ORBIT DETERMINATION USING GPS CONSIDERING DIRECT SOLAR RADIATION PRESSURE MODEL FOR THE TOPEX/POSEIDON SATELLITE

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Abstract. *The main target here is to determine the orbit of an artificial satellite, using signals of the GPS constellation and least squares algorithms through sequential Givens rotations as the method of estimation, with the aim of improving the performance of the orbit estimation process and, at the same time, minimizing the computational procedure cost. Perturbations up to high order geopotential and direct solar radiation pressure were taken into account. It was also considered the position of the GPS antenna on the satellite body that, lately, consists of the influence of the satellite attitude motion in the orbit determination process. An application has been done, using real data from the Topex/Poseidon satellite, whose ephemeris are available at Internet. The best accuracy obtained in position was smaller than 5 meters for short period (2 hours) and smaller than 28 meters for long period (24 hours) orbit determination. In both cases, the perturbations mentioned before were taken into consideration and the analysis occurred without selective availability on the signals measurements.*

Keywords*: Orbit Determination, GPS, Estimation, Least Squares Algorithms, Solar Radiation Pressure*

1. INTRODUCTION

The problem of orbit determination consists essentially of estimating parameters values that completely specify the body trajectory in the space, processing a set of information (measurements) from this body. Such observations can be collected through a tracking network on Earth or through sensors, like the GPS receiver onboard Topex/Poseidon (T/P).

The Global Positioning System (GPS) is a powerful and low cost means to allow computation of orbits for artificial Earth satellites. The T/P satellite is an example of using this system for space positioning.

The orbit determination of artificial satellites is a nonlinear problem in which the disturbing forces are not easily modeled, like geopotential and direct solar radiation pressure. Throughout an onboard GPS receiver is possible to obtain measurements (pseudo-ranges) that can be used to estimate the state of the orbit.

Usually, the iterative improvement of the position parameters of a satellite is carried out using the least squares methods. On a simple way, the least squares estimation algorithms are based on the data equations that describe the linear relation between the residual measurements and the estimation parameters. In this work, the algorithm was implemented through orthogonalization.

2. LEAST SQUARES METHODS

Parameters estimation aim at estimating things that are constant along the estimation process. It is necessary a set of measurements to mathematically shape the relation between these measurements and the parameters or state to be estimated.

One of the most used parameter estimator is the least squares algorithm. Basically, the algorithm minimizes the cost function of the residuals squared (Kuga, 2005). The recursive least squares algorithms, when applied to parameters or state estimation, presents two advantages: avoids matrix inversion in the presence of uncorrelated measurement errors; and needs smaller matrices, which means less need of memory storage.

2.1 Recursive Least Squares Using Sequential Givens Rotations

The Givens rotations are used when it is fundamental to cancel specific elements of a matrix. Alternative formulations were developed, based on the QR factorization methods, to solve this deficiency. Using orthogonal transformations, the equation matrix of data can be transformed on a triangular higher form, to which the least squares solution is obtained by a simple substitution. The aim of applying orthogonal transformations in matrices and vectors on the least squares problem is to substitute the matrices inversion by a stronger method, with less numerical erros. The Givens rotations (Givens, 1958) are a method to solve recursive least squares through orthogonal transformations (Silva, 2001).

The Givens rotations are used when is essential to annul especific elements of a matrix. In this procedure, a given matrix becomes triangular by a series of orthogonal matrices. The full transformation generically can be given by:

$$\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} U_m & U_{m-1} & \ldots & U_3 & U_2 \end{pmatrix} H = Q^T H$$
$$\begin{pmatrix} d \\ r \end{pmatrix} = \begin{pmatrix} U_m & U_{m-1} & \ldots & U_3 & U_2 \end{pmatrix} y = Q^T y \tag{1}$$

where R is triangular. At each step, the orthogonalization of the H matrix is performed (producing a transformed measurement vector d and r) and the results are stored to the next set of measurements. At the end, the final solution is computed. See details in Silva (2001), and Montenbruck and Suarez (1984).

