COMPARATIVE STUDY BETWEEN THE TECHNIQUES OF WAVELET DECOMPOSITION AND ENVELOPE DETECTION FOR BEARING DAMAGE DIAGNOSIS

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Abstract. Bearings are the key components responsible for the premature rotating equipment damages. To monitor its operatinal condition through predictive techniques is essential to prevent unexpected damages to occur. In this work, a comparative study between vibration analysis techniques of bearing faults detection (Wavelets Decomposition and Envelopes Detection) is presented. Initially, these techniques were applied to inner and outer races mathematical models of faults, using MATLAB environment. Then, a specific study was held in order to compare these techniques. Finally, through acquisition of real faults signals, these techniques were applied in roll bearings with induced damages in each races and both races simultaneously under variation of the amount of lubricant and submitted to different load levels. The tests were held using an experimental bench. The techniques were validated through the signals, collected by Microlog SKF GX-E and the SKF Basic Vibration Analysis System software.

Keywords: Bearing Faults, Wavelet Decomposition, Envelope Detection

1. INTRODUCTION

Bearing faults are one of the most common types of faults in several industry segments. In many applications, the economic impact of faults in bearings can extremely exceed the cost of the machine. Monitoring its operational condition, through predictive techniques, is essential to prevent unexpected damages to occur, thus increasing the equipment availability inside the industrial plant.

Among the most common causes of bearing faults are distinguished: incorrect selection, inadequate stockpiling, extreme or insufficient lubrication, imperfection sealing and parasite currents through the bearing housing.

In this work the Wavelets Decomposition and Envelope Detection techniques are presented. A comparison between the efficiency of these methods is made demonstrating its advantages and limitations, determining the techniques behavior according to lubricant variation, load variation and damages localization.

Initially the techniques were applied to mathematical fault models, implemented in MatLab environment, for the inner race and outer race of the bearings. A comparative study was done between these techniques through the generated artificial signals. Through the fault real signals acquisition, the techniques were applied to bearing with induced damage in each race and both races simultaneously, under lubricant variation and still submitted to three different load levels.

2. EXPERIMENTAL BENCH

The experimental tests were held using an instrumented test desk, represented in Fig. 1, set up in LASID - Laboratório de Sistemas Dinâmicos, in Mechanical Department of the UFSJ - Universidade Federal de São João del-Rei.

Before starting the tests, the bench was balanced dynamically and aligned with laser technology to eliminate the undesirable vibration sources. The flexible test desk structure turned possible that the induced defects became more perceivable in the collected signals.

The damages were induced in a set of bearing series 6206. The bearing [1] is mounted in a clamped axle of a device [2] that, together with an industrial belt A-43 [3], allows radial load variation. The bearing is put into motion by the belt that is activated by an induction motor WEG [4], squirrel-cage with 1/2 cv, 4 poles and 1720 rpm, through a pulley [5].

Fault real signals were collected using a data acquisition plate and an accelerometer, located in the axle in radial direction in relation to the load region. It was possible to apply the techniques in bearings with induced damages in each race and both races simultaneously, under lubricant variation and still submitted to three different load levels equal 3 kgf, 4 kgf and 6 kgf. The load values were obtained through an industrial belt tension measurer (Industrial Goodyear V-Belt Tension Tester). The used plate is a multifunction data acquisition (DAQ) *National Instruments* NI-6251. It has 16 canals with maximum sampling frequency of 200 KHz.

The techniques had been validated by the collector and signals analyzer Microlog SKF GX-E [6].



Figure 1 - Instrumented test desk

2.1. Introduced Damages

The 6206 series corresponds to the fixed bearing of spheres (rolling) career. As they belong to a representative type, they cover an extensive amount of applications. They allow not just radial load but also axial load in both directions. They are indicated for applications that require low or moderate load, low noise, low vibration and high rotation speed, (SKF, 2001).

