MODAL PARAMETERS IDENTIFICATION BY ORTHOGONAL FUNCTIONS CONSIDERING NOISES EFFECTS

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Abstract. *Many methods can be used to determine characteristics of vibrations in mechanical structures. In general, the methods use inputs and output signals. Currently, many researches and text books have been published in this area. In this context, this paper propose to identify modal parameters trhough orthogonal functions. Generally, papers present stifiness, mass and damping identification considering the motion equations like a second order differencial equation. In this situations input forces are necessary in all degree of freedom (dof). Modal parameters also can be identified using orthogonal functions and state space realization (SSR). This formulation presents some advantages as: small number of expansion terms; is necessary only once input sensor, etc. However, it was not found in literature, information about cases with noises effects in input and output signals. This paper presents a modal parameters identification using orthogonal functions considering the motion equations represented by the state space realization. Noise effects are considered in the output signals and Legendre and Chebyshev orthogonal functions are used. The paper concludes with a numerical application.*

Keywords*: Orthogonal Functions, Parameters Identification, Space State Realization and Noise Effect.*

1. INTRODUCTION

Modal Analysis comprises nowadays a vast range of different areas, like Modal Correlation and Updating, Modal Testing methods, Nonlinear Modal Analysis, Sub structuring, Structural Modification, to name some of them. However, one of the fundamental areas is, precisely, the identification of the dynamic properties of a structure from the measured data. In the seventies and more so in the eighties one has witnessed the development of a vast range of identification methods and techniques. Some of the best-performing have been integrated into commercial software, and nowadays one can find entire modal analysis packages running in computers linked to modal testing systems and doing not only modal parameter estimation but also other pre-processing and post-processing calculations, like geometry definition, mode animation, structural modification prediction, coupling of substructures, etc (Maia et al., 1997).

During the last three decades or so, many researchers have devoted their efforts to the development of techniques that aim to produce a reliable identification of the dynamic properties of structures. Those efforts have been fruitful due largely to the introduction of the Fast Fourier Transform (FFT) and to the development in recent years of very powerful multi-channel spectrum analysers, computers and instrumentation in general that permit the acquisition and treatment of large quantities of data. In this way, it was possible to evolve from very simple techniques, where analyses were based on data from single-input excitation and single-output response to highly sophisticated ones, where data from multi-input excitation and multi-output responses are treated simultaneously. In this context, modal parameters identification by orthogonal functions has been considered in many applications (Melo, 1992; Pacheco, 2001; Pacheco and Steffen, 2004). Morais (2006) showed the application of orthogonal functions series for excitation force identification in a robotic arm.

With orthogonal functions the main idea is to use integration and derivation properties. These properties perform a key role in the identification procedure, since, through it’s application, it’s possible to transform a set of differential equations of motion into a set of algebraic equations, facilitating the identification of the unknown parameters. This can be considered as the characteristic of the present method, which differs form other orthogonal function based on identification techniques (Pacheco and Steffen, 2004). Many orthogonal functions series are found in the literature as: Fourier, Legendre, Chebyshev, Jacobi, Block-Pulse and others; and also many works were published proving the efficiency of this methodology (Chang and Wang, 1985; Melo and Steffen, 1993). This paper presents the modal parameters identification in a system represented by a state space model, recently shown in Marqui et al., 2006, considering noises effects in the input and output signals.

2. ORTHOGONAL FUNCTIONS

A set of real functions $\varphi_k(t), k = 1, 2, 3...$ defined in the interval $[a,b] \in \Re$. The set is said orthogonal in the same interval if (Spiegel, 1976):

$$\int_a^b \varphi_m(t)\varphi_n(t)dt = K \tag{1}$$

where K is a constant equal zero if $m \neq n$ and different of zero if $m = n$.

The set of functions $\phi_k(t)$ is said orthonormal if will be valid the relation (Spiegel, 1976):

$$\int_a^b \phi_m(t)\phi_n(t)dt = \delta_{mn} \tag{2}$$

where δ_{mn}, is the Kronecker delta, i. e. "0" if $m \neq n$ or "1" if $m = n$ and $\phi_k(t)$ is the set of functions orthonormal.

