ON THE OBERBECK-BOUSSINESQ APPROXIMATION IN A ROTATING FRAME WITH DISSIPATIVE HEATING

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Abstract. *In this paper we obtain the Oberbeck-Boussinesq approximation in a rotating frame, including the dissipative heating effect. Basic for this derivation are the mechanical incompressibility constraint and a perturbation technique, in which a measure of compressibility is chosen as the small parameter* (ε). *We arrive at the Oberbeck-Boussinesq approximate equations by considering the zeroth order perturbations of the continuity and heat equations, along with the third order perturbation of the momentum equation. We note that the usual Oberbeck-Boussinesq equations in a inertial frame appears like a particular case in our derivation.*

Keywords: *Oberbeck-Boussinesq Approximation, mechanical incompressibility constraint, rotating frame, dissipative heating*

1. INTRODUCTION

The problem of natural or free convection appears in many important areas of knowledge. This problem is often approached via an approximation for Navier-Stokes-Fourier equations known as the Oberbeck-Boussinesq approximation (OBA) after the pioneering contribution of Oberbeck (1879) and Boussinesq (1903). Boussinesq (1903) simplified the governing equations for a atmospheric compressible flow, subject to gravitational and thermal expansion, mainly by considering order-of-magnitude argues. He noted, for example, that the density fluctuations are perceptible mainly due to the thermal effects. The approximated governing equations represents a quasi-incompressible system of equation. Many works have been made to provide justifications or the applicability of this approximation.

Spiegel and Veronis (1960) provides a dynamical study for the thermal convection, in which is used a order-of-magnitude argues too. Mihaljan (1962) utilized a expansion of the dimensionless in term of four numbers obtained by the Buckingham-Pi theorem, and discusses the role of the OBA energetics. Gray and Giorgini (1976) present a deduction of the range of validity of the Boussinesq approximation for liquids and gases using a first-order perturbation method for the physical properties of the fluid in terms of temperature and pressure. It is important to point out that in this work, it is used also a scale argue ad hoc. Recently, Ramos and Vargas (2005) extended the last work by considering a rotating frame. Rajagopal, Ružička and Srinivasa (1996) provided a new procedure for justification of the OberbeckŰBoussinesq by using the constraint of mechanical incompressibility along with a perturbation technique. The procedure extend by Kagei, Ružička and Thäter (2000), Passerini and Thäter (2005) in order to include dissipative heating and non-Newtonian fluids, respectively.

Below, we present the OBA in the context of geophysical flows (see Hutter and Jöhnk 2004), written as

$$
\begin{aligned}
\mathrm{div}\mathbf{v} &= 0,\\
\dot{\mathbf{v}} &= -\frac{1}{\rho_R}\mathrm{grad}(p-p_R)+\nu\triangle\mathbf{v}+\alpha(\theta-\theta_R)\mathbf{g}^*-2\omega\times\mathbf{v},\\
\dot{\theta} &= \kappa\triangle\theta,
\end{aligned}
\tag{1}
$$

where: $\mathbf{v}$, θ and p are the velocity, temperature and pressure fields; ρ_R, θ_R and p_R are, respectively, reference values for the mass density, temperature and pressure; $\mathbf{g}^*$ is the gravity acceleration, defined by the sum of gravitational and centrifugal acceleration: $\mathbf{g}^* := \mathbf{g}-\omega\times(\omega\times\mathbf{r})$; $2\omega\times\mathbf{v}$ is the Coriolis specific force; with ω the angular speed of rotation vector and $\mathbf{r}$ is position vector with origin in the rotate axes; the constants ν, α and κ are the cinematic viscosity and thermal expansion and diffusivity coefficients, respectively.

The role of the viscous dissipation in the natural convection would be investigated, because this effect can be appreciable when the induced kinetic energy becomes appreciable compared to the amount of heat transferred. This occurs when either the equivalent body force is large or when the convection region is extensive (Gebhart 1962). Another important feature of dissipative heating occurs when we analyze the entropy inequality, because with the usual Boussinesq equations result in an unbalanced irreversibility budget (Pons and le Quéré 2004, Pons and le Quéré 2005a,b). In these works the thermodynamic analysis are made by shows that these equations represent a system that exchanges with the surroundings, not only two heat fluxes, but also two fluxes of mechanical energy: an input, that generates the fluid motion, and an output, due to viscous term. The thermodynamic inconsistencies appearing in the usual Boussinesq calculations are solved when the heat equation includes the work of pressure forces, leading to the thermodynamic Boussinesq equations.