In all the tests, the damages introduced in bearings races simulate typical damages, as the False Brinel and the Axial Trines. The damages were done using drills used in dental treatment. The artificial damages are perpendicular to the bearing diameter and have dimensions of $1 \ge 7,5$ mm, with depth of approximately 0,4 mm.

2.2. Data Analyzer Microlog SKF GX-E

In Fig. 2, it's shown the collector and signals analyzer Microlog SKF GX-E with its accelerometer SKF CMSS2200. The vibration signals are transmitted to the computer through the support program SKF Basic Vibration Analysis System. This procedure allows to carry analyses more detailed, to build databases and to generate technical reports.



Figure 2 - Microlog SKF GX-E

3. VIBRATION MODEL FOR BEARING FAULTS

In bearing housing, the local damages cause a series of impacts that can excite resonances in the box structure or in an engine, for example, between 1 kHz and 20 kHz.

The characteristic frequencies in (Hz) of the external race (BPFO - Ball Pass Frequency Outer Race), of the internal race (BPFI - Ball Pass Frequency Inner Race), of the rolling element (BSF - Ball Spin Frequency) and of the roller cage (FTF - Basic Train Frequency), are given, respectively, for the Equations (1), (2), (3) e (4), where $\mathbf{f_r}$ is the rotation frequency (Hz), \mathbf{d} is the rolling diameter (mm), \mathbf{D} are the primitive bearing diameter (mm), \mathbf{n} is the number of rolling and $\boldsymbol{\beta}$ is the contact angle of the bearing, as shown in Fig. 3, (Brito, 2002).

According to mathematical models, the bearing of series 6206 has BPFI approximately 145 Hz and BPFO approximately 95 Hz.



Figure 3 - Bearing elements

$$BPFO = \frac{n}{2} f_r \left(1 - \frac{d}{D} \right) \cos \beta \tag{1}$$

$$BPFI = \frac{n}{2} f_r \left(1 + \frac{d}{D} \right) \cos \beta \tag{2}$$

$$BSF = \frac{d}{2D} f_r \left[1 - \left(\frac{d}{D}\right)^2 \cos^2 \beta \right]$$
(3)

$$FTF = \frac{1}{2} f_r \left(1 + \frac{d}{D} \right) \cos \beta \tag{4}$$

3.1. Modulation

There are several modulations types, among them could be distinguished: Amplitude Modulation (AM), Frequency Modulation (FM) and Phase Modulation (PM). When the bearings are submitted to the constant speed, the damages that occur in race points or rolls that dislocate in relation to the load region, experience amplitude modulation, (McFadden, 1984; Braun, 1986; Mitchell, 1993; Bezerra and Pederiva, 2004).

To obtain a frequency modulation in the bearing is necessary that the bearing rotation frequency suffers variation during the acquisition process. During the assays done for this work, the bearing rotation speed was kept constant. Then if there is modulation in some signal of fault, it is going to be in amplitude.

Two waves are necessary to have modulation: a modulating and a carrier. In the amplitude modulation the carrier will have its amplitude modified proportionally to the modulator signal. In this case, the modulator signal is the external race rotation 1800 rpm (30 Hz), with frequency smaller than the frequency of the carrying signal, fault frequency in the internal race (145 Hz) and external (95 Hz).



Figure 4 - Carrying function (a), modulating (b), the modulation (c) and the modulated specter function (d)

In Fig. 4, for the generated artificial signals proceeding from damages in the external race (BPFO = 95 Hz), are represented a carrying function (a), a modulating function (b), the modulation (c) and the specter of the modulated function (d), after being submitted to Butter Worth filter band -pass with cut band between 2.000 Hz and 8.000 Hz. The intention of the filter is the elimination of the noise (15%) added to the artificial signal, in order to describe its behavior faithfully. It is verified that in the modulated function specter, the frequency of the carrying signal (central resonance frequency adopted is 4.000 Hz) appears surrounded by spaced frequencies from main one, with the same value of the modulation frequency signal (BPFO).