The set $\varphi_k(t)$ is orthonormal with respect to the weight function $w(t)$ in which $w(t) \geq 0$, then the set of orthonormal functions is whiting by:

$$\phi_k(t) = \sqrt{w(t)}\varphi_k(t), \qquad \mathrm{k} = 1, 2, 3,... \tag{3}$$

and verify the relation:

$$\int_a^b \varphi_m(t)\varphi_n(t)w(t)dt = \delta_{mn} \tag{4}$$

If a function $f(t)$ is continuous or partially continuous in the interval [a,b], then $f(t)$ can be expanded in series of orthogonal functions, that is:

$$f(t) = \sum_{n=1}^{\infty} c_n \phi_n(t) \tag{5}$$

Such series, called orthonormal series, constitute the generalizations of the Fourier series. Admitting that the sum in Equation (5) converges to $f(t)$, we can multiply both for $\phi_m(t)$ and, integrating in the interval [a,b]. In this equation c_n are the generalized coefficients of Fourier. The following property, related to be successive integration of the vectorial basis, holds for a set of r orthonormal functions in the interval [0,t]:

$$\int_0^t \underbrace{\cdots}_{n\, times} \int_0^t \{\phi_m(\tau)\}(d\tau) \cong [P]^n \{\phi_m(t)\} \tag{6}$$

where $\{\phi_m(t)\} = \{\phi_0(t)\ \ \phi_1(t)\ \ ...\ \ \phi_r(t)\}^T$ is the finite set of orthogonal series, [P] is a square matrix of order "r" with constants elements called operational matrix integration.

Tables 1 and 2 describe the main characteristics and the operational matrix integration of Legendre and Chebyshev orthogonal functions, respectively.

Table 1. Main characteristics of Legendre series.

Recursive formula in the interval $t \in [0, t_f]$	Operational Matrix of Integration
$(n-1)\Phi_{n+1}(t) = (2n+1)\left(\frac{2t}{t_f} - 1\right)\Phi_n(t) - n\Phi_{n-1}(t)$ $\Phi_0(t) = 1$ $\Phi_1(t) = \frac{2t}{t_f - 1}$ n=1, 2, 3, …, r-1	$[P] = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \dots & 0 & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{2r-3} \\ 0 & 0 & 0 & 0 & \dots & \frac{-1}{2r-3} & 0 \end{bmatrix}$

Table 2. Main characteristics of Chebyshev series.

Recursive formula in the interval $t \in [0, t_f]$	Operational Matrix of Integration
$\Phi_{n+1}(t) = 2n\left(\frac{2t}{t_f} - 1\right)\Phi_n(t) - n\Phi_{n-1}(t)$ $\Phi_0(t) = 1$ $\Phi_1(t) = \frac{2t}{t_f - 1}$ n= 1, 2, 3, …, r-1	$[P] = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ -1/4 & 0 & 1/4 & 0 & \dots & 0 & 0 & 0 \\ -1/3 & -1/2 & 0 & 1/6 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{(-1)^{r-1}}{(r-1)(r-3)} & 0 & 0 & 0 & \dots & \frac{-1}{2(r-3)} & 0 & \frac{1}{2(r-1)} \\ \frac{(-1)^r}{r(r-2)} & 0 & 0 & 0 & \dots & 0 & \frac{-1}{2(r-2)} & 0 \end{bmatrix}$

2. STATE SPACE MODEL IDENTIFICATION USING ORTHOGONAL FUNCTIONS

A mechanical system can be represented using the state space realization efficiently (Equation (7)). Using input and output signals, in general, this representation is obtained through methods as: ERA (Eigensystem Realization Algorithm); N4SID (Numerical Subspace State Space System Identification); PEM (Prediction Errors Method); CE (Complex Exponential); or orthogonal functions.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

$$y(t) = Cx(t) \tag{8}$$

where A, B and C, shown in Equation (9), are the dynamic, input and output matrices, respectively; also, $x(t)$ is the state vector, $y(t)$ is the output vector and $u(t)$ is the input vector.

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -M^{-1}B_0 \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} C_{oq} & C_{ov} \end{bmatrix}$$

where A is 2n x 2n, B is 2n x s, C is r x 2n; n is the number of modes, s is the number of inputs and r is the number of outputs. The matrix of input position, B_0, is n x s, C_{oq} is the displacement output matrix (r x n) and C_{ov} is the velocity output matrix (r x n); and n is the number of degree of freedom.