3. DISTURBING EFFECTS CONSIDERED

The main disturbing forces that affect the orbit of an artificial satellite are: the non-uniform distribution of Earth's mass; ocean and terrestrial tides; gravitational attraction of sun and moon; and relativistic effects. There are also the nongravitational effects, such: Earth atmospheric drag; direct and reflected solar radiation pressure; electric drag; electromagnetic effects; and impacts of meteorites.

The disturbing effects are included according to the physical situation presented and to the accuracy that is intended for the orbit determination.

3.1 Perturbations due to Earth Gravitational Field

Earth gravitational field and the attraction force associated with it are studied in the case of an artificial satellite. The geopotential is a force of gravitational origin that disturb the orbits of artificial Earth satellites. Earth gravitational field represents one of the main perturbations on the motion of artificial satellites. The principal term due to Earth oblateness is J2, and the others terms are considered according to the mission accuracy.

Earth is not a perfect sphere with homogeneous mass distribution, and on that account, it cannot be considered as a material point. Such irregularities disturb the orbit of an artificial satellite, since the keplerian elements that describe the orbit do not stay constants. The potential function is given by (Kaula, 1966):

$$U(r,\phi,\lambda) = \frac{\mu}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R_{\mathrm{T}}}{r} \right)^n P_{nm}(\sin\phi)(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) \tag{2}$$

where μ is Earth gravitational constant; R_{T} is Earth radius; r is the spacecraft radial distance; ϕ is the geocentric latitude; λ is the longitude on Earth fixed coordinates system; C_{nm} and S_{nm} are the normalized harmonic spherical coefficients, with n degree and m order; P_{nm} are the normalized Legendre associated functions, with n degree and m order. The constants μ, R_{T}, C_{nm}, and S_{nm} determine a particular gravitational potential.

3.2 Perturbations due to Direct Solar Radiation Pressure

The solar radiation pressure is a force of nongravitational origin that disturbs the translational motion of an artificial satellite. Solar radiation pressure is engendered throughout a continuous flux of photons that stumble at satellite surfaces, which can absorb or reflect such flux. The rate which all incident photons reach the satellite surfaces origins the solar radiation pressure force, what can cause perturbations on the orbital elements.

The way as the perturbations due to solar radiation pressure will affect the keplerian elements depends on the pressure model adopted (if it includes or not shadow, for example). Meanwhile, in the general case, it causes secular and periodic perturbations on the angular variables (Ω, ω, and M), and periodic perturbations on the metric variables (a, e, and i).

The components of solar radiation pressure force can be expressed in several systems. Throughout these systems, the orbital elements of the satellite can be connected with sun's position. This procedure was used here, for the direct solar radiation pressure model adopted for the Topex/Poseidon satellite (Marshall and Luthcke, 1994).

4. DIRECT SOLAR RADIATION PRESSURE MODEL FOR THE TOPEX/POSEIDON SATELLITE

In this Section, it will be described an intrincate direct solar radiation pressure model applied to Topex/Poseidon satellite, for precise computation of such perturbation in the orbital motion.

4.1 Topex GPS Antenna Offset

The force model describes the motion of a satellite's center of mass, but the range measurements are seldom considered at this point. In the case of T/P, they are taken from the location of the center of the antenna. For this reason, it is important the knowledge about satellite attitude motion. Figure 1 shows Topex's antenna in relation to rest of the spacecraft and Tab. 1 gives the antenna offsets with respect to the center of mass (Binning, 1996).

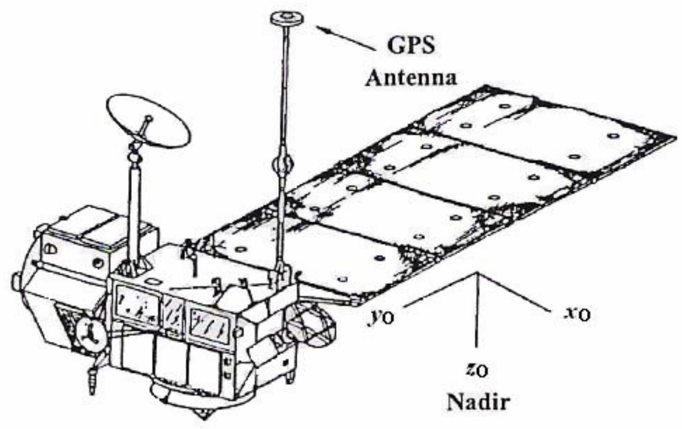

Figure 1. Topex GPS antenna location.

where (x_O, y_O, z_O) is the orbit fixed coordinates system, with its origin in the satellite's center of mass.