According to Ponci and Wedge (2005), when the rotating race has a defect, the vibration intensity increases when the race damaged region passes by the load zone, provoking an amplitude modulation of BPFO component for the race rotational frequency.

3.2. Demodulation

As seen in Fig. 5, from the generated artificial signal for damage in external race, after the modulation, it's obtained a signal whose peaks are linked by a curve, represented by the Fig. 5 (b), called as envelope. The amplitude demodulation process consists in extracting the envelope. When obtaining the envelope, an analogical process can be used through plates called "envelope detector" or digitally through the Hilbert Transform, (Haykin, 1989; Haykin and Veen, 2001; Bezerra and Pederiva, 2004).

For being a procedure of easy implementation and versatile, it was opted the use of the Hilbert Transform. The Hilbert Transform expresses a relation between the real and imaginary components of the Fast Fourier Transform of a signal that is null for the negative time, (Bendat and Piersol, 1986; Randal, 1987).

The graph (a) of Fig. 5 represents a signal that, after the modulation process, "was enveloped", and the graph (b), represents the envelope obtained after the application of the Hilbert Transform.



Figure 5 - Artificial signal modulated (a) and its envelope (b)

4. ENVELOPE DETECTION

If the central frequency of resonance is unknown the envelope technique is composed of the following procedures. The first one is the application of the Fast Fourier Transform to the signal. With the application of the Fast Fourier Transform the filtering band can be defined. The intention of this procedure is to know the central frequency of resonance (fc). For the bearing used in the test, is around 3.000 - 7.000 Hz, according to Fig. 6 of the real signal processing of damages in the external race, respectively without and with defect.



Figure 6 - Identification of the resonance central frequency through the FFT

The second stage is the application of the band-pass filter, represented for the Fig. 7 (a) through the treatment of artificial signals. The idea is to eliminate frequencies of undesirable vibrations, as the low frequencies of high amplitude, which in general, are related to the misalignment and unbalancing. A band-pass filter with cut band between 3.000 and 7.000 Hz was used for the processing of the defect real signals.



Figure 7 - Envelope Detection through artificial signals BPFO

The following step is the application of the Hilbert Transform that aims to obtain the defect signal envelope, Fig. 7 (b). After that, the high-pass filter is applied to eliminate the component DC. Finally the Fast Fourier Transform is applied to the signal envelope to obtain the characteristic defect frequencies, showed in Fig. 7 (c).

5. WAVELETS DECOMPOSITION

The Wavelet Transform (WT) does not use a fixed scale in the analysis, but it varies the scale to prevent commitment with a specific one. This transform is independent of the scale. It makes possible to the signal to be analyzed with good resolution in the time or frequency. For a narrow window, there is a good resolution in the time and a low resolution in frequency. For a wide window, there is a good resolution in frequency and a low resolution in the time.

The Wavelet Transform represents an advance in relation to the Fourier Transform, because it is one technique that uses variable scales. The analysis by wavelet allows the use of a smaller scale when a greater resolution of the information contained in the signal in high frequency is desired, and a bigger scale when a greater resolution of the information contained in the signal in low frequency is desired. Frequency and scale are related inversely, that means, a smaller scale implies high frequency and bigger scale implies low frequency, (Staszewski and Tomlinson, 1994; Satish, 1998).

There are several wavelets functions that can be used as wavelet mother, among them are distinguished: Haar Wavelets, Morlet, Daubechies, Meyer, Coiflet, Gabor, Cauchy, Bessel, Marr ('Mexican hat'), Poisson, Bessel, Shannon, Franklin and others.

In this work, it is used Daubechies wavelets because it is available in wavelet toolbox of the Matlab® software.

According to Santiago and Pederiva (2004), one of the wavelet analysis advantages is that it allows the use of a smaller scale when better resolution in the time of the information contained in the signal in high frequency is desired and bigger scale when better resolution in frequency of the information contained in the signal in low frequency is desired. This is clearly observed in the frequency domain, as showed in Fig. 8.



