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HYDROGENERATOR DYNAMICS CONSIDERING THE UNBALANCED MAGNETIC PULL AND MECHANICAL UNBALANCE

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Abstract. *Unbalanced magnetic pull is frequently observed in hydropower generators, as a result of faults like geometric distortions at the rotor or the stator, mechanical unbalances, bearing imperfections and bowed shafts. These problems result in an asymmetric airgap distribution at the generator, and strong radial forces between the rotor and the generator. The unbalanced magnetic pull can cause severe vibrations, even bringing the generator to collapse. It is important to account for this force when designing and analyzing hydropower plants. The hydropower generator is a part of a more complex system that comprises also the shaft and the turbine. Depending on the configuration, the rotor is supported by two or three bearings. Based on a finite element model of an existing hydropower generator plant, this work analyzes the dynamic behavior of the system considering a static eccentricity for the stator of the generator. Numerical simulations considering the nonlinear magnetic forces of the generator are shown, using the harmonic balance method to find the steady state vibrations of the system. Based on the orbits, it is possible to put in evidence the anisotropic behavior of the system and also the appearance of 2X components on the response. The later can be used to detect static eccentricity in field, at real machines.*

Keywords: *unbalanced magnetic pull, rotordynamics, nonlinear response*

1. INTRODUCTION

Hydropower generators have small airgap dimensions, around 0.2% of the stator radius (Lundström and Aidanpää, 2007). This airgap often has non-uniformities due to manufacturing and operational problems. Geometric distortions, mechanical unbalances, bearing imperfections and bowed shafts are among the most common factors that cause asymmetries in airgap. These asymmetries distort the airgap density flux resulting in a radial unbalance of magnetic forces called Unbalanced Magnetic Pull (UMP). The UMP can assume high levels, causing strong vibration which is dangerous to the machine.

The unbalanced magnetic pull influence on rotordynamics has been studied mainly with Jeffcott rotors. Guo et al. (2002) modeled the UMP based on an approach where the MMF (magnetomotive force) waves are modulated by the airgap permeance in a Fourier series. This procedure leads to closed form expressions for the magnetic forces in a three-phase machine with an arbitrary number of poles. From this model, it is easy to see that the oscillating component of the UMP is not important for a number of pole pairs greater than three. The simulated system is a Jeffcott rotor model with the UMP, whose equations of motion are solved with a Newmark integrator. The authors showed that UMP can generate 2X and 3X super harmonics, as well as 2X frequencies plus or minus the synchronous frequency when the generator has one pole pair.

The UMP model can be improved to consider electrical unbalance. Yang et al. (2004) consider the unbalance between the phases in the magnetic flux density. The UMP is modeled using the airgap permeance in function of the eccentricity. The system is an electric motor and it is modeled as a Jeffcott rotor with two poles and three phases. The equations of motion are integrated by a Newmark schema. The stability is found by Floquet technique. The results clearly show how the UMP affects the critical speed of the rotor, and how transient operation conditions and fluctuating charges can lead to unstable regimes.

Although Jeffcott rotors can give valuable insights on the dynamics of an electric machine, finite element models are indispensable for describing precisely the eletromechanical phenomena associated to unbalanced magnetic pull and rotordynamics. (Pennacchi and Frosini, 2005) used such a model to study a multi degree of freedom generator. The mechanical model is coupled with a detailed model of magnetic forces, resulting in responses containing super and sub harmonics that are not multiples of the rotation frequency.

Gustavsson and Aidanpää (2006) studied a hydropower generator system where the generator is modeled as a rigid body with four degrees of freedom (rotation and translation in two orthogonal planes), a vertical shaft and two linear bearings supporting the rotor. For this system, they studied the stability in function of the positions of the generator spider hub over the shaft and the generator offset, as well as the static eccentricity. Among other interesting findings, the authors showed that instability can appear even for small values of static eccentricity.

Usually, in hydropower design, the effect of UMP is taken into account though a negative stiffness. This way the traditional machine design tools can still be used in a linear approach. However this procedure can underestimate the

UMP influence on vibration and on machine stability. The main objective of this paper is to develop a more precise dynamic finite element model of a hydropower generator that considers the UMP and mechanical unbalance in a non linear approach. As an application example, this model will provide a particular study case for the influence of static eccentricity in the global hydropower dynamics. This case is based in a real Kaplan hydropower plant in North's region of Brazil.

2. MODEL OF THE HYDROPOWER GENERATOR

The hydropower generator studied here has its dimensions and physical properties closely inspired by a real Kaplan turbine/generator system. The model is created using the finite element methodology of Ferraris and Lalanne (1990).

Figure 1 shows the dimensions of the rotor. The shaft is 9.04 m long and its diameter is 710 mm. The shaft is discretized in 12 Euler-Bernoulli beam elements, with factors of correction that take into account the rotation of the cross sections and shear stress. The shaft is made of steel ($\rho = 7850\,\text{kg/m}^3$, $E = 2\times10^{11}$ Pa).

The generator is modeled as a rigid disk with four degrees of freedom and the same density as the shaft. The dimensions of this disk are such that the inertia properties of the real generator can be precisely reproduced. The disk is placed at the center of mass of the real generator, lying 472 mm above the shaft. The connection between the disk and the shaft is made by a rigid and massless element. This element is used to maintain the generator at the correct position relative to the shaft. The same arrangements are valid for the turbine.