In estimating the Topex state there is a complex, predetermined attitude model being applied. This model was created to maneuver the solar array towards the sun for the most sun-facing surface area while still pointing the altimeter in the nadir direction (Antresian and Rosborough, 1992). In basic terms, this model gives rotation angles about the orbit local coordinates to allow for positioning of the antenna with respect to the center of mass. In truth, the recursive least squares algorithm returns position and velocity coordinates in relation to antenna location. Since the update is only a coordinate translation it is instead applied to the center of mass.

Table 1. Attitude information in Topex case study.

Antenna X coordinate	2.104949 m
Antenna Y coordinate	-0.45854 m
Antenna Z coordinate	-4.53263 m
Roll bias	-0.015 deg
Pitch Bias	-0.15 deg

For completeness, it needs to be stated that the attitude model also gives the orientation of the solar array. This orientation, along with the spacecraft position and the sun's position are used to compute the direct solar radiation pressure.

4.2 Marshall and Luthcke's Model Development

Topex spacecraft was launched on an Ariane rocket on August 10, 1992 and had ceased operations on January 5, 2006. It was a dedicated altimetry mission to precisely measure the ocean topography. The spacecraft was in a circular "frozen" orbit at an altitude of 1336 km and an inclination of 66 deg, resulting in a groundtrack that repeats every 10 days.

The commom method for computing the radiation pressure upon orbiting satellites within the orbit determination software had been to ignore rotating, attitude controlled, geometrically complex shapes and to treat the satellite form as a symmetrically perfect and rotationally invariant sphere, or so-called cannonball. The approaches of the cannonball radiation pressure model were not adequate to meet the required 6-cm rms error budget for modeling the radiation forces acting on T/P over a 10-day period. After considerable analysis of all surface force contributions, resultant models to be used in Topex orbit determination were present by Marshall and Luthcke (1994).

The first step in a detail analysis of the radiation forces acting on Topex was to accurately compute all the radiation forces upon T/P with the use of a finite element model of the spacecraft. Since a precise thermal and radiative model of a satellite is necessarily computationally intensive, this micromodel, which served as a “truth” model, was computed offline. A relatively simple and less computationally intensive model, called macromodel, more suitable for precise orbit computations, was devised and tested to emulate the micromodel. One representation of this development is shown in Fig. 2.

This concept is based on approximating the satellite shape with a combination of flat plates. For Topex, a box-wing shape was chosen, with the plates aligned along the satellite body-fixed coordinate system (x_B, y_B, z_B).