Figure 8 - Wavelets Decomposition

Figure 9 - Wavelet Filtering Process

Similar to the Fast Fourier Transform (FFT), there is an algorithm for implementation of the Discrete Wavelet Transform (DWT) based in the Fast Wavelet Transform (FWT) decomposition, that normally it is used and known as Multi Resolution Analysis (MRA) or Mallat Pyramidal Algorithm, which was developed by Mallat in 1988, (Misiti *et al*, 1997; Mallat, 1989). This algorithm uses a special filtering process to decompose the signal, where, the content of the signal in low frequency is called *Approach*, and high frequency it is called *Detail*. This filtering process decomposes the original signal in *Approaches* and *Details*, relating the filter according to the wavelet family, and can be understood as low-pass and high-pass filters, respectively, as its shown in Fig. 9, where **S** represents the signal, **A** and **D** represents, respectively the *Approach* and the *Detail*.



Figure 10 - Wavelet Decomposition Tree of a signal in three levels.

Summarizing, the multi resolution theory allows the decomposition of a signal as follows: first, a discrete original signal (S) is decomposed in the first level in two components and for a low-pass and high-pass filter, respectively. A1 is called approach of the signal and D2 is called detail of the signal. For the second level, the approach now is decomposed in a new approach, A2 and a detail D2. This procedure can be repeated up to seven levels for the family db order 4 (db4) and up to ten levels for the family db order 1 (db1). Fig. 10 shows the wavelet decomposition tree of a signal in three levels.

The DWT shown in Fig. 10 only decomposes the signal in low frequencies or from the *Approach* coefficient. Observing the Fig. 11, the vectors hold information of the original signal in different frequencies bands. In this case the signal sampling frequency is 40.000 Hz, and then the frequency band of analysis related to the A1 vector is 0-20.000 Hz, for D1 is 20.000-40.000 Hz, for A2 is 0-10.000 Hz, for D2 is 10.000-20.000 Hz, for A3 is 0-5.000 Hz, for D3 is 5.000-10.000 Hz and so on. As the central frequency of the artificial signal 4.000 Hz was adopted, the Detail level 4, as much for db1 as well as for db4, demonstrates better signal for BPFI. D4 has a frequency band between 2.500 Hz and 5,000 Hz. Inferior and superior levels return signals with smaller amplitudes. For BPFO and defects in both races simultaneously the behavior is similar.



Figure 11 - Detail Levels BPFI for db4 through artificial signals

6. RESULTS

The same procedures used for Envelope Detection and Wavelet Decomposition through the artificial signals were used for the damage real signals processing in the races internal, external and in both simultaneously. The real defects signals were collected in a sampling frequency equal 40,000 Hz and 20,000 points for second.

The greater the load the greater will be the carrier frequency amplitude and greater will be the specter amplitude. These amplitudes are directly related to the load levels amplitude in the loading region. According the graph of Fig. 12, the bigger the load levels, as well as smaller lubricant levels, the greater will be the signal energy. According to Abreu *et al.* (2006) the variation in load levels and lubricant affect significantly the vibration and the development rate of damages in the bearing. In this case the energy was measured in RMS and Peak - Peak from db4 through the Detail level 3, Fig. 12 (a), for BPFI and from the signal in the time, Fig. 12 (b), also for BPFI. Lub 1 corresponds to the plant sufficiency. Lub 2 approximately corresponds 60% of the plant sufficiency. The energy related to BPFO and to both faults simultaneously has the same behavior.



Figure 12 - Energy variation of the Wavelet due the Load variation and Lubricant level

6.1. Envelope Detection

According to the envelope showed in Fig. 13, obtained by the implementation in Matlab environment of the damage real signals, it's noticed that the amplitude of characteristic damage frequency in the external race, 94,6 Hz, are smaller when compared to the defect signals envelope in the internal race with damage frequency of 142,7 Hz, Fig. 14. This

behavior is due to the amplitude modulation of the damage signal caused by the bearing external race frequency of rotation, in a load region of 3 kgf.



