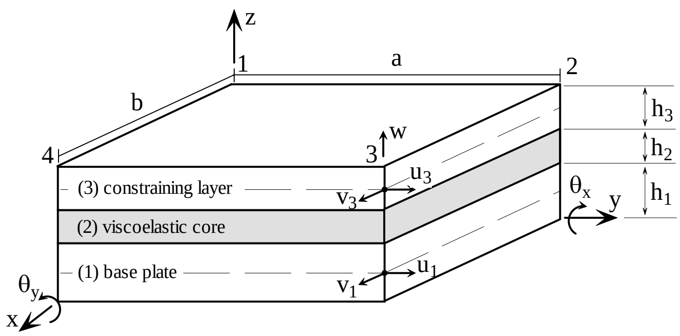



Figure 2. Illustration of the three-layer sandwich plate finite element.

The stiffness matrix of the FE model can be expressed into the following form :

$$\boldsymbol{K}(\omega,T) = \boldsymbol{K_e} + \boldsymbol{K_v}(\omega,T) \tag{3}$$

with:

$$\boldsymbol{K_e} = \boldsymbol{K}^{(1)} + \boldsymbol{K}^{(3)}\,; \quad \boldsymbol{K_v}(\omega,T) = G(\omega,T)\overline{\boldsymbol{K}}^{(2)} \qquad (\,\overline{\boldsymbol{K}}^{(2)}\text{: constant matrix}) \tag{4}$$

where $\boldsymbol{K_e}$ and $\boldsymbol{K_v}(\omega,T)$ are the stuffiness matrices corresponding to the elastic and viscoelastic layers, respectively.

Details of the formulation and the finite element modeling procedure are given by Kathua and Cheung (1973) and Lima et al. (2003).

The viscoelastic damping can be introduced into the viscoelastic matrix, by using a viscoelastic model that takes into account the frequency and temperatures dependence. The frequency and the temperature are recognized as being the most important factors which exerts influence upon the properties of viscoelastic materials. Thus, it becomes important to account for the variations of these parameters in the modeling of structural systems containing viscoelastic materials. This can be done by making use of the so-called *Frequency-Temperature Superposition Principle – FTSP*, which establishes a relation between the effects of the excitation frequency and temperature on the properties of the thermorheologically simple viscoelastic materials (Nashif et al., 1985). This implies that the viscoelastic characteristics at different temperatures can be related to each other by changes (or shifts) in the actual values of the excitation frequency. This leads to the concepts of *shift factor* and *reduced frequency*. Symbolically, the *FTSP* is expressed as:

$$G(\omega,T) = G'(\omega_r,T_0) = G(\alpha_T\omega,T_0)\,; \quad \eta(\omega_r,T_0) = \eta(\alpha_T\omega,T_0) \tag{5}$$

where $\omega_r = \alpha_T(T)\omega$ is the reduced frequency, $\omega$ is the excitation frequency, $\alpha_T$ is the *shift* factor, and $T_0$ is a reference value of temperature. Figure 3 illustrates the *FTSP*, showing that having the modulus and loss factor of a given viscoelastic material for different temperature values, $T_{-1}$, $T_0$, $T_1$, if horizontal shifts along the frequency axis are applied to each of these curves, all of them can be combined into a single one, called master curves.

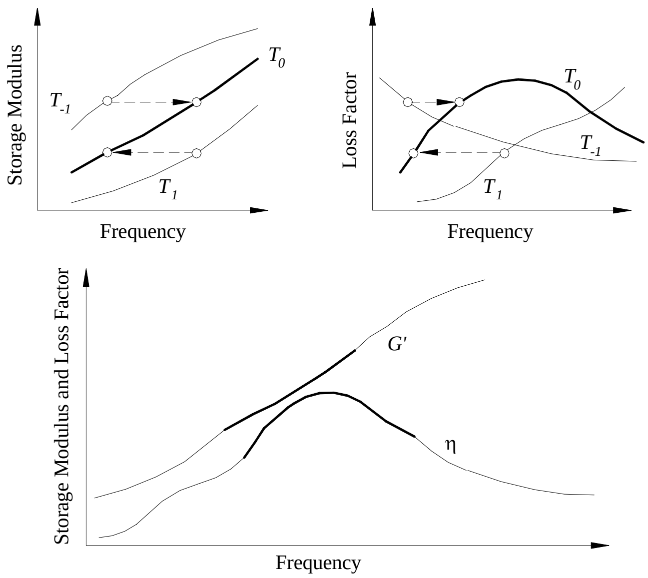



Figure 3 – Illustration of the frequency-temperature superposition principle

The functions $G(\omega_r)$ and $\alpha_T(T)$ can be obtained from experimental tests for specific viscoelastic materials (Nashif et al., 1985). As the result of a comprehensive experimental work, Drake and Soovere (1984) suggest analytical expressions for the complex modulus and shift factor for various commercial viscoelastic materials. The following equations give these functions for the 3M ISD112™ viscoelastic material, which is considered in the numerical application that follows, as provided by those authors:

$$G(\omega_r) = B_1 + B_2\big/\left(1 + iB_5\omega_r/B_3\right)^{-B_6} + \left(i\omega_r/B_3\right)^{-B_4} \tag{5}$$

$$log(\alpha_T) = a(1/T - 1/T_0) + 2.303(2a/T_0 - b)log(T/T_0) + (b/T_0 - a/T_0^2 - S_{AZ})(T - T_0) \tag{6}$$

Figure 4 depicts the standardized curves representing the variations of the storage modulus, loss modulus and loss factor as functions of the reduced frequency, and a plot of the shift factor as a function of temperature for 3M ISD112™.

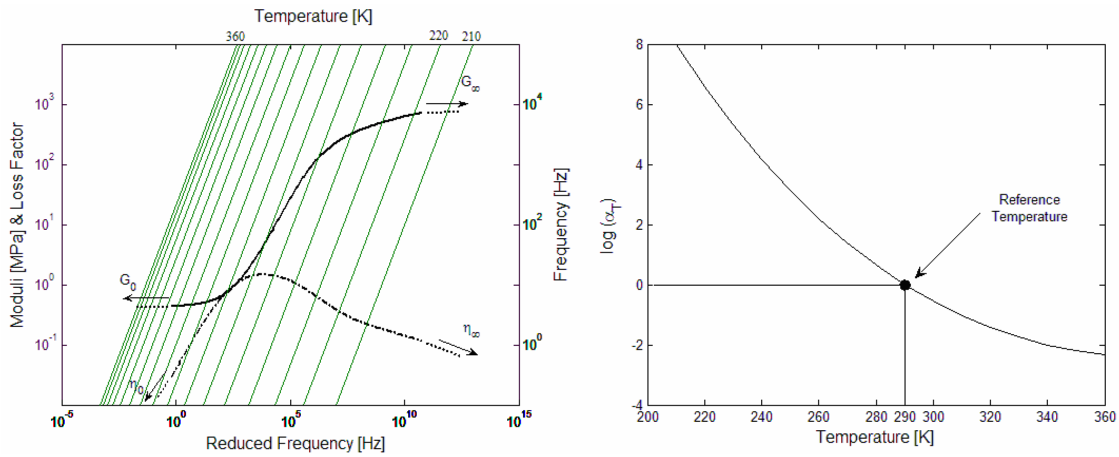



Figure 4 – Master (left) and shift factor curves (right) for the 3M ISD112™.

## 4. NUMERICAL APPLICATION

The following numerical example is presented to illustrate the application of the proposed method to obtain a robust design of a stiffened panel treated with constraining damping layer. Figure 5 depicts the test structure consisting of a free stiffened panel containing 4 stringers, whose geometric dimensions, in millimeters are: internal radius: 938, length: 720, arc length: 680, thicknesses of the panel and the stringers: 1.5 and 0.75, respectively, and height of stringers: 30. The material properties for both panel and stringers are: Young modulus $E=2.1\times10^{11}$ N/m$^2$, mass density $\rho=7800$ Kg/m$^3$ and Poisson's ratio, $\upsilon=0.3$. The FE without viscoelastic treatment is composed by 928 shell finite elements having 6840 d.o.f, and the viscoelastic treatment is composed by 10 viscoelastic patches, each one composed by 16 three-layer sandwich plate FE developed accordingly to the theory previously presented. Material properties for the base-plate and the constraining layer are the same of the stiffened panel, and for the viscoelastic core, one uses the modulus function of the ISD112 3M ($\rho$=950Kg/m$^3$), as shown in Fig. 4. The design parameters and their admissible variations are illustrated on Table 1. These ranges were chosen arbitrarily in such a way to avoid large variations from the nominal values. Only the ranges of continuous variables are taken as constraints.