Integrating the Equation (7) in the interval [0,t], it can obtain:

$$\int_0^t \dot{x}(t)dt = \int_0^t Ax(t)dt + \int_0^t Bu(t)dt$$
$$x(t) - x(0) = A\int_0^t x(t)dt + B\int_0^t u(t)dt \tag{10}$$

The displacement responses $x(t)$ and the vector of exciting forces $u(t)$ can be expanded in "r" number of terms truncated series orthogonal functions as follows:

$$x(t) \cong \underset{(2n\ x\ r)}{X} \underset{(r\ x\ 1)}{\Phi(t)}$$
$$u(t) \cong \underset{(i\ x\ r)}{U} \underset{(r\ x\ 1)}{\Phi(t)} \tag{11}$$

where X and U are the matrix of the coefficients of expansion. Substituting in Equation (10), it can obtain:

$$X\Phi(t) - x(0) = A\int_0^t X\Phi(\tau)d\tau + B\int_0^t U\Phi(\tau)d\tau \tag{12}$$

Applying $1 = e^T \phi(t)$ and the integral property given by Equation (7), it can obtain:

$$X\Phi(t) - x(0)e^T\Phi(t) = AX\int_0^t \Phi(\tau)d\tau + BU\int_0^t \Phi(\tau)d\tau$$
$$X\Phi(t) - x(0)e^T\Phi(t) = AXP\Phi(t) + BUP\Phi(t) \tag{13}$$

Developing the Equation (13) and grouping in matrices systems, have the following equation:

$$\begin{bmatrix} A & B & x(0)e^T \end{bmatrix} \begin{bmatrix} XP \\ UP \\ e^T \end{bmatrix} = X \tag{14}$$

Considering H_{ss} = [A B $x(0)e^T$]; J_{ss} = [XP UP e^T]$^{\mathrm{T}}$; and E_{ss} = [X] it is possible to write $H_{ss}J_{ss} = E_{ss}$. Solving this equation it is possible obtain H_{ss} and, so, obtain the triple (A, B, C).

3. NUMERICAL APPLICATION

The proposed methodology was numerically applied in a mass-spring system, as shown in Figure (1). The values of physical parameters were considered as 20 Kg, 1000 N/m and 20 Ns/m, for mass (M1 = M2), stiffness (K1 = K2) and damping coefficient (C1 = C2), respectively. It was considered random noises of 2% and 5% of energy in the output and input signals. Three sines with 5 Hz, 15 Hz and 25 Hz were considered as input excitations separately, all located on mass M1.

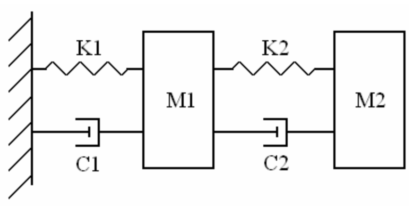

Figure 1. Mechanical system with two degree of freedom.

Using Legendre series, Table 3 presents the real and estimated natural frequency for the first and second vibration modes. It is possible observe that relative errors between them was small when was considered the first case of noises effect. Considering 5 % of the energy as noises the relative errors were smaller when it was considered the input excitation with low frequency. There is the relative error of approximately 13 % in the second estimated natural frequency when it was used the input excitation with 25 Hz. Table 3 also shows the optimal number of terms used to expansion of the signals. This number was obtained minimizing the RMS between the real and the expanded signal. Table 4 shows identified damping factors for the first and second modes, considering the same noises effects; excitation inputs and terms of expansion. Also, it is possible observer the relative errors *Ef1* and *Ef2* – in percentage.

Table 3. Natural frequencies (Hz) identified using Legendre.

Noise (%)	Frequency Excitation (Hz)	Terms of Expansion (r)	1ª Natural Frequency **(Real)**	1ª Natural Frequency **(Estimated)**	*E1* (%)	2ª Natural Frequency **(Real)**	2ª Natural Frequency **(Estimated)**	*E2* (%)
2	5	60	1.1254	1.1240	**0.1275**	1.9492	1.9356	**0.6986**
	15	65		1.1344	**0.8037**		1.9523	**0.1546**
	25	80		1.1274	**0.1770**		1.9028	**2.3817**
5	5	95		1.1360	0.9383		1.9070	2.1653
	15	100		1.1456	1.7935		1.8067	7.3134
	25	90		1.1280	0.2297		1.6993	12.821

Table 4. Damping factors identified using Legendre.