Here, our main concern is the description of natural convection in a rotating frame of reference, having in mind the problem of atmospheric convection (Holton 1992, Emanuel 1994). We present a justification for the Oberbeck-Boussinesq approximation adding the rotational and dissipative heating effects for a thermally convective atmosphere. The main features of this derivation rely on the use of the mechanical incompressibility constraint, and a perturbation technique that employs a small parameter defined as a measure of compressibility, following the methodology of Rajagopal, Ružička and Srinivasa (1996) and sequently Kagei, Ružička and Thäter (2000).

2. FORMULATION

In this Section we present the governing equations that will be starting point for the derivation of the OBA in a rotating frame with dissipative heating.

Following the framework of continuum mechanics, we first introduce the basic laws, namely, the mass conservation, the momentum and energy balances and the Clausius-Duhem inequality, along with a thermodynamically compatible constitutive theory, which includes the mechanical incompressibility constraint. Finally, the governing equations for the velocity, temperature and pressure fields are obtained by combining the aforementioned basic laws and constitutive theory.

2.1 Basic laws

The local statements of the basic laws expressing the mass conservation, the momentum and energy balances and the Clausius-Duhem inequality are, respectively, given by

$$\begin{aligned} \dot{\rho} + \rho \mathrm{div}\mathbf{v} &= 0, \\ \rho\dot{\mathbf{v}} &= \mathrm{div}\mathbf{T} + \mathbf{b}, \\ \rho\dot{e} &= \mathbf{T}.\mathbf{L} - \mathrm{div}\mathbf{q} + \rho r, \\ \rho(\dot{\varphi} + \dot{\theta}\eta) &\geq \mathbf{T}.\mathbf{L} + \frac{\mathbf{q}\cdot\mathbf{g}_\theta}{\theta}. \end{aligned} \tag{2}$$

In the above equations, ρ, e, φ and η are the mass, internal energy, free energy and entropy densities, $\mathbf{T}$ is the symmetric Cauchy stress tensor, $\mathbf{b}$ is the body force density, $\mathbf{q}$ (r) is the heat flux $(supply)$, $\mathbf{L}$ $(\mathbf{g}_\theta)$ is the velocity $(temperature)$ gradient. In this work we consider that the body force density is given by

$$\mathbf{b} = \rho\mathbf{g} - \rho\omega \times (\omega \times \mathbf{r}) - 2\rho\omega \times \mathbf{v}. \tag{3}$$

2.2 The constitutive equations

Our first constitutive assumption is concerned with a particular class of generalized incompressible fluids (Hills and Roberts 1991, Rajagopal, Ružička and Srinivasa 1996, Kagei, Ružička and Thäter 2000, Passerini and Thäter, 2005) defined by the mechanical incompressibility constraint:

$$\mathrm{tr}\mathbf{L} = \mathrm{div}\mathbf{v} = \alpha\dot{\theta}, \tag{4}$$

where the material parameter α is the volumetric expansion coefficient. From the above equation, it follows that a fluid constrained by (4) is incompressible in isothermal processes and compressible otherwise.

From (4) and $(2)_4$ it follows that

$$\rho\dot{\varphi} + \pi\dot{\theta} \geq -\mathbf{T}^d.\mathbf{D}^d + \frac{\mathbf{q}\cdot\mathbf{g}_\theta}{\theta}, \tag{5}$$

where $\pi := \rho\eta - \frac{1}{3}p\alpha$, $p := \frac{1}{3}(\mathrm{tr}\mathbf{T})$ and $\mathbf{T}^d$ $(\mathbf{D}^d)$ is the deviatoric part of $\mathbf{T}$ $(\mathbf{D})$.

The inequality (5) suggests that response functions for φ, π, $\mathbf{T}^d$ and $\mathbf{q}$ should be provided. Therefore, we assume response functions of the kind

$$\sigma = \hat{\sigma}(\theta, \mathbf{D}^d, \mathbf{g}_\theta). \tag{6}$$

We remark that (6) defines also an incompressible fluid with viscosity and heat conduction when $\mathbf{D}^d = \mathbf{D}$.