There are three bearings in the system. Two of them are radial bearings, situated close to the generator and to the turbine. The stiffness of the generator's radial bearing is 1.37×10^9 N/m . For the turbine's radial bearing, the stiffness is 1.64×10^9 N/m . The third bearing is a thrust bearing near the middle span of the shaft. The effect of the thrust bearing on the transversal vibrations of the shaft can be approximated by a linear angular spring of stiffness 5.0×10^7 Nm/rad .

The rotor is axisymmetrical and the whole system is linear. Under these conditions, the equation of motion for transversal vibrations of the system is written as:

$$\mathbf{M}\ddot{\mathbf{X}} + \left(\mathbf{C} + \Omega\mathbf{G}\right)\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}_1\Omega^2 \sin\left(\Omega t\right) + \mathbf{F}_2\Omega^2 \cos\left(\Omega t\right) \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{G}$ is the gyroscopic matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{F}_1$ and $\mathbf{F}_2$ are the unbalance vectors, Ω is the frequency of rotation and $\mathbf{X}$ is the displacement vector. As usual, the overdot means time differentiation.

The damping matrix is written in the form $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, where the constants α and β are such that the modal damping ratios of the first and second distinct modes are equal to 0.01. There is mechanical unbalance in both generator and turbine, and they are such that the amplitude of the displacement taken at the generator's rotor center is 200 µm.

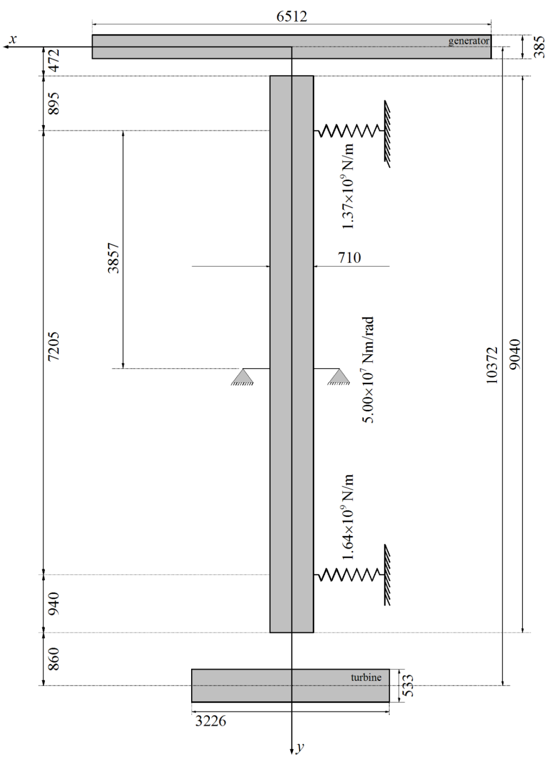

Figure 1. Schema of the rotor

3. MAGNETIC FORCES

This section presents the model of electromagnetic forces at the generator.

3.1. Eccentricity

In this work, the eccentricity at the generator is defined as the radial distance between the outer surface of the rotor and the inner surface of the stator, measured from the center of the rotor. In this section an approximate expression for the eccentricity will be developed. Similar expressions can be found in the works of Lundström and Aidanpää (2007) and Yang et al. (2004).

For simplicity, the stator and the rotor are supposed to have circular shape. The fixed frame of reference is represented by the $Oxyz$ set of axis, as shown in Figure 2. At the generator level, the stator is represented by a circle centered at O_E with its radius given by R_E. The rotor is represented by a circle centered at (x,z) with radius R_R. Let S be the intersection point between the stator and a straight line containing the center of the rotor with inclination given by the ϕ angle. The coordinates of S are:

$$x_S = x + h\sin(\phi) \tag{2}$$

$$z_S = z + h\cos(\phi) \tag{3}$$

For the stator,

$$R_E{}^2 = (x_S - x_E)^2 + (z_S - z_E)^2 \tag{4}$$

Equations (2) and (3) are inserted in (4), and assuming that x_E, z_E, x and z are small compared to R_E, h can be written approximately as:

$$h \approx (x_E - x)\sin(\phi) + (z_E - z)\cos(\phi) + R_E \tag{5}$$

Finally, from the definition of eccentricity, one can write:

$$g(x,z,t,\phi) = R_E - R_R + (x_E - x)\sin(\phi) + (y_E - y)\cos(\phi) \tag{6}$$