Noise (%)	Frequency Excitation (Hz)	Terms of Expansion (r)	1° Damping Factor **(Real)**	1° Damping Factor **(Estimate)**	*Ef1* (%)	2° Damping Factor **(Real)**	2° Damping Factor **(Estimate)**	*Ef2* (%)
2	5	60	0.0707	0.0664	6.0436	0.1225	0.1038	15.285
	15	65		0.0664	6.0342		0.1122	8.4111
	25	80		0.0695	1.6948		0.1187	3.1037
5	5	95		0.0562	20.524		0.0610	50.194
	15	100		0.0641	9.3444		0.973	20.573
	25	90		0.0625	11.612		0.0883	27.925

Only for a comparison, Figures 2a and 2b show the identification realized using the Legendre series without and with noises, respectively. The signal is correctly estimated using the orthogonal series when there is not noise, but considering one the results are not very good. So, the noises effects can generate errors in the modal parameters identification. In this first test was considered in each input and output signal 2 % of the energy as noises and an input excitation with 15 Hz; also, the number of terms expansion used was r = 74. Figures 3 and 4 show the displacement and the velocity of the first and second degree of freedom (dof), respectively. To use the orthogonal series to represent the real signals is the most important step for a good identification process using this methodology.

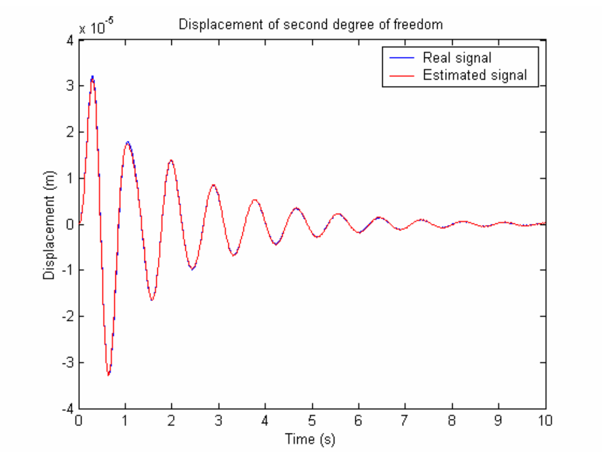

Figure 2a

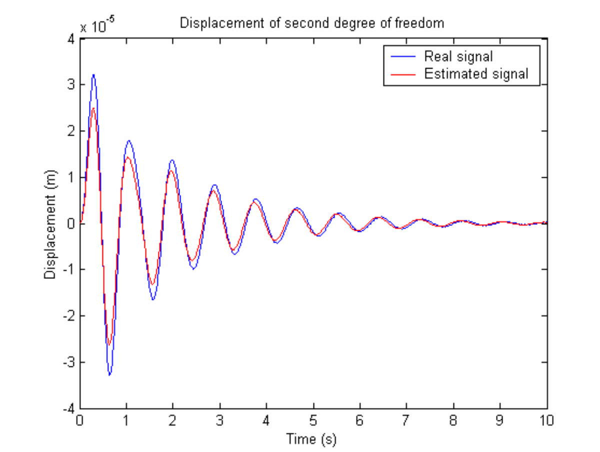

Figure 2b

Figure 2. Signals identified by Legendre – (a) without noises; (b) with noises (2 % of the energy and input excitation with 15 Hz).

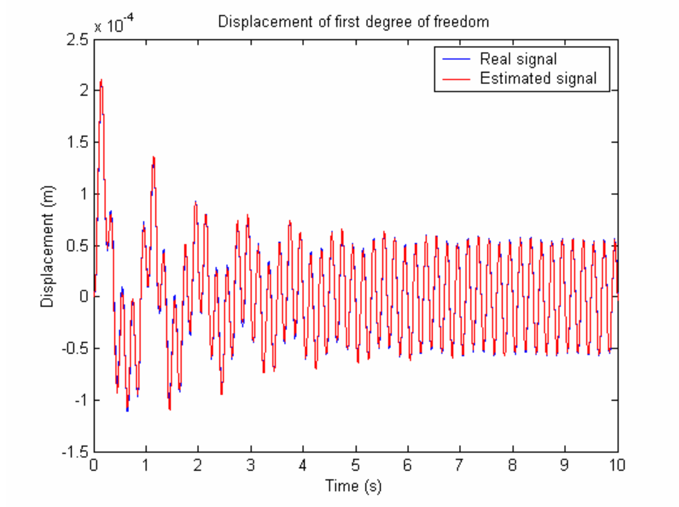

Figure 3. Signals identified by Legendre – 2 % of noise for 5 Hz. Displacement of the first dof.