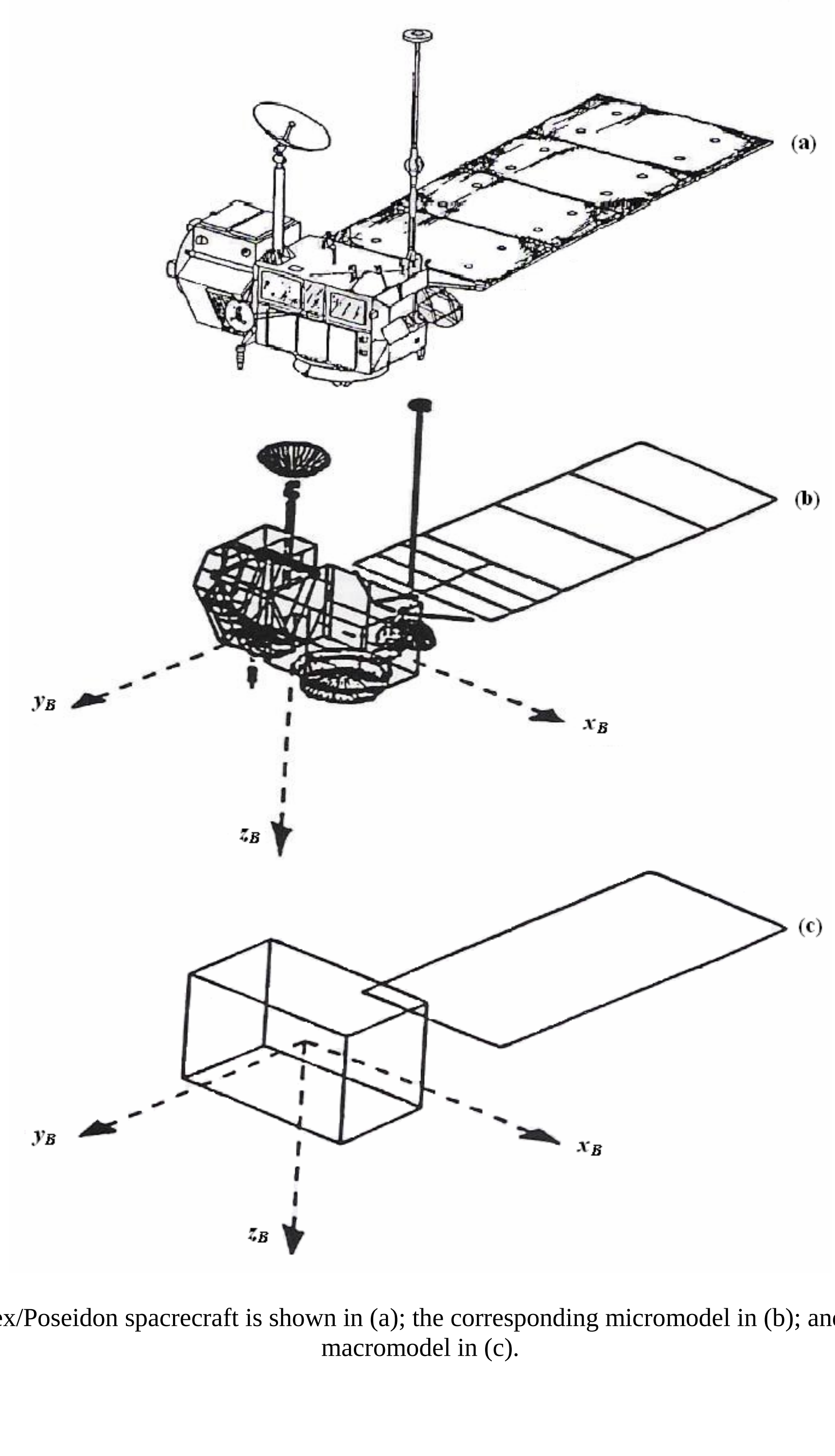

Figure 2. The Topex/Poseidon spacrecraft is shown in (a); the corresponding micromodel in (b); and the corresponding macromodel in (c).

4.3 Radiant Energy of the Sun

The major source of radiante energy which T/P will encounter is the sun. The sun emits a nearly constant amount of photons per unit of time, varying less than 0.2%, that acts on the surfaces of artificial satellites. The force produced by this radiation is by far the largest of the radiative effects (Marshall, et al., 1991). Also, for Topex, it is the largest nongravitational force acting on the satellite. This is the reason why we considered only this parcel of the radiation forces herein.

It was necessary to develop an algorithm for computing the direct solar radiation pressure, following Marshall and Luthcke's model. Routines for computation the force due to direct solar radiation pressure on the satellite's surfaces were developed.

According to Marshall and Luthcke's model, the force acting on each plate is given by (Marshall and Luthcke, 1994):

$$\vec{F} = \frac{GA\cos\theta}{c}\left[2\left(\frac{\delta}{3} + \rho\cos\theta\right)\hat{n} + (1-\rho)\hat{s}\right] \tag{3}$$

where G is solar radiant flux (W/m^2); A is the surface area of each plate (m^2); δ is difusive reflectivity, percentage of the total incoming radiation; ρ is specular reflectivity, percentage of the total incoming radiation; $\hat{n}$ is surface normal vector; $\hat{s}$ is source incidence vector; θ is the angle between surface normal and solar incidence; and c is the speed of light (m/s).