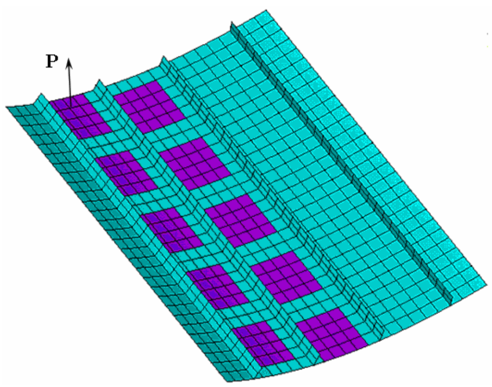



Figure 5. FE model of the stiffened panel treated with PCLD.

Table 1. Design variables and their admissible variations

| Design parameters | Nominal values | Variations | Uncertainties |
|---|---|---|---|
| $h_2$ | 0.0254 mm | ± 60% | $\Delta h_2 = 10\%$ |
| $h_3$ | 0.5 mm | ± 30% | $\Delta h_3 = 10\%$ |
| $T$ | 25ºC | ± 15% | $\Delta T = 5\%$ |

$h_2$ : thickness of the viscoelastic core ; $h_3$ : thickness of the constraining layer; $T$ : temperature of the viscoelastic material

The deterministic optimization problem is composed by two objective functions: the first cost function is the sum of the amplitudes of the FRFs of the damped system corresponding to the natural frequencies of modes 10 (M10) and 11 (M11), respectively, and the second objective function is the total mass of the viscoelastic treatment.

$$minimize: \begin{cases} f_1 = [amplitude(M10) + amplitude(M11)] \\ f_2 = total\ \ mass\ \ of\ \ viscoelastic\ \ treatment \end{cases} \tag{7}$$

The FRFs are computed for an excitation force applied at point P, and the responses acquired at the same point, which is indicated on Fig. 5. The stochastic optimization problem is composed by the defined objective functions (7) and the additional robustness functions associated with each cost function, expressed by equation (8). The interest is to minimize the cost functions and to maximize the robustness functions simultaneously.

$$minimize: \quad \boldsymbol{F}(x) = \left(f_1, f_1^v, f_2, f_2^v\right) \quad where \quad f_1^v = \left(\sigma_1/\mu_1\right)^{-1};\ f_2^v = \left(\sigma_2/\mu_2\right)^{-1} \tag{8}$$

The parameters of NSGA are defined in Table 2.

Table 2. NSGA parameters

| | |
|---|---|
| Probability of selection | 0.25 |
| Probability of crossover | 0.25 |
| Probability of mutation | 0.25 |
| Number of generations | 100 |
| Number of individus/generation | 30 |
| Sharing coefficient ( $\sigma$ ) | 0.2 |

The FRFs of the damped system are computed based on the reduced viscoelastic model, in which the nominal basis of reduction is composed by 115 vectors (60 eigenvectors, 1 vector related to the static residue, and 54 vectors related to the viscoelastic forces). Details of the robust condensation adapted to viscoelastic systems are given in (Lima et al., 2006).

Figure 6 shows the NSGA solutions obtained by applying the proposed robust method, representing the cost functions and their vulnerabilities. In practice, the vulnerability functions consist in minimizing the dispersion around each cost function. By this figure, one can notice that the interval of dispersion for each cost function is:

- From 0.04% to 0.14% for the optimal solutions corresponding to the first cost function $f_1$;
- From approximately 0% to 0.07% for the optimal solutions corresponding to the second cost function $f_2$;