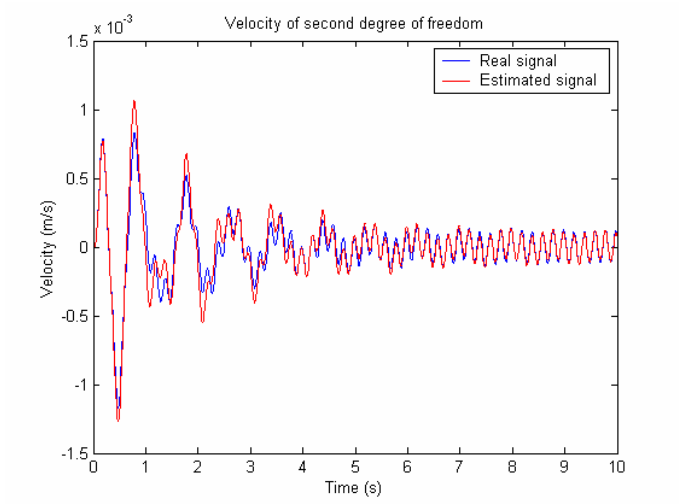

Figure 4. Signals identified by Legendre – 5 % of noise for 5 Hz. Velocity of the second dof.

Using Chebyshev series, Table 5 presents the real and estimated natural frequency for the first and second vibration modes. It is possible observe that using this orthogonal series, the relative errors between real and estimated was small when was considered the first case of noises effect. Considering 5 % of the energy as noises there is considerable identification errors in the second natural frequency for all input excitations. Table 5 also shows the optimal number of terms used to expansion of the signals, obtained minimizing the RMS between the real and the expanded signal. Table 4 shows identified damping factors for the first and second modes, considering the same noises effects; excitation inputs and terms of expansion. Also, it is possible observer the relative errors *Ef1* and *Ef2* – in percentage.

Table 5. Natural frequencies (Hz) identified using Chebyshev.

Noise (%)	Frequency Excitation (Hz)	Terms of Expansion (r)	1ª Natural Frequency **(Real)**	1ª Natural Frequency **(Estimated)**	*E1* (%)	2ª Natural Frequency **(Real)**	2ª Natural Frequency **(Estimated)**	*E2* (%)
2	5	85	1.1254	1.1227	0.2417	1.9492	1.9512	0.1029
	15	105		1.1282	0.2536		2.0515	5.2449
	25	95		1.1281	0.2396		1.8779	3.6592
5	**5**	95		1.1139	1.0223		1.7036	**12.604**
	15	105		1.0911	3.0511		1.5035	**22.869**
	25	95		1.0642	5.4347		1.3650	**29.972**

Table 6. Damping factors identified using Chebyshev.

Noise (%)	Frequency Excitation (Hz)	Terms of Expansion (r)	1° Damping Factor **(Real)**	1° Damping Factor **(Estimate)**	*Ef1* (%)	2° Damping Factor **(Real)**	2° Damping Factor **(Estimate)**	*Ef2* (%)
2	**5**	85	0.0707	0.0565	**20.141**	0.1225	0.0733	**40.125**
	15	105		0.0368	47.934		0.0166	86.410
	25	95		0.0587	**16.937**		0.0897	**26.772**
5	5	95		0.0649	8.1548		0.0156	87.302
	15	105		0.0840	18.742		0.0107	91.259
	25	95		0.1687	138.64		0.0153	87.521

Figures 5a and 5b show the identification realized using the Chebyshev series without and with noises, respectively, as was shown in Figure 2a. In this test was considered in each input and output signal 2 % of the energy as noises and an input excitation with 5 Hz; also, the number of terms expansion used was r = 80. Figures 6 and 7 show the displacements of the first and dof, respectively.

