LOW THRUST TRANSFERS FROM THE EARTH TO HALO ORBITS AROUND THE LIBRATION POINTS OF THE SUN-EARTH/MOON SYSTEM

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Abstract. *As a natural step of the advance of the human explorations, the limits of low-Earth and geostationary orbits are being exceeded. The libration points, defined as the equilibrium points of the Restricted Three Body Problem, are one of the most promising candidates of being part of future missions, receiving great attention in the last 25 years. The use of the Dynamical Systems Theory, in special the determination of invariant manifolds, provides us with sets of orbits that can be used as a fast and cheap way to transfer a spacecraft from the a vicinity of the Earth to the libration points. On the other hand, the use of solar electric low thrust thrusters, with their outstanding efficiency, arises as one of the most promising technologies in order to support the increasing need for larger payloads in space missions. Considering this, the main objective of this project was the determination and study of transfers from low Earth orbits to Halo orbits around the libration points of the Sun-Earth/Moon system by means of low thrust and using the stable manifolds associated to the target orbits. The interesting transfers were the ones that start in low eccentricity and low inclination parking orbits. This study was made with a series of increasing complexity models, the last one based on the JPL Ephemerides.*

Keywords: *libration points, dynamical systems theory, low-thrust, restricted three body problem, orbital transfers*

1. INTRODUCTION

Due to their unique characteristics, the libration points have been receiving much attention in the last 25 years. The Lagrange relative positions known as points 1 and 2 of the Sun-Earth and Earth-Moon systems present nowadays the most practical applications, since they generate and control many trajectories with interesting characteristics to space missions and planetary science (Canalias et al, 2004) (Folta et al., 2002). The International Sun-Earth Explorer 3 (ISEE-3) launched in 1978 was the first libration point mission and orbited a halo orbit around Sun-Earth libration point 1 (SEL1) for around 4 years.

Interesting trajectories around these points are called stable and unstable manifolds, surfaces in the trajectory design space consisting of orbits that wind on and off periodic orbits such as halo orbits. The use of these manifolds in the transfer trajectory design is known as the *Dynamical Systems Approach to the Transfer Problem*. Due to the important saves of propellant, this method is receiving great attention (Masdemont, 2005) (Gomez et al., 1993). The Genesis mission, launched in 2001, was the first designed with this method.

By other hand, the most famous and used spacecraft propulsion system type is the chemical, where the propellant is heated and pressurized by a chemical reaction (usually its own combustion). The result is a strong thrust applied for seconds or minutes that is able to lift a spacecraft directly from the Earth's surface to an orbit around it.

The major disadvantage of chemical rockets is their small *specific impulse* in order of 200 to 465 s (Humble, Henry and Larson 2005). Defined as the total impulse per unit weight of propellant, it represents how much mass of propellant is needed to make a certain mass (of the spacecraft) to perform an orbital transfer or movement. Therefore, when the spacecraft mass is huge, the amount of propellant mass needed to change its movement by using a chemical rocket becomes prohibitive.

In order to solve these problems, the study of low-thrust thrusters have been increasing in the last decades. Their main characteristic is to provide a very low thrust value but with high efficiency, i.e., using very few propellant mass. Numerous technologies are being studied in order to developed such thrusters (Gilland, Fiehler and Lyons, 2004) (Brophy et al. 2000). The most promising is the use of electrostatic ion thrusters that use Xenon as propellant and are able to provide specific impulses of the order of 10^5 s with a thrust-weight ratio between 10^{-4} to 10^{-6} for months or years continuously (Humble, Henry and Larson 2005).

The very low thrust provided by the electrostatic thruster makes it impossible to be used as the propulsive system of a launcher, however its enormous specific impulse makes the solution ”launch a spacecraft to a Low-Earth orbit and then raise it by using a low-thrust thruster” very interesting to the future of the space missions.

The use of the "Dynamical Systems Approach" to design libration point missions considering a low-thrust thruster to transfer the spacecraft from a low-Earth orbit to a libration point orbit is the object of this study. The first studies with this method were more concerned with high impulse transfers but already considered the use of low-thrust (Canalias et al., 2004) (Folta et al., 2002). We present a simple method to design such missions.

2. DESCRIPTION OF THE METHODOLOGY

2.1 The Restricted Three Body Problem and the Libration Points

The Circular Restricted Three Body Problem (CRTBP) consists in describing the motion of a very light particle under the influence of two massive bodies (known as the primaries) revolving in circular motion around their center-of-mass. The major simplification hypothesis are the circular motion of the primaries and that the third particle has such a light mass that its influence on the primaries is neglegible. The equations of the motion are derived straightforward from the well know Newton’s equation (see Szebehely, 1967) and are usually expressed in a dimensionless coordinate frame (*synodic*) that is fixed in relation to the primaries with origin in their center-of-mass, as presented below.