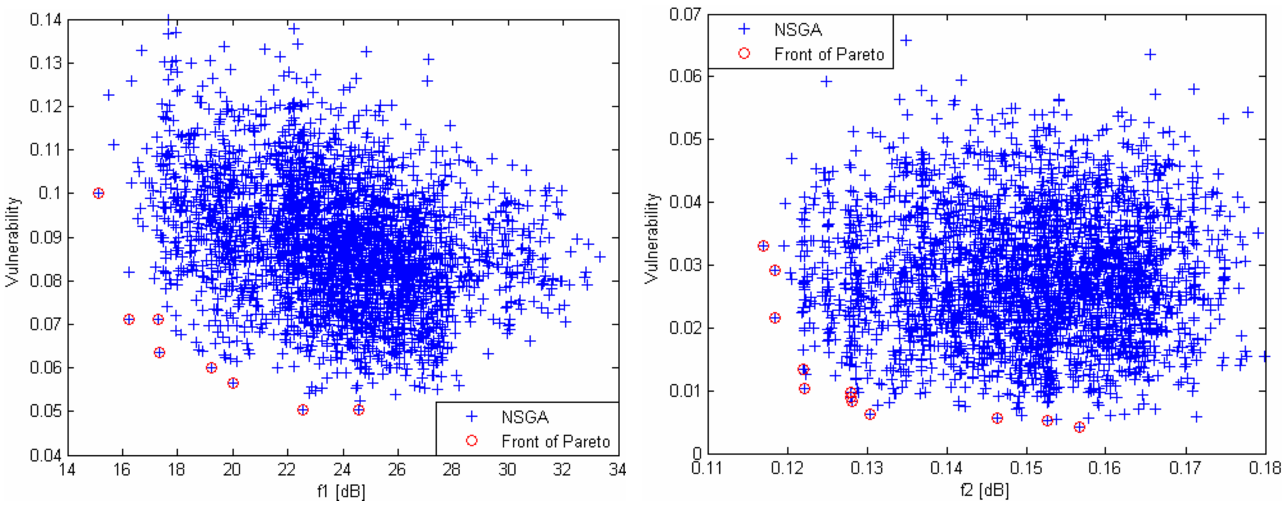



Figure 6 – NSGA solutions and the first front of Pareto for both objective and vulnerability functions

Figure 7 enables to compare the robust solutions and the deterministic solutions. One can conclude that the deterministic solutions exhibit better performance than the robust solutions. Nevertheless, the deterministic solutions are not robust enough with respect to uncertainties on the design parameters.

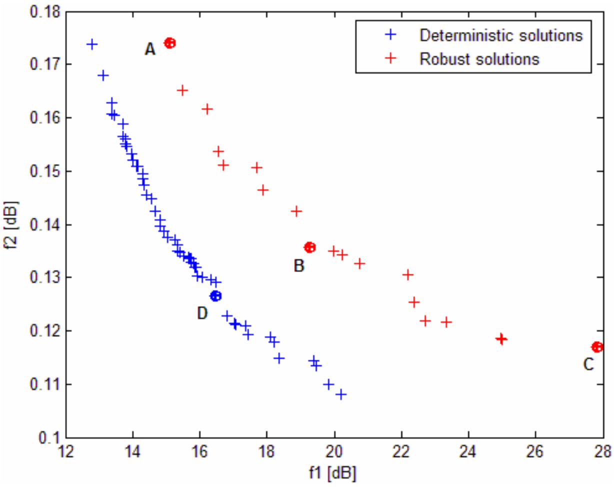
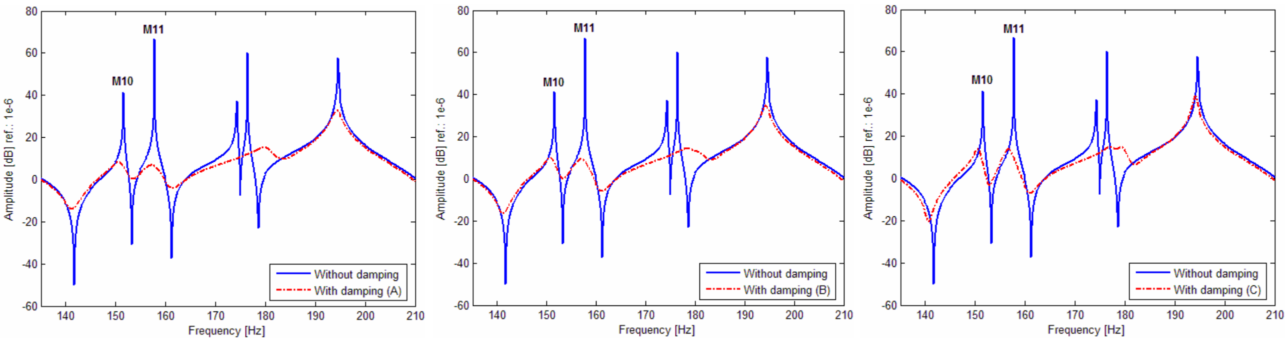



Figure 7 – Comparison between the deterministic solutions and the robust solutions

For the robust optimal solutions corresponding to points **A**, **B** and **C** (indicated in Fig. 7) and listed on Tab. 3, one can compare the amplitudes of the driving point FRFs of system, related to point ***P*** (see Fig. 5) with and without viscoelastic treatment. By comparing Figs. 8(c) and 7 and Tab. 3, one can conclude that point **B** is the best optimal robust solution. In the opposite, point **A**, is the best solution in terms of damping performance, but it is the less robust in the first front of Pareto, as indicated in Tab. 1.