By using the Coleman-Noll procedure we arrive at the following restrictions on the responses functions:

i) The free energy depends only on the temperature, i.e.,

$$\varphi = \hat{\varphi}(\theta); \tag{7}$$

ii) The response function for π is obtained from the free energy response:

$$\hat{\pi} = -\rho\frac{\partial\varphi}{\partial\theta}; \tag{8}$$

iii) The response functions $\hat{\mathbf{T}}^d$ and $\hat{\mathbf{q}}$ must comply with the reduced dissipation inequality

$$\hat{\mathbf{T}}^d.\mathbf{D}^d - \frac{\hat{\mathbf{q}}.\mathbf{g}_\theta}{\theta} \geq 0, \tag{9}$$

and

$$\hat{\mathbf{T}}^d(\theta, 0, 0) = 0, \quad \hat{\mathbf{q}}(\theta, 0, 0) = 0. \tag{10}$$

From now on, we assume that the response functions $\mathbf{q}$ and $\mathbf{T}^d$ are linear in $\mathbf{g}_\theta$ and $\mathbf{D}^d$, respectively, as in the case of an incompressible Navier-Stokes-Fourier fluid, i.e.,

$$\begin{aligned} \mathbf{q} &= K(\theta)\mathbf{g}_\theta, \\ \mathbf{T}^d &= 2\mu(\theta)\mathbf{D}^d, \end{aligned} \tag{11}$$

where the thermal conductivity K and the viscosity μ are non-negative valued functions.

2.3 Governing Equations

To obtain the governing equations we proceed as follows. First, we rewrite the stress tensor as

$$\mathbf{T} = -p\mathbf{I} + 2\mu(\theta)\mathbf{D} - \frac{2}{3}\mu(\mathrm{tr}\mathbf{D})\mathbf{I}, \tag{12}$$

after taking into account the definitions of p and $\mathbf{D}^d$ as well as the constitutive equation $(11)_2$. Next, we substitute eqs. (3), (7)-(10), $(11)_1$ and (12) in eq. (2) to arrive at the governing equations:

$$\mathrm{div}\mathbf{v} = \alpha\dot{\theta}, \tag{13}$$

$$\dot{\rho} + \rho\mathrm{div}\mathbf{v} = 0, \tag{14}$$

$$\rho\dot{\mathbf{v}} = -\mathrm{grad}p + 2\mu[\mathrm{div}\mathbf{D} - \frac{1}{3}\mathrm{grad}(\mathrm{div}\mathbf{v})] + \rho\mathbf{g} - \rho\omega \times (\omega \times \mathbf{r}) - 2\rho\omega \times \mathbf{v}, \tag{15}$$

$$[C(\theta) - \theta\alpha^2 p]\dot{\theta} - \dot{p}\alpha\theta = K\mathrm{div}\mathbf{g}_\theta + 2\mu\left[\|\mathbf{D}\|^2 - \frac{1}{3}|\mathrm{tr}\mathbf{D}|^2\right], \tag{16}$$

where $C(\theta) := -\rho\theta\partial_{\theta\theta}\varphi$ is the specific heat.

By inserting eq. (13) into eq.(14), the final equation can be integrated to give

$$\rho = \rho^0 \exp[-\alpha(\theta - \theta^0)], \tag{17}$$

where ρ^0 and θ^0 are reference values and α is assumed constant.

3. SCALING

It is considered a fluid layer with depth L heated from below, whose temperature difference between the base (θ_B) and the top (θ_T), $\Theta := \theta_B - \theta_T$, is maintained constant. With these choices, the dimensionless quantities, which are denoted by using bars overwritten, are given by

$$\begin{gathered} \bar{t} = \frac{t}{t_R}, \bar{\mathbf{r}} = \frac{\mathbf{r}}{r_R}, \overline{\mathbf{v}} = \frac{\mathbf{v}}{v_R}, \\ \overline{\theta} = \frac{1}{\theta_R}(\theta - \Theta_M), \Theta_M := \frac{1}{2}(\theta_B + \theta_T), \\ \overline{p} = \frac{p}{p_R}, \overline{\rho} = \frac{\rho}{\rho_R}, \overline{C} = \frac{C}{C_R}, \\ \overline{\mathbf{g}} = \frac{\mathbf{g}}{g_R}, \overline{\omega} = \frac{\omega}{\omega_R}, \end{gathered} \tag{18}$$