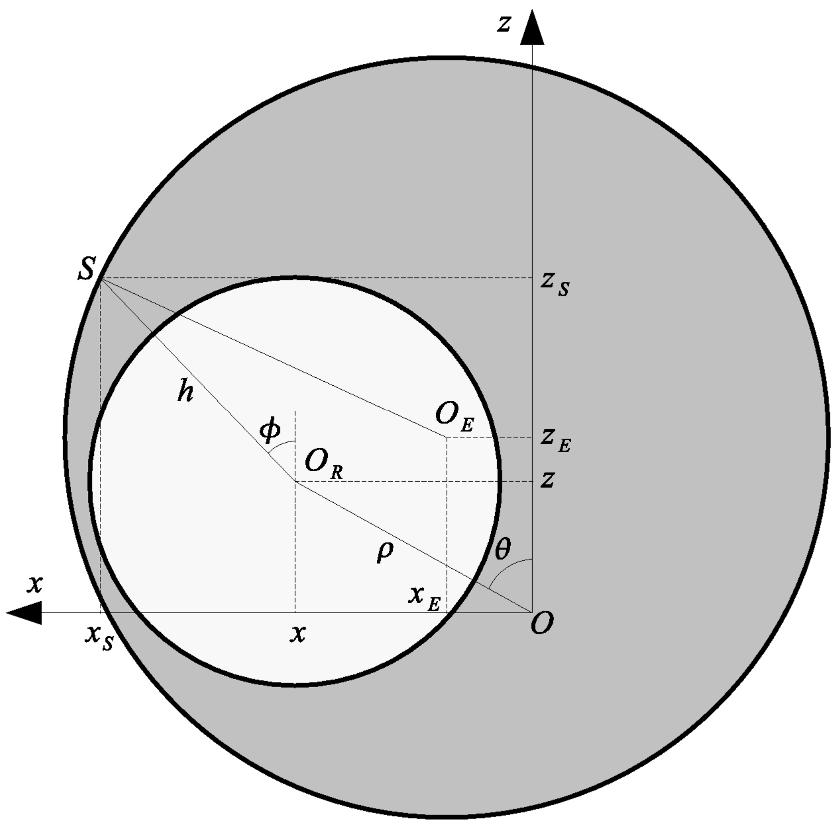

Figure 2. Geometry of the generator

3.1. Magnetic forces

This work uses the UMP model of Lundström and Aidanpää (2007), considering that at the generator there is no slope between the centerlines of the rotor and the stator and the magnetic field is constant. With these assumptions, the magnetic forces are given by:

$$F_x = \frac{B_0^2 g_0^2 u_0 l_0}{2\mu_0}\int_0^{2\pi} \frac{\sin(\phi)}{g(x,z,t,\phi)^2} d\phi \tag{7}$$

$$F_z = \frac{B_0^2 g_0^2 u_0 l_0}{2\mu_0}\int_0^{2\pi} \frac{\cos(\phi)}{g(x,z,t,\phi)^2} d\phi \tag{8}$$

where B_0 is the non-perturbed magnetic field, $u_0 = 0.5(R_R + R_E)$ is the mean radius of the airgap, $g_0 = R_E - R_S$ is the initial airgap, l_0 is the height of the generator and μ_0 is the permeability of vacuum.

Depending on the nature of the eccentricity, the magnetic forces are constant or oscillating. When the rotor is perfectly balanced, without whirling, the magnetic force is on the smallest gap direction. When the rotor whirls, the smallest gap direction depends on the position of the rotor, what makes the magnetic forces oscillate.

4. DETERMINATION OF THE STEADY STATE SOLUTION FOR THE EQUATIONS OF MOTION

The final equations of motion for the hydropower system are obtained from Eq. (1) combined with Eq. (7) and Eq. (8):

$$\mathbf{M}\ddot{\mathbf{X}} + (\mathbf{C} + \Omega\mathbf{G})\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}_1\Omega^2 \sin(\Omega t) + \mathbf{F}_2\Omega^2 \cos(\Omega t) + \mathbf{F}_{NL} = \mathbf{F}(\mathbf{X}, \Omega, g) \tag{9}$$

where $\mathbf{F}_{NL}$ is the vector of the nonlinear forces. The first and second elements of the nonlinear forces vector are given by Eq. (7) and Eq. (8). The remaining elements are equal to zero. As the eccentricity depends on the radial coordinates of the generator, Eq. (9) is a set of nonlinear ordinary differential equations. The steady-state solution of Eq. (9) can be found by a sufficient long direct integration. If the analysis involves the variation of a parameter to evaluate its influence on the steady-state response, this procedure can be very time consuming. In this case, it is advisable to use a frequential method like the harmonic balance method (Cameron and Griffin, 1989, Cardona et al., 1998).

In the harmonic balance method, one assumes that the response of the system is a m terms truncated Fourier series with the fundamental frequency being the rotor's rotation frequency:

$$\mathbf{X} = \mathbf{B}_0 + \sum_{i=1}^{m}\left(\mathbf{A}_i \sin(i\Omega t) + \mathbf{B}_i \cos(i\Omega t)\right) \tag{10}$$

The first and second time derivatives of the displacement are:

$$\dot{\mathbf{X}} = \sum_{i=1}^{m}\left(i\Omega\mathbf{A}_i \cos(i\Omega t) - i\Omega\mathbf{B}_i \sin(i\Omega t)\right) \tag{11}$$

$$\ddot{\mathbf{X}} = \sum_{i=1}^{m}\left(-i^2\Omega^2\mathbf{A}_i \sin(i\Omega t) - i^2\Omega^2\mathbf{B}_i \cos(i\Omega t)\right) \tag{12}$$

The excitation forces are also expanded in Fourier series:

$$\mathbf{F}(\mathbf{X}, \Omega, g) = \mathbf{C}_0 + \sum_{i=1}^{m}\left(\mathbf{S}_i \sin(i\Omega t) + \mathbf{C}_i \cos(i\Omega t)\right) \tag{13}$$