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} \qquad ; \qquad \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} \qquad ; \qquad \ddot{z} = \frac{\partial \Omega}{\partial z}$$

where x, y and z are the dimensionless positions of the third particle and Ω is given by

$$\Omega(x,y,z) = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu) \tag{1}$$

and r_1 and r_2 by

$$r_1^2 = (x-\mu)^2 + y^2 + z^2 \qquad ; \qquad r_2^2 = (x-\mu+1)^2 + y^2 + z^2 \tag{2}$$

where μ is the only parameter of the system, defined by the ratio between the mass of the two primaries. In this work we follow the assumption of the book by Szebehely (1967) and use $\mu < 0.5$. This implies that the heavier primary is located at the point $(\mu; 0; 0)$ while the lighter is at the point $(-1+\mu; 0; 0)$.

The system described by Eq. 1 has five equilibrium points known as *libration* or *Lagrangian* points. The first three points are collinear with the primaries, while L_4 and L_5 form equilateral triangles with them, as can be seen in Fig. 1. As an illustration of their distance, the points L_1 and L_2 of the Sun-Earth/Moon system are 1.5 millions of kilometers away from the Earth.

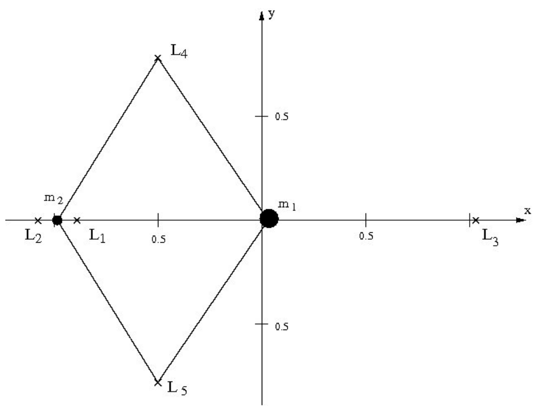

Figure 1. The Lagrangian points position in the usual CRTBP synodic reference system.

In a first approximation, the motion of the third particle in a vicinity of them can be seen as a composition of two oscillators and some hyperbolic behavior. For a specific amplitude both frequencies of the oscillators are equal and the resulting motion is periodic, known as Halo orbits.

Although, the hyperbolic behavior of the third body around the libration points causes the Halo orbits to be unstable, it is responsible for the existence of *invariant manifolds*. These are global families of trajectories that wind on and off periodic orbits such as Halo (see Fig 2). The name *stable* is used to the ones that tends to the periodic orbits when time tends to the infinity. A spacecraft that is inserted in one stable manifold will approach the periodic orbit without the use of any thrust.

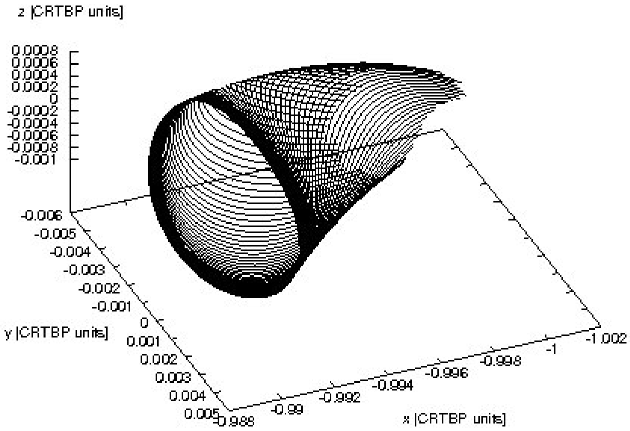

Figure 2. Stable manifolds of a L_1 Halo orbit of the Sun-Earth/Moon system

The manifold can be determined by the method described by Masdemont (2005) that uses the solution of the linear part of the CRTBP equations in terms of Legendre polynomials and proceeds an Lindsteck-Poincare expansion to express the global solution. The inputs to the method are one halo orbit amplitude (in-plane or out-of-the-plane) and a amplitude of the hyperbolic part of the solution which can be understood as the initial distance of the manifold to the target orbit.

2.2 Low-Thrust Thrusters

In order to model the effect of a low-thrust thruster in the motion of the spacecraft, we use in this work two basic hypotheses. The first is that the thrust magnitude can only assume one fixed value that is a parameter of the problem or zero in the coasting arcs. The second is that the thrust direction is always aligned with the velocity of the spacecraft with respect to the Earth in the inertial frame. It is shown by Kluever (1998) that, in this way, the variation of the semi-major axis of the orbit is maximized.