The spectra of damages in both races simultaneously have the same behavior. The envelope of the real signal processing corresponds to the generated artificial signals envelope, Fig. 7 (c), and to the Microlog, Fig. 15. All the fault real signs processed for damage in races and in the three load levels, corresponded with the generated artificial signals and with the Microlog data. This is a validation of the routines used for damages models implementation.

Through the artificial signal envelope for defects in the internal and external race simultaneously, it was possible to identify the BPFO, equal to 92,5 Hz, after the increase of the definition by 25 times.

In the artificial signals and fault real signals processing, the amplitudes in BPFO frequency, equal to 94,67 Hz, are presented as lateral bands of the BPFI harmonic.

For BPFI, with the increase of the load, the system operating from 3 kgf to 6 kgf, the number of amplitudes in faults frequencies increased, corresponding to the load increase ratio, as well as the value of the amplitudes.





For BPFO, with a load increasing, from 3 kgf to 6 kgf, there was a slightly increase in the amplitudes of the fault frequencies and the number of harmonics. There was not an increase of harmonics and amplitudes proportional to the load rise due the modulation in amplitude caused by the external race rotation speed.

For BPFI and BPFO simultaneously, with a load increasing from 3 kgf to 6 kgf, the amplitude of the characteristic fault frequencies in the internal race were higher in relation to the same procedure followed for internal race fault only. This behavior is due to the existence of damage in the race under movement, which favors the fault frequencies excitement of the stationary race, what leads to an amplitude increase. The BPFO kept similar behavior to the analysis done for the damage in the external race. In all the load levels, the BPFO are presented as lateral bands of the BPFI.

The envelope analysis of the real signal for a bearing with inserted damage outside of the load region (internal race), using the same filter with the same filtering bands and submitted to the three load levels, according to Fig. 16 for 3 kgf, does not detect the characteristic fault frequencies in the internal race. It was only possible to detect the fault frequencies, by the Fig. 17, after band-pass filter adjustment, with cut band between 5.500 Hz and 7,000 Hz. This behavior kept constant for the others load levels, with exception for the amplitude value.



Figure 16 - BPFI

Real Signal Envelope - 3 Kgf Lub - BPFI

Figure 17 - BPFI, after pass-band filter adjustment

6.2. Wavelets Decomposition

Using the Daubechies family, composed by 10 Wavelets, results proceeding from the families db1, db4, db6 and db8, for BPFI under 3 kgf of load were compared. A good approach was reached with the use of db1 and db4. Between these two orders db4 showed a greater energy in the Detail level 3, Fig. 19, in relation db1 - D3, Fig. 21. The Details only bring the information of high frequency, as the damage introduced in the bearing races. The Detail level 3 correspond to the frequency band between 5.000 Hz and 10.000 Hz, which contain the fc. For the used bearing, fc is approximately 3.000 - 7.000 Hz, more precisely around 6.000 Hz. In the tests with artificial signals the Detail level 4 was more efficient as an fc of 4.000 Hz was adopted. The Detail level 4 corresponds to the frequency band, between 2.500 Hz and 5,000 Hz.

The details of smaller levels, as D1, must present less energy, proceeding from the defects impacts in the races. This Detail has a frequency band distant of fc, varying between 20.000 Hz and 40.000 Hz, consequently it must have less interferences proceeding from the defects in the races. According to Fig.18 and Fig.20, db4 presented a D1 more coherent, with less interference.



The family db order 4 also demonstrated the Approach (A) more coherent in relation to db1, this, due to its function that represents an Orthogonal Wavelet, Fig. 22. The family db1, also known as Wavelet Haar, is discontinuous and represents a function with a step, Fig. 23. The Approaches only bring low frequency information, as the component DC equal to 60 Hz. The bigger the Approach level, the more susceptive the identification of the low frequencies components it becomes and the smaller will be the high frequency interferences. Still for db1 the decomposition is necessary in more a level (A6), in frequency band between 0 and 625 Hz, so that it does not suffer interference from the high frequency components proceeding from the impacts generated by the damages in the races, presenting, according to Fig. 24, a characteristic signal of a function step. According to Fig. 25, in db4 A5, with frequency band between 0 and 1.250 Hz, returns better approach besides needing one level less of decomposition.