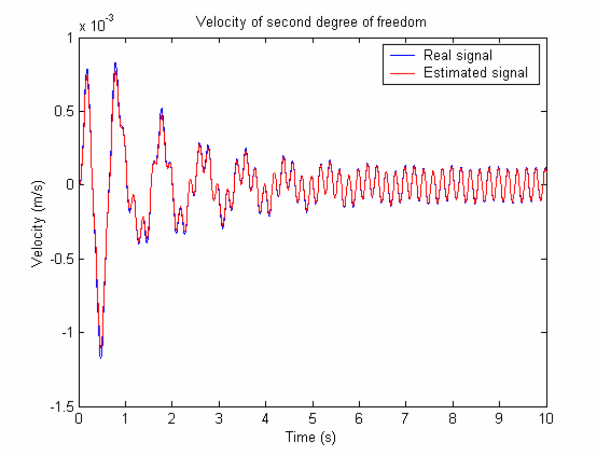

Figure 5a

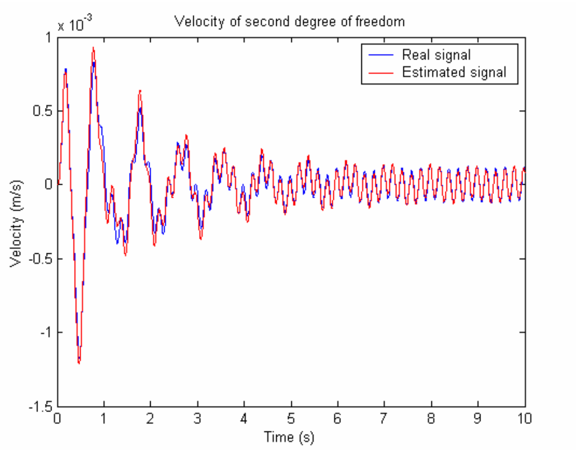

Figure 5b

Figure 5. Signals identified by Chebyshev – (a) without noises; (b) with noises (2 % of the energy and input excitation with 5 Hz).

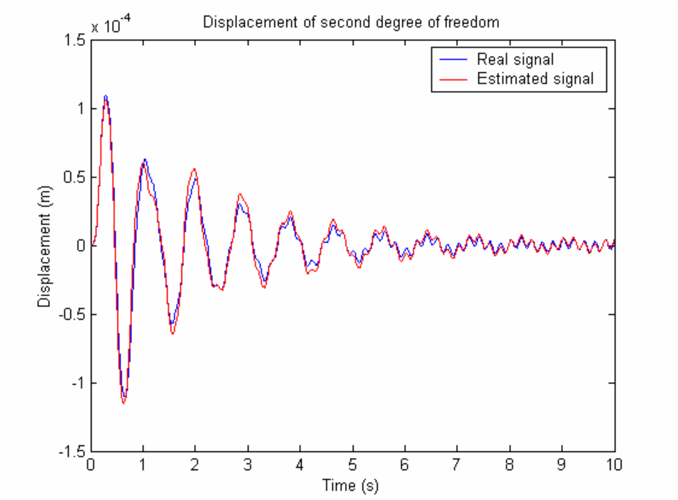

Figure 6. Signals identified by Chebyshev – 2 % of noise and input excitation with 5 Hz (Displacement of first dof).

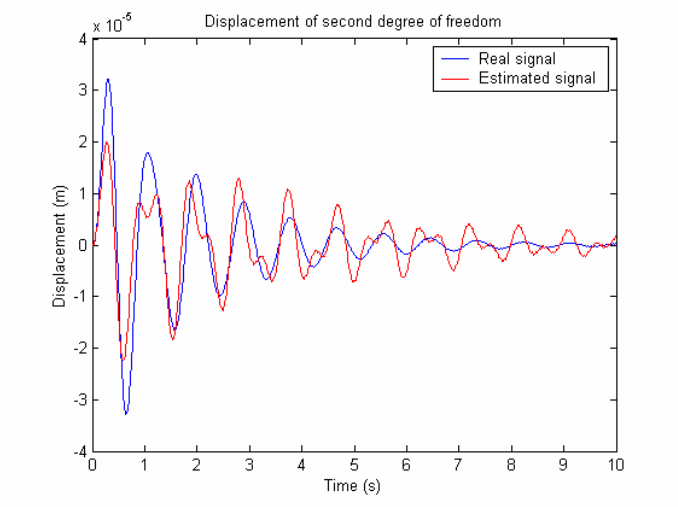

Figure 7. Signals identified by Chebyshev – 2 % of noise and input excitation with 15 Hz (Displacement of second dof).

4. FINAL REMARKS

In this paper a methodology of parameters identification through orthogonal functions series in the state space model was applied in a mechanical system. In the input and output signal were considered random noises of 2% and 5% of the energy of signals. The results showed that the methodology efficient for systems identifications using experimental data with noises. There was limitation to identify the damping factors. It is a complex problem and further work will be done to verify if it is a draw back of this methodology. In future works others series of orthogonal functions will be analyzed to minimize the difference between natural frequencies from experimental data and numerical model.

5. ACKNOWLEDGEMENT

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