(a) (b) (c)

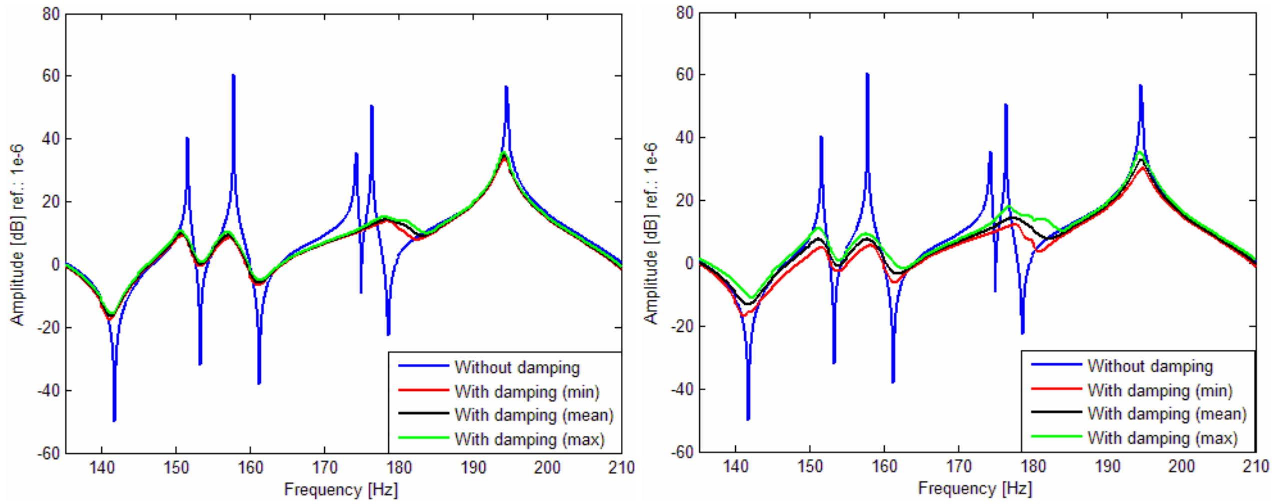
Figure 8 – Amplitudes of FRFs for the robust optimal design solutions **A** (a), **B** (b) and **C** (c)

Table 3 – First front of Pareto for the objective functions (Fig. 7)

| | $f_1$ : Amplitude [dB] | $f_2$ : Mass [Kg] | $f_1^v$ | $f_2^v$ |
|---|---|---|---|---|
| Point A | 15.129 | 0.1739 | 0.1002 | 0.0415 |
| **Point B** | **19.234** | **0.1357** | **0.0738** | **0.0274** |
| Point C | 27.848 | 0.1171 | 0.0768 | 0.0330 |

For the deterministic and robust set of solutions corresponding to points **D** and **B** indicated in Fig. 7, one can check the robustness (in terms of the level of dispersion) of the frequency responses (FRFs) of the viscoelastic system. To introduce uncertainties on the optimal and robust solutions, one uses the Latin Hyper Cube (HCL) simulation, by generating 200 samples for each design variable that will be evaluated to generate the response envelopes. The amount of uncertainties introduced for each design variable is: $\Delta h_3 = 5\%$ for the thickness of the constraining layer; $\Delta T = 2\%$ for the temperature; $\Delta h_2 = 5\%$ for the thickness of the viscoelastic layer.

Figure 9 shows that the robust solutions (Fig. 9.a) are more robust when compared with the deterministic solutions (Fig. 9.b), as demonstrated by the minimal dispersion (vulnerabilities) of the minimal and maximal responses around mean response.



(a) (b)

Figure 9 – Envelopes of dynamic responses for robust optimal solutions (a) and deterministic optimal solutions (b)

## 5 CONCLUSIONS

In this paper a stochastic multiobjective optimization algorithm method which takes into account uncertainties on the design variables was suggested and implemented. The originality of the optimization methodology suggested consists in introducing robustness functions as additional objective functions to be maximized. Each robustness function is associated to an original objective function and is defined to be inversely proportional to the dispersion. Uncertainties on the design variables are introduced directly through a parametric approach, by performing a Monte Carlo simulation.

In the numerical application, an optimization problem involving two objective functions and two robustness functions was considered with the interest oriented towards a relatively complex engineering structure. The choice of the design parameters (thicknesses of the viscoelastic and constraining layers and temperature of viscoelastic material) is based on previous knowledge that these parameters of the multilayer sandwich plate FE are those that influence the most the effectiveness of the viscoelastic damping treatments. This fact can be confirmed by a sensitivity analysis as demonstrated in reference (Lima et al., 2005).

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