There are 8 plates in the model developed for Topex, according to the box-wing shape chosen. So, it is necessary to compute independently the direct solar radiation force acting on each surface. All plate interaction effects, such as shadowing, reflection, and conduction are ignored. This yields vector forces which are summed to compute the total effect on the spacecraft's center of mass. Mathematically, Eq. (4) shows it and Tab. 2 and 3 gives specific informations about each of the surfaces, which are necessary to compute the total effect of direct solar radiation force on T/P.

$$\vec{F}_k = \frac{GA_k\cos\theta_k}{c}\left[2\left(\frac{\delta_k}{3} + \rho\cos\theta_k\right)\hat{n}_k + (1-\rho_k)\hat{s}\right] \quad \Rightarrow \quad \vec{F} = \sum_{k=1}^{8}\vec{F}_k \tag{4}$$

where subscript k varies from 1 to 8, representing each plate, and $\vec{F}$ is the total direct solar radiation force acting on the satellite.

Table 2. Macromodel plate normal vectors in the satellite body-fixed system.

	X+	**X-**	**Y+**	**Y-**	**Z+**	**Z-**	**SA+**	**SA-**
Area [m**2]	3.74	3.77	8.27	8.07	8.67	8.44	21.4	21.44
Specular ref.	0.201	0.244	0.886	0.782	0.239	0.275	0.05	0.17
Diffuse ref.	0.375	0.386	0.302	0.339	0.390	0.363	0.22	0.66
Emissivity	0.769	0.995	0.876	0.714	0.770	0.746	0.87	0.88

Table 3. Plates characteristics for direct solar radiation pressure model.

Plate	x_B	y_B	z_B
X+	1.0	0.0	0.0
X-	-1.0	0.0	0.0
Y+	0.0	1.0	0.0
Y-	0.0	-1.0	0.0
Z+	0.0	0.0	1.0
Z-	0.0	0.0	-1.0
SA+	1.0	0.0	0.0
SA-	-1.0	0.0	0.0

5. RESULTS

Here, the tests and analysis from the algorithm developed to compute direct solar radiation pressure are presented. On the analysis of direct solar radiation pressure is already included Topex's GPS antenna location that, lately, consists of the influence of the satellite attitude motion in the orbit determination process. The algorithm were implemented through FORTRAN 77 language (Pardal, 2007).

To validate and to analyze the purposed method, real data from the T/P satellite were used. Position and velocity to be estimated were compared with Topex's precise orbit ephemeris (POE), from JPL/NASA. The test conditions considered pseudo-range real data, collected by GPS receiver onboard Topex, on November 18, 1993. The tests ocurred at the same day, for a short period (2 hours) and a long period (24 hours) of orbit determination.

The force model included perturbations due to high order geopotential (50 × 50), with harmonic coefficients from JGM-2 model, and due to direct solat radiation pressure. The measurements model considered ionospheric correction, although the accuracy on position and velocity is not meaningful (Chiaradia et al., 2000).

The obtained data were evaluated through one parameter: error in position. Such parameter is given by:

$$\Delta \vec{r} \equiv \begin{bmatrix} x - \hat{x} \\ y - \hat{y} \\ z - \hat{z} \end{bmatrix} \qquad (5)$$

which are after translated to radial, normal, and tranverse components of orbit fixed system. In Eq. (5), x_i and $\hat{x}_i$ are the position reference and estimated components, respectively, in the orbit fixed reference frame.

First, only geopotential were considered for the mentioned periods of orbit determination. After, the direct solar radiation pressure force acting on Topex's center of mass and the way such force acts on satellite orbit determination were analyzed. This analysis already includes Topex's GPS antenna location, which is one of the steps to determine the direct solar radiation pressure effects.