Following, we present the derivation of the analytical expression of this thrust model. Let $\vec{r}$ and $\vec{r}_i$ be the position's coordinate vectors of the spacecraft in the dimensionless synodic and sidereal frames, respectively. The relation between the two coordinate frames is given by

$$\vec{r}_i = \mathbf{A}\vec{r} \quad , \quad \mathbf{A} = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

Where $t = nt^*$ is the time span in synodic units, with n being the *mean motion* of the primaries. Then,

$$\dot{\vec{r}}_i = \dot{\mathbf{A}}\vec{r} + \mathbf{A}\dot{\vec{r}} \quad \text{and} \quad \dot{\vec{r}} = \dot{\mathbf{A}}^T\vec{r}_i + \mathbf{A}^T\dot{\vec{r}}_i \tag{4}$$

From now on, the subscripts 's' and 'E' will denote *spacecraft* and *Earth* respectively. Then, the relative position and velocity of the spacecraft with respect to the Earth in the synodic frame is

$$\vec{r}_{sE} = \vec{r}_s - \vec{r}_E \quad \text{and} \quad \dot{\vec{r}}_{sE} = \dot{\vec{r}}_s - \dot{\vec{r}}_E \tag{5}$$

From the definition of the synodic reference system, we have that $\dot{\vec{x}}_E = 0$. Therefore, from Eq. 4 and Eq. 5, the relative position and velocity in the inertial frame is given by the expression

$$\dot{\vec{r}}_{i_{sE}} = \dot{\mathbf{A}}\vec{r}_{sE} + \mathbf{A}\dot{\vec{r}}_{sE} \tag{6}$$

As we use the synodic reference system in the system equations (Eq. 1), the last step is to transform the Eq. 6 using the matrix shown as Eq. 3, therefore

$$\mathbf{A}^T\dot{\vec{r}}_{i_{sE}} = \mathbf{A}^T\dot{\mathbf{A}}\vec{r}_{sE} + \mathbf{A}^T\mathbf{A}\dot{\vec{r}}_{sE} = \hat{\mathbf{A}}\vec{r}_{sE} + \dot{\vec{r}}_{sE} \tag{7}$$

where

$$\hat{\mathbf{A}} = \mathbf{A}^T \dot{\mathbf{A}} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

Therefore, using Eq. 7 and Eq. 8 we got the following expression for the thrust direction $\vec{a}_T = (a_{Tx}, a_{Ty}, a_{Tz})^T$

$$\vec{a_T} = (a_{Tx}, a_{Ty}, a_{Tz})^T = (\dot{x} - y, \dot{y} + x + 1 - \mu, \dot{z})^T \tag{9}$$

2.3 The Low-Thrust Dynamical System Approach

The objective of our work was finding simple transfers with the following characteristics:

- Start in a Low-Earth parking orbit, with small eccentricity and inclination;
- With the use of low-thrust it describes a spiral trajectory around the Earth increasing its altitude;
- At a certain moment the trajectory meets a stable manifold of the target Halo orbit;
- From this moment the thruster is turned off and the spacecraft follows the stable manifold until the target orbit.

The system used was the Sun-Earth/Moon where the Sun is the heavier primary and the barycenter of the Earth-Moon is the lighter one. For this system we have the mass constant of $\mu = 3.040423398444176.10^{-06}$ according to JPL Ephemerides. We were interested in Halo orbits around its librations points 1 and 2 that are closer to the Earth.

The reason for low eccentricity and inclination of the target parking orbit is the economy in propellant for the chemical rocket that is needed to raise the spacecraft from the Earth's surface to the initial parking orbit.

Combining Eq. 1 with Eq. 9 we have a perturbed Restricted Three Body Problem that was used in this work, described by the following equations:

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \frac{\partial \Omega(x,y,z)}{\partial x} + F_T(t) \frac{a_{Tx}}{||\vec{a}_T||} \\ \ddot{y} + 2\dot{x} &= \frac{\partial \Omega(x,y,z)}{\partial y} + F_T(t) \frac{a_{Ty}}{||\vec{a}_T||} \\ \ddot{z} &= \frac{\partial \Omega(x,y,z)}{\partial z} + F_T(t) \frac{a_{Tz}}{||\vec{a}_T||} \end{aligned} \tag{10}$$

where $F_T(t)$ is the thrust magnitude that is equal to zero during the phase where the spacecraft is following the stable manifold and to a constant value during the thrust phase.