Equations (10), (11) and (12) are inserted into the equations of motion (9), and the terms on sine and cosine of same frequency are collected. This procedure leads to:

$$\mathbf{K}\mathbf{B}_0 + \sum_{i=1}^{m}\begin{pmatrix}\left((\mathbf{K} - i^2\Omega^2\mathbf{M})\mathbf{A}_i - i\Omega(\mathbf{D} + \Omega\mathbf{G})\mathbf{B}_i\right)\sin(i\Omega t) + \\ +\left(i\Omega(\mathbf{D} + \Omega\mathbf{G})\mathbf{A}_i + (\mathbf{K} - i^2\Omega^2\mathbf{M})\mathbf{B}_i\right)\cos(i\Omega t)\end{pmatrix} = \mathbf{C}_0 + \sum_{i=1}^{m}\left(\mathbf{S}_i \sin(i\Omega t) + \mathbf{C}_i \cos(i\Omega t)\right) \tag{14}$$

Then, for the constant term, one has:

$$\mathbf{K}\mathbf{B}_0 = \mathbf{C}_0 \tag{15}$$

And for the i^{th} ($1 \le i \le m$) harmonic:

$$\begin{bmatrix}(\mathbf{K} - i^2\Omega^2\mathbf{M}) & -i\Omega(\mathbf{D} + \Omega\mathbf{G}) \\ i\Omega(\mathbf{D} + \Omega\mathbf{G}) & (\mathbf{K} - i^2\Omega^2\mathbf{M})\end{bmatrix}\begin{Bmatrix}\mathbf{A}_i \\ \mathbf{B}_i\end{Bmatrix} = \begin{Bmatrix}\mathbf{S}_i \\ \mathbf{C}_i\end{Bmatrix} \tag{16}$$

Equations (15) and (16) for all harmonics can be put in the form:

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$$\begin{bmatrix} \mathbf{K} & & & & \\ & \ddots & & & \\ & & \mathbf{\Lambda}_i & & \\ & & & \ddots & \\ & & & & \mathbf{\Lambda}_m \end{bmatrix} \begin{bmatrix} \mathbf{B}_0 \\ \vdots \\ \mathbf{Y}_i \\ \vdots \\ \mathbf{Y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{C}_0 \\ \vdots \\ \mathbf{Z}_i \\ \vdots \\ \mathbf{Z}_m \end{bmatrix} \tag{17}$$

where:

$$\mathbf{\Lambda}_i = \begin{bmatrix} \mathbf{K} - \mathbf{M}(i\Omega)^2 & -(\mathbf{D} + \Omega\mathbf{G})(i\Omega) \\ (\mathbf{D} + \Omega\mathbf{G})(i\Omega) & \mathbf{K} - \mathbf{M}(i\Omega)^2 \end{bmatrix}, \mathbf{Y}_i = \begin{bmatrix} \mathbf{A}_i \\ \mathbf{B}_i \end{bmatrix}, \mathbf{Z}_i = \begin{bmatrix} \mathbf{S}_i \\ \mathbf{C}_i \end{bmatrix} \tag{18}$$

The vectors $\mathbf{Y}_i$ and $\mathbf{Z}_i$ contain respectively the non-DC Fourier series coefficients of the response and the excitation forces. From equations (10) and (13), it is not difficult to see that $\mathbf{Z}_i$ depends on $\mathbf{Y}_i$, meaning that the system of equations (17) is nonlinear. To solve this system, one defines:

$$\mathbf{H}(\mathbf{Y}) = \mathbf{\Lambda}\mathbf{Y} - \mathbf{Z}(\mathbf{Y}) \tag{19}$$

where the quantities involved are readily identifiable. Then, solving the system of equations (17) is equivalent to find the zeros of Eq. (19). It is not practical or even possible to search analytical solutions for this nonlinear algebraic system of equations. One technique to solve it is the AFT (Alternating Frequency Time) (Cameron and Griffin, 1989), where the relationship between $\mathbf{Y}_i$ and $\mathbf{Z}_i$ is constructed as follows:

1) From an initial vector $\mathbf{Y}$, the response $\mathbf{X}(t)$ is evaluated (Eq. (10)) on the time domain;
2) The excitation forces $\mathbf{F}(\mathbf{X}, \Omega, g)$ are evaluated over one period of the excitation force;
3) The excitation forces are transformed into the frequency domain, and the vector $\mathbf{Z}_i$ is constructed;

With this procedure, a nonlinear algebraic system of equations solver can be used to find the zeros of Eq. (19). In the present work, the solver is based on the Powell dog leg method (Madsen et al., 2004).

5. STUDY OF THE STATIC ECCENTRICITY

The objective of this study is to find how static eccentricity at the generator position affects the dynamics of the whole hydropower generator. The hydropower generator system rotates at 150 RPM and has the general properties:

Table 1. Properties of the generator

Property	Value
B_0	0.5 T
u_0	2.9945 m
g_0	0.011 m
μ_0	$4\pi \times 10^{-7}$ H/m
l_0	0.97 m

The simulation is made with the following general procedure:

1. Determination of the steady-state solution for the non-eccentric generator;
2. The horizontal coordinate of the stator, x_E, is increased;
3. The steady-state solution of this new situation is calculated, with the last known solution as the initial guess for the nonlinear system of algebraic equations solver;
4. If the solver has converged, go to step 2.
5. Stop.

The next paragraphs will show the results of the simulation. First we show the validation of the harmonic balance method. Then, the analysis concerning the static eccentricity will be shown.