The same considerations made for damages in the internal race, are made for damages in the external race and both races simultaneously. The amplitudes of the Details for BPFO are smaller, in relation to BPFI. As well as for the Envelope Detection, the Wavelets Decomposition also suffers modulation influence submitted by the damage signals proceeding from the race in movement.

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Through the envelope of db1 D3 and db4 D3, for damages outside the load region of 3 kgf and without the use of the band-pass and high-pass filters, was not possible to detect the faults characteristic frequencies, surrounded by the carpet formed by the noise presence, Fig. 26. Through db4 D2 and db1 D2 the deterministic frequency for BPFI presented itself detectable. According to Fig. 27 with BPFI equal 145,7 Hz, the envelope of db4 D2 resulted in a signal with greater energy. For db4 D1 and db1 D1 the frequencies are identified with less interference of noises, however with low amplitude.



Figure 26 - Envelope D3 db4, BPFI

Figure 27 - Envelope D2 db4, BPFI

Outside the load region of 4 kgf, the envelope of db4 D1 and db1 D1 detect the frequencies with less noise interference in relation to db4 D2 and db1 D2. The envelope of db4 D1 presented less random interference noises. As well as for the load region of 3 kgf, it was not possible to identify the fault frequencies proceeding from the envelopes of db1 D3 and db4 D3.

Still for damage outside of the load region, now submitted to 6 kgf, through the envelope of db1 D1 and db4 D1, it is possible to identify the fault frequency for internal race, with its harmonic and without great interference of noise. db1 D1 presents greater energy. The envelope of db4 and db1 D2 presents considerable noise and only identifies the first harmonic. From db1 and db4 D3, the characteristic frequencies with identical energy to the envelope proceeding from the standard procedure of envelope detection is identified, already with new filtering band.

7. CONCLUSION

The results show that the analysis based on the signal decomposition through the Wavelet and on the Envelope Detection allows the detection of damages in bearing races.

For the energy quantification through waves in time domain and the wavelet details it is possible to verify how much the lubricant insufficiency, as well as the load variation influence the signals energy levels. This variation significantly affects the rate of bearings damages development.

The disadvantage of the envelope detection is the choice of the filter and its cut bands. The wavelet decomposition has, due to the Mallat Pyramidal Algorithm, an automatic association of the filter, according to the mother wavelet adopted. In both techniques is necessary to know the central frequency of resonance. To know the fc, is what will determine the cut band of the filters used in the technique of envelope detection. In the wavelet decomposition technique the decomposition levels are determined according to the value of the fc, which will determine the best Detail level to be evaluated.

The damage placed outside the load region does not promote an efficient excitement of the fault frequencies. The real signal envelope analysis for a bearing with inserted damages outside the load region, only detect the fault

characteristic frequencies after the band-pass filter adjustment. It is practically necessary to know the exact fc of the bearing.

After the wavelet decomposition, the details obtained from the damages outside of the load region, were submitted to the envelope detection without the use of the high-pass and band-pass filters. From wavelets db1 and db4 were possible to identify the BPFI, according to the load variation.

The wavelet decomposition application as well as the envelope detection (already a well known technique) is very important in the condition monitoring and diagnosis of bearing damages.

In future works, it is intended to apply the wavelets decomposition for mechanical faults detection, as misalignment and unbalancing. A study will be held using the Wavelet Packet Transformed (WPT) theory, which is a generalization of the Discrete Wavelet Transformed (DWT), for the identification of faults incipient in the races, roll cage and bearing spheres. It will be also studied the viability of the Short-Time Fourier Transformed (STFT) application. The STFT transforms a signal in time domain into a bi-dimension function in the time-frequency domain, represented through a spectrogram, for bearings damages diagnosis.

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