The dimensionless governing equations are written as

$$\frac{v_R}{L}\overline{\mathrm{div}}\overline{\mathbf{v}} = \alpha\Theta\left[\frac{1}{t_R}\partial_{\bar{t}}\overline{\theta} + \frac{v_R}{L}\overline{grad\theta}.\overline{\mathbf{v}}\right], \tag{19}$$

$$\frac{v_R}{t_R}\overline{\rho}\partial_{\overline{t}}\overline{\mathbf{v}}+\frac{v_R^2}{L}\overline{\rho}(\overline{grad}\overline{\mathbf{v}})\overline{\mathbf{v}}=-\frac{p_R}{\rho_R L}\overline{grad}\overline{p}$$
$$+g_R\overline{\rho}\overline{\mathbf{g}}+2\frac{\nu v_R}{L^2}\left[\overline{div}\mathbf{D}-\frac{1}{3}\overline{grad}(\overline{div}\overline{\mathbf{v}})\right]$$
$$-2\omega_R v_R\overline{\rho}\overline{\omega}\times\overline{\mathbf{v}}-\omega_R^2 L\overline{\rho}[\overline{\omega}\times(\overline{\omega}\times\overline{\mathbf{r}})], \tag{20}$$

$$\left[\overline{C}-\frac{(\alpha\Theta)^2 p_R}{C_R\Theta}\left(\overline{\theta}+\frac{\Theta_M}{\Theta}\right)\overline{p}\right]\left[\frac{1}{t_R}\partial_{\overline{t}}\overline{\theta}+\frac{v_R}{L}\overline{grad}\overline{\theta}.\overline{\mathbf{v}}\right]$$
$$-\frac{\alpha p_R}{C_R}\left(\overline{\theta}+\frac{\Theta_M}{\Theta}\right)\left[\frac{1}{t_R}\partial_{\overline{t}}\overline{p}+\frac{v_R}{L}\overline{grad}\overline{p}.\overline{\mathbf{v}}\right]$$
$$=\frac{K}{L^2 C_R}\overline{div}(\overline{grad}\overline{\theta})+2\frac{\mu}{C_R\Theta}\frac{v_R^2}{L^2}\left[\|\overline{\mathbf{D}}^2\|-\frac{1}{3}|(\mathrm{tr}\overline{\mathbf{D}})^2|\right]. \tag{21}$$

The choice of characteristic values is very important to describe correctly a problem. In the case of atmospheric convection, the convenient chooses are $r_R := L$, $g_R := |\mathbf{g}|$, $p_R := \rho_R g_R L$, $\theta_R := \Theta$, C_R and ρ_R are characteristic values for the specific heat and density. The characteristic scale of velocity is chosen as $v_R = (\alpha|\mathbf{g}|\theta_R L)^{\frac{1}{2}}$, that represents a velocity when the buoyancy has a same order of inertial term (see Chandrasekhar, 1961). The characteristic time (t_R) and angular velocity (ω_R) scales will be open at now. Substituting the above characteristic scales, and we can rewritten eqs. (19)-(21) as

$$\overline{div}\overline{\mathbf{v}}=(\alpha\Theta)\left[St\partial_{\overline{t}}\overline{\theta}+\overline{grad}\overline{\theta}.\overline{\mathbf{v}}\right], \tag{22}$$

$$(\alpha\Theta)\overline{\rho}\left[St\partial_{\overline{t}}\overline{\mathbf{v}}+(\overline{grad}\overline{\mathbf{v}})\overline{\mathbf{v}}\right]=\left[-\overline{grad}\overline{p}+\overline{\rho}\overline{\mathbf{g}}\right]$$
$$+\frac{(\alpha\Theta)}{(Gr)^{\frac{1}{2}}}\left[\overline{div}\mathbf{D}-\frac{1}{3}\overline{grad}(\overline{div}\overline{\mathbf{v}})\right]+(\alpha\Theta)\frac{\overline{\rho}}{Ro}\left[-2\overline{\omega}\times\overline{\mathbf{v}}+\frac{1}{Ro}(\overline{\omega}\times(\overline{\omega}\times\overline{\mathbf{r}})\right], \tag{23}$$

$$\left[\overline{C}-(\alpha\Theta)Di\left(\overline{\theta}+\frac{\Theta_M}{\Theta}\right)\overline{p}\right]\left[St\partial_{\overline{t}}\overline{\theta}+\overline{grad}\overline{\theta}.\overline{\mathbf{v}}\right]-Di\left(\overline{\theta}+\frac{\Theta_M}{\Theta}\right)$$
$$\left[St\partial_{\overline{t}}\overline{p}+\overline{grad}\overline{p}.\overline{\mathbf{v}}\right]=\frac{1}{PrGr^{\frac{1}{2}}}\overline{\mathrm{div}}\overline{\mathbf{g}}_\theta+2\frac{Di}{Gr^{\frac{1}{2}}}\left[\|\overline{\mathbf{D}}^2\|-\frac{1}{3}|(\mathrm{tr}\overline{\mathbf{D}})^2|\right], \tag{24}$$