5.1. Validation of the harmonic balance method

The validation is made for the static eccentricity equal to 8.8% of the initial airgap and the above-mentioned speed of rotation. The direct integration is made with the centered finite difference (Géradin and Rixen, 1997), considering 1000 periods of rotation of the hydropower generator. The initial condition imposed is the steady state solution of linear problem. To avoid strong transients during the integration, the magnetic field is applied following a ramp that stabilizes at 0.5 T when the number of periods reaches 400. The harmonic balance method retains the first four harmonics of the response, i. e., $m = 4$ in equation (8).

After the integration, it is considered that the steady state is reached. Figure 3 shows the orbit constructed with the three last periods of the integration. The solution obtained by the harmonic balance method is plotted in the same figure, showing good agreement between the two methods. The same conclusion can be drawn by inspecting the time evolution of the x and z components in Figure 4.

This validation was made in strong nonlinearity conditions. Then, it is reasonable to expect that the harmonic balance method will give correct results for other cases. So, the study of the influence of the static eccentricity on the steady state response of the system will be carried out using the harmonic balance method.

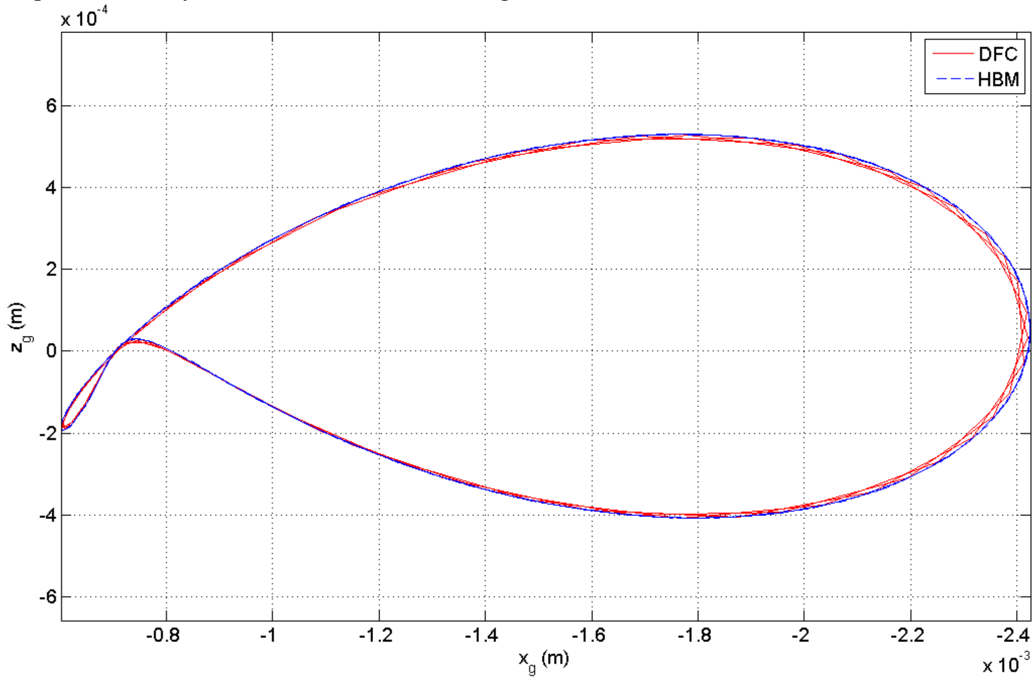

Figure 3. Orbits for the validation of the harmonic balance method (DFC: Centered Finite Differences, HBM: Harmonic Balance Method)

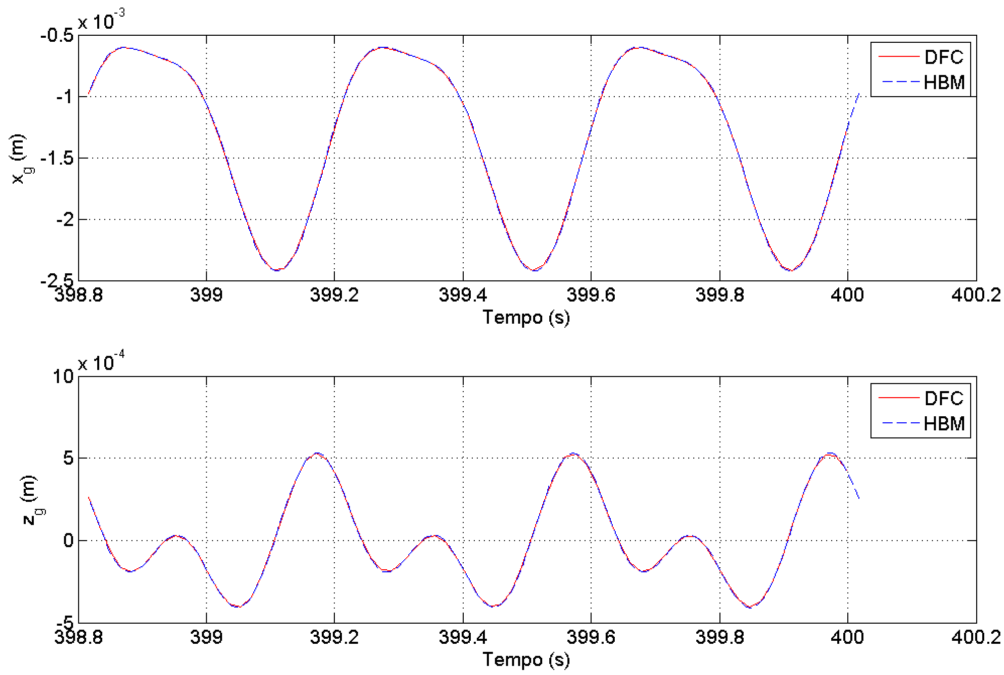

Figure 4. Time evolution of the x and z components of the response (DFC: Centered Finite Differences, HBM: Harmonic Balance Method)