As said before, the obtained data were after translated to RTN (radial, transverse, and normal) system. In this system, "R" points to the nadir direction, "N" is perpendicular to orbital plane, and "T" is orthogonal to "R" and "N", and is also the velocity component. Thus, it is possible to analyze what happens with the orbital components, and with the orbit evolution too. This is better than analyze from an Earth referential, where is more difficult to see the phyical situation.

Figure 3 shows the behavior of the error in position, in meters, along time, in seconds, considering only geopotential, and Fig. 4 considers geopotential and direct solar radiation pressure effects, shown in two different curves. In the legends of Fig. 3 and 4, "R" means radial component; "N" normal component; and "T" transverse component of orbit fixed system. The subscript "geo" means perturbations due to geopotential only, and subscript "prs", perturbations due to geopotential and direct solar radiation pressure.

Table 4 shows the maximum and minimum amplitudes of the curves from Fig. 3 and 4, for short period (2 hours) and long period (24 hours) of orbit determination.

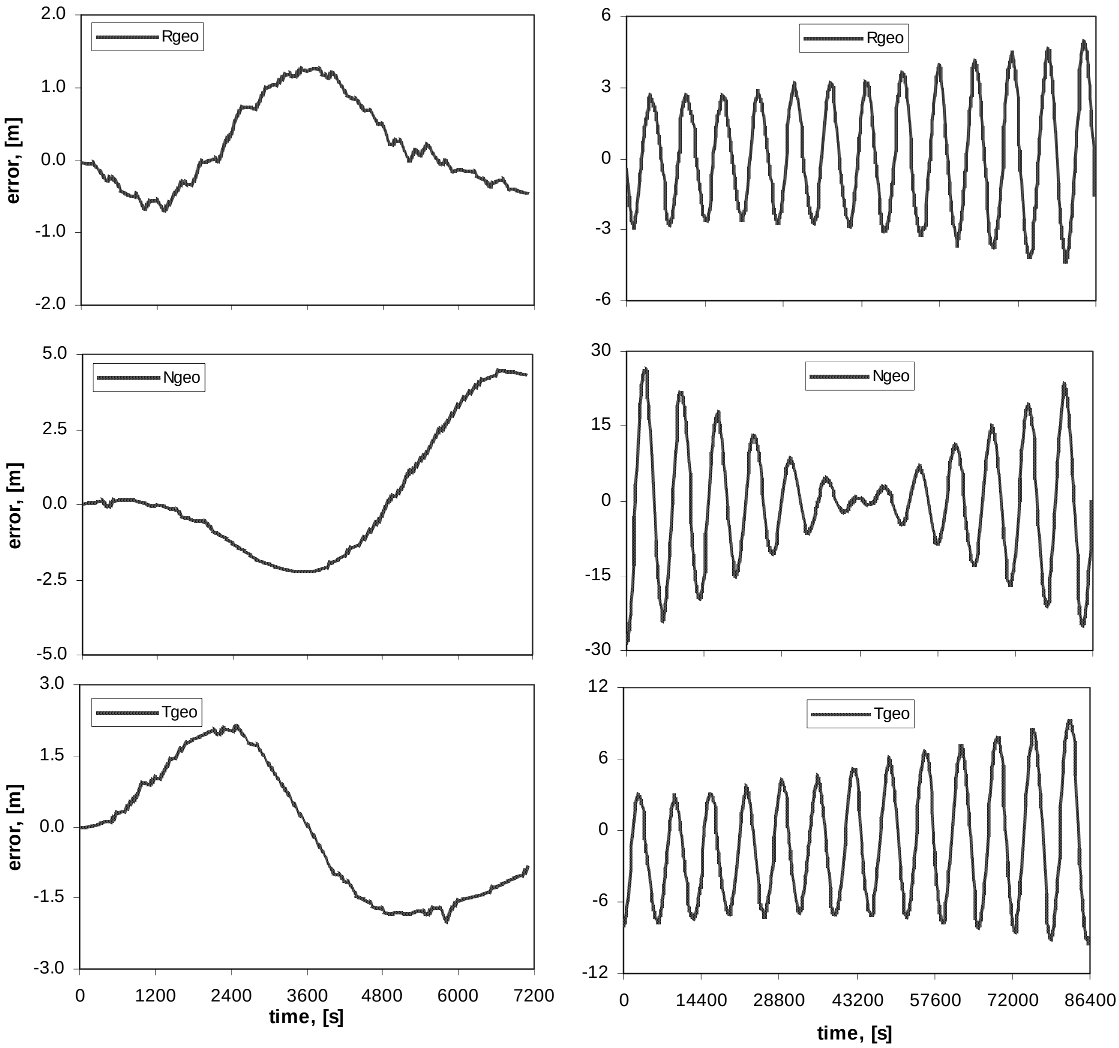

Figure 3. Errors in position, given in RNT coordinates, for 2 hours (left side) and 24 hours (right side), considering perturbations due to geopotential.

Figure 3, specially the long period graphics (right side graphics), shows a sinusoidal behavior of the errors in RTN coordinates, with a period near once per revolution of the satellite orbit (around 2 hours). Following verification of all know dynamic models, there may exist a residual signature in the orbit as a result of unmodeled accelerations, which come in many forms (Soyka and Davis, 2001). In the case of geopotential, the acceleration is due to truncation of geopotential field. The used model is JGM-2 50×50, while the full model is 70×70, which is computationally intensive and may cause numerical problems, due to the order of the some terms (10^{-127}), that it was not suitable in this work.

The same sinusoidal behavior appears in long period graphics of Fig. 4 (right side graphics), for the curves corresponding to geopotential and direct solar radiation pressure. It is also result of unmodeled accelerations, but solar radiation pressure is responsible for another acceleration, caused by limitations in modeling solar pressure as a function of the satellite attitude, surface properties and space environment. So, in Fig. 4, there is one unmodeled acceleration to each disturbing effects, in the shown curves.