where was adopted the dimensionless numbers: Strouhal (St), Rossby (Ro), Grashof (Gr), Dissipation (Di) and Prandtl number (Pr) defined by

$$St:=\frac{L}{v_R t_R},\quad Ro:=\frac{v_R}{\omega_R L}$$
$$Gr:=\frac{\alpha\Theta|\mathbf{g}|L^3}{\nu^2},\quad Di:=\frac{\alpha|\mathbf{g}|L\rho_R}{C_R},\quad Pr:=\frac{C_R\nu}{K}. \tag{25}$$

The above numbers relates, respectively, the time with the length and the velocity of the fluid; is a measures the relation of body forces to viscous forces; the product of thermal expansion coefficient, acceleration due to the applied body force and length scale to the specific heat at constant pressure; the relation between viscous and inertial forces; and finally a dimensionless number that relating the ratio of inertial to Coriolis forces for a given flow of a rotating fluid, respectively. We observe that $PrGr^{\frac{1}{2}}$ is a ratio of the conduction and the convection energy, and $Gr^{\frac{1}{2}}=\frac{Lv_R}{\nu}:=Re$ represents the Reynolds number.

4. APPROXIMATE EQUATIONS

The starting point for the justification of the Oberbeck-Boussinesq Approximation is the dimensionless governing equations in which their dependent variables are expanded asymptotically in in terms of the small parameter associated to the perturbation technique. Then, the OBA will be obtained collecting some perturbation orders of the governing equations.

4.1 Asymptotic Expansions

For the development of this Subsection, we first assume the following assumptions, as in Kagei, Ružička and Thäter (2000):

i) A measure of compressibility

$$\varepsilon^3 := \alpha\Theta \ll 1, \tag{26}$$

is chosen as the small parameter;

ii) The non-dimensional numbers are are supposed to be the order one compared to the ε,i.e.,

$$St \sim 1; \quad Ro \sim 1; \quad Gr \sim 1; \quad Di \sim 1.$$

The last assumption is required to retain on the one hand both inertial and viscous effects and on the other hand effects due to dissipative heating, and the rotation. The consequences of the above assumption is shown below:

$$Gr \sim 1 \Rightarrow L \sim \frac{1}{\varepsilon}; \quad St \sim 1 \Rightarrow t_R \sim \varepsilon^2 \; Ro \sim 1 \Rightarrow \omega \sim \frac{1}{t_R}; \quad \text{Di} \sim 1 \Rightarrow \Theta \sim \varepsilon^2. \tag{27}$$

Finally, we can written the dimensionless equations, omitting the bars in eqs. (22)-(24), as

$$div\mathbf{v} = \varepsilon^3 \left[St\partial_t\theta + grad\theta.\mathbf{v}\right], \tag{28}$$

$$\begin{aligned}\varepsilon^3(1-\varepsilon^3\theta)\left[St\partial_t\mathbf{v} + (grad\mathbf{v})\mathbf{v}\right] = \big[&-gradp + (1-\varepsilon^3\theta)\mathbf{g}\big] \\ &+\frac{\varepsilon^3}{(Gr)^{\frac{1}{2}}}\left[div\mathbf{D} - \frac{1}{3}grad(div\mathbf{v})\right] \\ &+\varepsilon^3\frac{(1-\varepsilon^3\theta)}{Ro}\left[-2\omega\times\mathbf{v} + \frac{1}{Ro}\omega\times(\omega\times\mathbf{r})\right]\end{aligned} \tag{29}$$

$$\begin{aligned}&\left[C - \varepsilon^3 Di\left(\theta + \frac{\Theta_M}{\Theta}\right)p\right]\left[St\partial_t\theta + grad\theta.\mathbf{v}\right] - Di\left(\theta + \frac{\Theta_M}{\Theta}\right) \\ &\left[St\partial_t p + gradp.\mathbf{v}\right] = \frac{1}{PrGr^{\frac{1}{2}}}\text{div}\mathbf{g}_\theta + 2\frac{Di}{Gr^{\frac{1}{2}}}\left[\|\mathbf{D}^2\| - \frac{1}{3}|(\text{tr}\mathbf{D})^2|\right],\end{aligned} \tag{30}$$

where is adopted $\overline{\rho} = \exp[-\alpha\Theta\overline{\theta}] \sim 1 - \alpha\Theta\overline{\theta}$, in which are choose $\rho^0 := \rho_R$ and $\theta^0 := \Theta_M$.