5.2. Evolution of the orbits

The study of the evolution of the steady state orbits is made based on the results calculated with the harmonic balance method. First we show the evolution of the amplitude of each harmonic (including the DC component). The amplitude of a harmonic is defined here as the square root of the sum of the squares of the sine and cosine coefficients. After the evolution of the harmonics, we show the corresponding orbits. These results are shown for the center of the generator's rotor, i.e., the first finite element node of the system. In order to extend the analysis, a global view of the rotor will be shown, illustrating how the system dynamics is affected by the static eccentricity of the generator's stator.

The evolution of amplitude of each harmonic component, as a function of gap, is shown in Fig. 5. Figure 6 shows some orbits constructed with the amplitudes shown in Fig. 5. These results are taken at the center of the rotor of the generator. For eccentricities smaller than 3% or 4%, the main influence of the imposed static eccentricity is the displacement of the center of the orbit of the rotor, as indicated by the evolution of the DC component. The evolution of the harmonics shows that the shape of the orbit experiments relatively small changes.

Beyond this value of static eccentricity, the orbit of the rotor continues to travel inside the airgap, and the corresponding orbit has significant changes on its shape, indicating a anisotropic behavior. When the static eccentricity goes beyond 8%, the amplitude of the considered harmonics grows rapidly. As the second harmonic grows, the orbit presents a loop. For eccentricities beyond about 8.9%, preliminary calculations show that no steady state solution exists. The curves suggest that a small increment on the static eccentricity will result in a huge increase of the amplitude of the components of the response.

The discussion above is found to be valid for the whole rotor. Figure 5 shows the orbits along the rotor for 1%, 5%, 7% and 8.9% static eccentricities. The 1% case shows that the rotor has an almost linear behavior, with mainly a displacement at the mean position of the orbits. For the 5% case, one can see that the displacement caused by the magnetic forces at the generator bends the shaft in a more pronounced way than the observed in the 1% case. When the static eccentricity is 7%, it is possible to see the shaft with a higher degree of bending and the orbits begins to show an elliptical shape. For the 8.9% static eccentricity, it is possible to see the effect of the second harmonic on the dynamics of the system, along with a highly bent shaft.

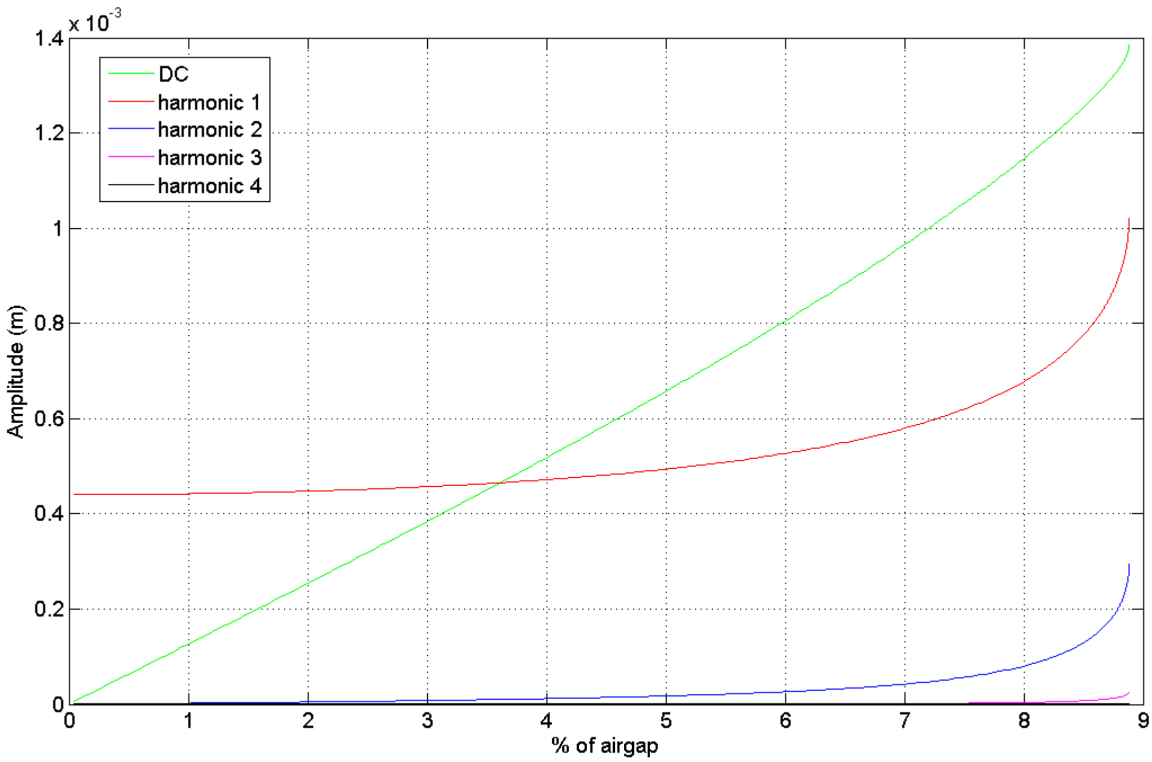

Figure 5. Evolution of the amplitude of the DC component and the first four harmonics of the response