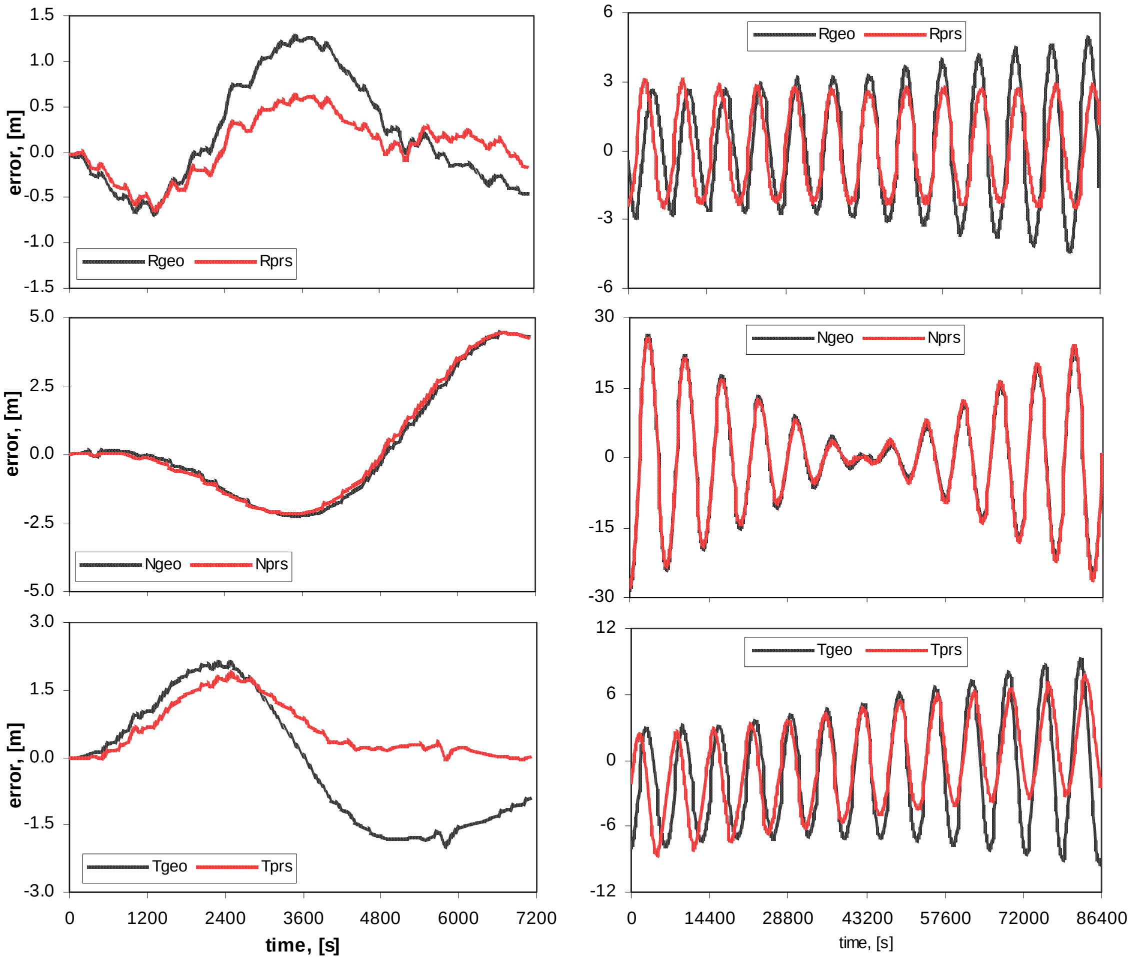

Figure 4. Errors in position, given in RNT coordinates, for 2 hours (left side) and 24 hours (right side), considering perturbations due to geopotential and direct solar radiation pressure.

According to Tab. 4, it is possible to see that the minor amplitude variation occurs in radial component, and the higher, in normal component, such for short and long periods of orbit determination. Table 4 also confirms the graphics information: solar radiation pressure has more meaningful effects in transverse component, and less in normal one. It was expected, because solar radiation pressure acts specially on alongtrack compenents, which are here represented by transverse component, the one associated with satellite's velocity.

Table 4. Maximum and minimum values of the errors in position, for the two cases of test, on 11/18/1993.

Error (m)		2 hours			24 hours		
	value	**R**	**N**	**T**	**R**	**N**	**T**
geo	*Maximun*	1.48	4.44	2.19	5.70	26.26	9.27
	Minimum	-0.75	-2.28	-2.09	-4.67	-28.63	-12.43
prs	*Maximum*	0.84	4.49	1.89	4.90	25.60	7.70
	Minimum	-0.71	-2.21	-0.16	-2.69	-27.92	-8.70

As Tab. 4 shows, for the short period, solar radiation pressure decreases up to 43 % the radial component value and up to 16 % the transverse one. For the long period, solar radiation pressure reduces up to 42 % the radial component value and up to 30 % the transverse one. In both cases, such perturbation does not act favorably on normal component.

6. CONCLUSIONS

The principal aim here was to determine the orbit of an artificial satellite, using signals of the GPS constellation and least squares algorithms using sequential Givens rotations as the method of estimation. The analysis period covered a short period (near once Topex's period) and a long period (24 hours) of orbit determination. Pseudo-range measurements were corrected from ionospheric effects, although the accuracy on orbit determination is not expresssive. Real time requirements were not present, meantime, it was appropriate to keep low computational cost, with accuracy enough to satellite positioning at 10 meters level for one day.

The results were compared with real data from Topex's POE/JPL (Precision Orbit Ephemeris/Jet Propulsion Laboratory), available at Internet. For short period orbit determination, the magnitude of error in position varied from 4.6 m to 4.2 m, and for long period, the magnitude varied from 29.3 m to 27.8 m, according to the model's complexness increase. As the numbers show, the model that includes direct solar radiation pressure decreases at most around 5 % the precision in position. It happens because of the appearance of residual unmodeled accelerations due both perturbations. Geopotential and direct solar radiation pressure were taken into consideration and the analysis occurred without selective availability on the signals measurements.

Throughout the results, it was found that least squares method through sequential Givens rotations and positioning using GPS showed trustfulness and accuracy enough for artificial satellites orbit determination.

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