Now, we proceed towards the asymptotic expansion to provide a justification for the OBA in a rotating frame with dissipative heating. The dependent variables $\mathbf{v}$, θ and p are expanded into power series with respect to the perturbation parameter in the form

$$\mathbf{v} = \sum_{n=0}^{\infty}\varepsilon^n\mathbf{v}_n, \quad \theta = \sum_{n=0}^{\infty}\varepsilon^n\theta_n, \quad p = \sum_{n=0}^{\infty}\varepsilon^n p_n. \tag{31}$$

We replace all quantities in in the dimensionless equations (28)-(30) by these power series and have a look at the first ε-powers. We not interesting terms with large exponents at ε. Moreover we assume that all quantities are as smooth as we want.

At in the zeroth order of equations, $O(\varepsilon^0)$, we collect

$$\begin{aligned}\text{div}\mathbf{v}_0 &= 0, \\ 0 &= -\text{grad}p_0 + \mathbf{g}, \\ C(\theta_0)\dot{\theta}_0 - Di\left(\theta_0 + \frac{\Theta_M}{\Theta}\right)\dot{p_0} &= \frac{1}{PrGr^{\frac{1}{2}}}\triangle\theta_0 \\ &+2\frac{Di}{Gr^{\frac{1}{2}}}\left[\|\mathbf{D}(\mathbf{v}_0)^2\| - \frac{1}{3}|\text{tr}\mathbf{D}(\mathbf{v}_0)^2|\right]\end{aligned} \tag{32}$$

For ε^1 and ε^2 we see that gradp_1 = grad$p_2 = 0$, which together with the zero boundary conditions leads to

$$p_1 = p_2 = 0 \tag{33}$$

Finally, at ε^3, the momentum equation turn into

$$\dot{\mathbf{v}}_0 = -\text{grad}p_3 + \theta_0\mathbf{g} + \frac{2}{Gr^{\frac{1}{2}}}div\mathbf{D}(\mathbf{v}_0) - \frac{1}{Ro}[2\omega\times\mathbf{v}_0 + \frac{1}{Ro}(\omega\times(\omega\times\mathbf{r}))]. \tag{34}$$

Is adopted that $\mathbf{v} := \mathbf{v}_0$, $\theta := \theta_0$ e $:= p_0 + \varepsilon^3 p_3$, since $p_1 = p_2 := 0$. Finally, the Oberbeck-Boussinesq approximation in a rotating frame with dissipative heating is obtained by writing the eqs. $(32)_{1,3}$ and (34) in their dimensional forms. In this case, $\theta_R = \Theta$, $\rho_R = \rho_0$ and p_R corresponds to the dimensional form of p_0.

The distinct effects present in this class of problem can considered or not, depending into the choice of the dimensionless numbers invoked here. For example, if we are interesting in recuperate the usual Oberbeck-Boussinesq equations for a rotating frame of reference, we can assume $Gr, St, Ro = O(1)$ and $Di = O(\varepsilon)$. Also, if we want work with the Oberbeck-Boussinesq system for a inertial frame, we can imposed $Di = O(\varepsilon)$, $Ro = O(\frac{1}{\varepsilon})$, and the other numbers maintained like before.

5. CONCLUSIONS

In this paper, a justification for the Oberbeck-Boussinesq approximation in a rotating frame with dissipative heating was provided following the procedure advanced by Rajagopal, Ružička and Srinivasa (1996) and Kagei, Ružička and Thäter (2000). To achieve this, a thermodynamically consistent theory for a viscous fluid with Fourier conduction constrained by the mechanical incompressibility constraint was obtained. Next, an asymptotic analysis of the governing equations was carried out, in which the small parameter was chosen as a measure of compressibility. The Oberbeck-Boussinesq approximation was obtained after collecting the zeroth order perturbations of the continuity and heat equations along with the third order perturbation of the momentum equation.

The main feature of the justification provided in this paper rely on its ability to account for both heating dissipation and rotational effects. In addition, the usual Oberbeck-Boussinesq approximation can be obtained as a particular case.

6. REFERENCES

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