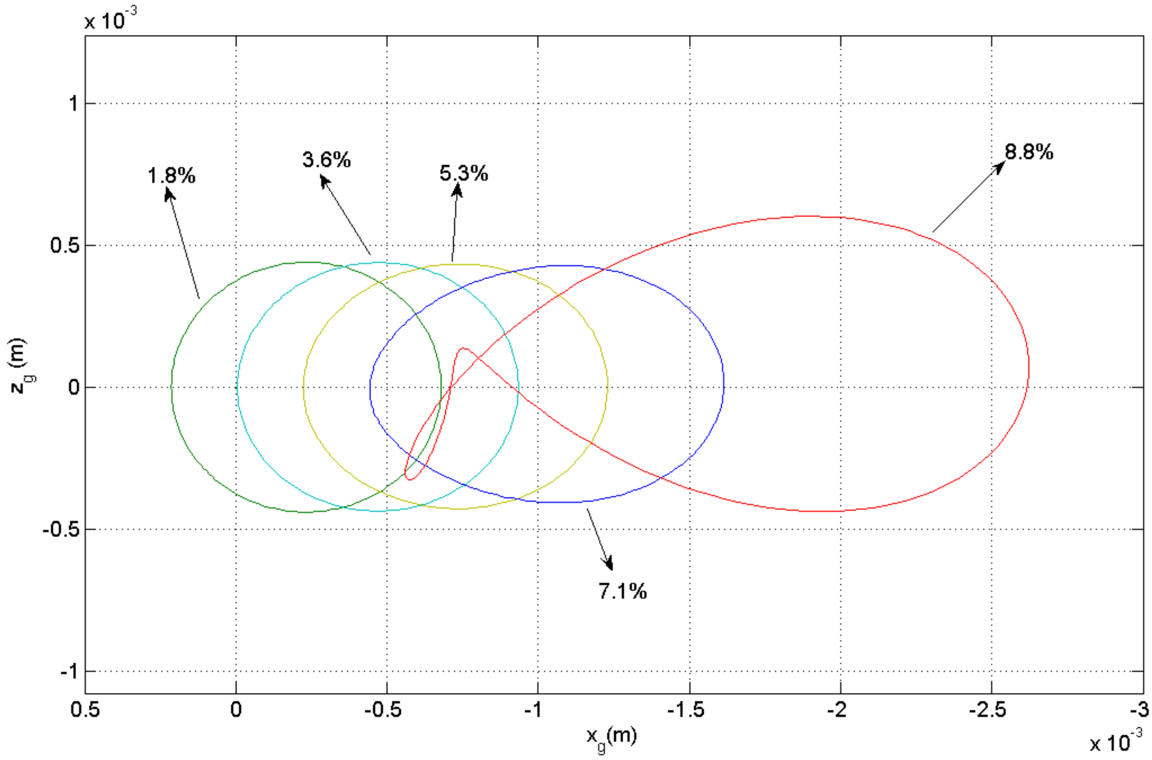

Figure 6. Orbits of the rotor at the generator's position in function of the static eccentricity percentage.

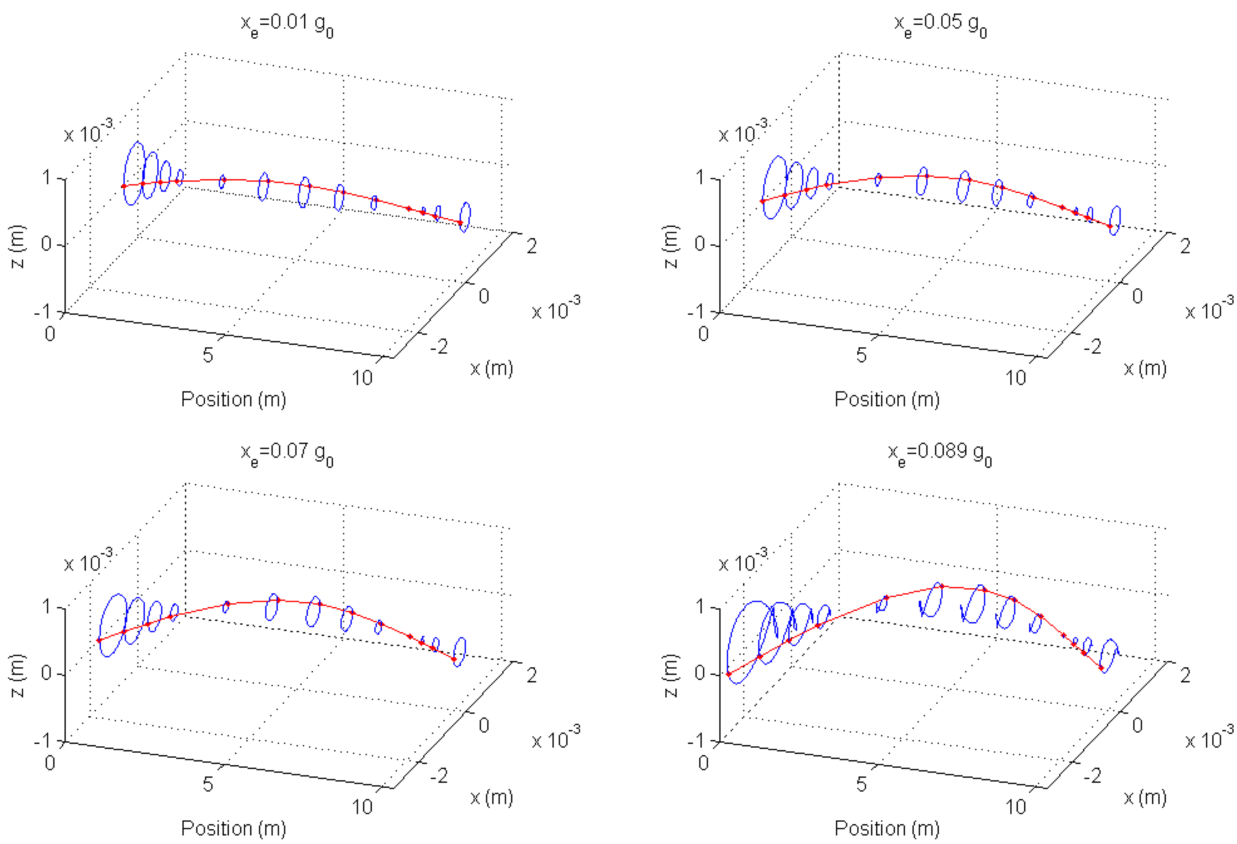

Figure 7. The rotor under the influence of magnetic forces at the generator

6. ACKNOWLEDGEMENTS

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7. CONCLUSIONS

This work studies the rotordynamics of a hydropower generator system working with a generator presenting static eccentricity. In particular the steady state behavior of the system was determined by the harmonic balance method, after a successful validation against a centered finite difference solution. From the results obtained, it was found that four harmonics and the DC component are quite enough to describe the system dynamics. Also, preliminary calculations with static eccentricity beyond 8.9% showed that there are no harmonic solutions. Indeed, the behavior of the system needs to be precisely analyzed when the static eccentricity falls beyond this threshold.

The system is closely based on the dimensions of an existing Kaplan turbine/generator system. It was found that static eccentricity, a common operational fault, can be hazardous to the hydropower generator system. In fact, as the static eccentricity grows, the system loses its axisimmetry and can develop 2X components. Also the amplitude of the vibrations can be doubled. When the system works with axisimmetry, there are fluctuating stresses acting on the shaft, what can contribute to the appearance of fatigue cracks.

The 2X component could be an indication of exaggerated static eccentricity. However, as the hydropower generator works in a complex environment, there are other factors that could generate 2X components, like nonlinearities in the bearings or transverse cracks. On the continuation of this work, the model of the bearings will be enhanced to include nonlinear effects. With this improvement, it will be possible to draw more precise conclusions about the effect of the static eccentricity of the stator of the generator.

8. REFERENCES

Cameron, T. M., Griffin, J. H., 1989, “An Alternating Frequency Time Domain Method for Calculating the Steady State Response of Nonlinear Dynamic Systems”, ASME Journal of Applied Mechanics, Vol. 56, No. 1, pp. 149-154.

Cardona, A., Lerusse, A. and Géradin, M., 1998, “Fast Fourier Nonlinear Vibration Analysis”, Computational Mechanics, Vol. 22, No. 2, pp. 128-142.

Géradin, M., Rixen, D., 1997, “Théorie des Vibrations – Application à la Dynamique des Structures”, 2ème édition, Masson.

Guo, D., Chu, F. and Chen, D., 2002, “The Unbalanced Magnetic Pull and its Effects on Vibration in a Three-Phase Generator with Eccentric Rotor”, Journal of Sound and Vibration, Vol. 254, No. 2, pp. 297-312.

Gustavsson, R. .K., Aidanpää, J. O., 2006, “The Influence of Nonlinear Magnetic Pull on Hydropower Generator Rotors”, Journal of Sound and Vibration, Vol. 297 No. 3-5, pp. 551-562.

Lalanne, M. and Ferraris, G., 1990, “Rotordynamics Prediction in Engineering”, John Wiley & Sons Inc.

Lundström, N.; Aidanpää, J. O., 2007, “Dynamic Consequences of Electromagnetic Pull Due to Deviations in Generator Shape”, Journal of Sound and Vibration, Vol. 301, No. 1-2, pp. 207-225.

Madsen, K., Nielsen, H. B. and Tingleff, O., 2004, “Methods for Nonlinear Least Squares Problems”, Technical University of Denmark, 2004.

Pennacchi, P., Frosini, L., 2005, “Dynamical Behaviour of a Three-Phase Generator due to Unbalanced Magnetic Pull”, IEE Proceedings on Electric Power Applications,Vol. 152, No. 6, pp. 1389-1400.

T. M. Cameron and J. H. Griffin, 1989, An Alternating Frequency Time Domain Method for Calculating the Steady State Response of Nonlinear Dynamic Systems.,ASME Journal of Applied Mechanics, Vol. 56, pp. 149-154.

Yang, B. S., Kim, Y. H., Son, B. G., 2004, “Instability and Imbalance Response of Large Induction Motor Rotor by Unbalanced Magnetic Pull”, Journal of Vibration and Control, Vol. 10, No. 3, pp. 447-460.

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