When stable manifolds are used in a transfer trajectory design, the method is known in literature as the *dynamical systems approach to the transfer problem*. Canalias et al. (2004) details the proceedings to find impulsive transfers through this approach.

In the case studied in this work, however, we are interested in low thrust trajectories, and, therefore, the methodology used was a little different than that, and it is presented below.

1. A target orbit is defined. This includes the definition of the aimed libration point, the type of orbit and its amplitudes and frequencies;
2. A local approximation of the stable manifold at a certain point of this orbit is taken using the method derived by Masdemont (2005). This includes the choice of a starting phase ϕ and a halo amplitude α_4. Moreover, it is necessary to choose the manifolds which goes towards your objective (in this case the Earth) and not in the other direction. The result is the determination of a line in the phase space based in a nominal point of the orbit;
3. With this starting state known, it is numerically integrated backwards in time for a defined period called t_{coast}, whose value is a chosen parameter. This integration is done with thrust magnitude equal to zero, i.e, only considering the CRTBP equations, thus we are going through the line determined in the phase space;
4. After t_{coast}, the vectorfield is changed to include the low thrust, and the integration proceeds backwards in time until a determined altitude with respect to the Earth is reached.
5. The last state obtained is used to calculate the orbital elements of the parking orbit.

As it can be seen, numerous parameters are involved in the problem. We tried to restricted our search for low thrust transfers limiting them to the interesting ones, whose parking orbits have small eccentricity and inclination. The Halo orbit amplitudes tested were restricted to the ones presented in Tab 1. Values for L_2 point are similar. The order of magnitude of the thrust/weight ratio used was 10^{-4}, which is consistent with real values.

Table 1. Halo orbit amplitude conversions for the L_1 libration point (Sun-Earth/Moon system)

Amplitude	Upper Furthest Point		Left Furthest Point		Bottom Furthest Point		Right Furthest Point	
[CRTBP units]	X Pos. [km]	Z Pos. [km]	X Pos. [km]	Y Pos. [km]	X Pos. [km]	Z Pos. [km]	X Pos. [km]	Y Pos. [km]
0.04	-1.483e+08	5.391e+04	-1.482e+08	6.614e+05	-1.479e+08	-6.689e+04	-1.482e+08	-6.614e+05
0.08	-1.483e+08	1.076e+05	-1.482e+08	6.696e+05	-1.479e+08	-1.340e+05	-1.482e+08	-6.696e+05
0.12	-1.484e+08	1.610e+05	-1.482e+08	6.832e+05	-1.479e+08	-2.015e+05	-1.482e+08	-6.832e+05

2.4 The CRTBP with Near-Earth Perturbations

Once interesting transfers were selected with the simple perturbed CRTBP model, the work evolved to the study of the impact of near-Earth perturbations in theses transfers. The three main near-Earth perturbations are the gravitational harmonics, the atmospheric drag and the solar radiation pressure.

Each one of this effect was modeled using the models suggested by NASA (2006) and ECSS (2000). The contribution of each one of them was then added to the Eq. 10 with attention to the reference frame as other perturbing forces as the low thrust.

The gravitational harmonics were modeled using the coefficients of the EGM96 model and the algorithm presented by Montenbruck (2002). The expansion used in the work was with 7 terms which was considered to be enough to our objectives and accuracy. The atmospheric drag was calculated using a standard drag coefficient of 2.3 and the density model MSISE of NASA. The relative velocity of the flow to the spacecraft was calculated with the hypothesis of atmosphere rotating together with the Earth, i.e. no wind model was used. Finally, the solar radiation pressure was modeled using the constant solar radiation coefficient at 1AU. This hypothesis was adopted, since the farther distance used in this work was around 0.01AU.

The same procedure described in the low thrust dynamical system approach was used again in this part of the work, but now with the new perturbed model and the parameters of interesting orbits found with the simpliest model as inputs.

2.5 The JPL Ephemerides Model

The final and most complex model used in this work consisted of the gravitational influence of all Solar System bodies in the spacecraft, calculated with the use of the JPL Ephemerides DE403 model.

A discussion must be made about the existence of the Halo orbits. When the perturbations of all Solar System bodies are considered, a periodic Halo orbit is no longer observed. It is substituted by a quasi-periodic orbit around the original periodic one. Thus, the target orbit depends also on time and not only of the phase.

This fact introduces a difficulty in adapting the parameters of the interesting transfers obtained with the simplest models to this complex one. The solution used in this work was the comparison of the point were the stable manifold passes through the $y = 0$ plane of the synodic frame from positive to negative in order to establish a relationship between the halo orbit phase in the CRTBP model and the so called manifold index, a parameter to identify the manifold in the JPL model.

The determination of the starting point in a vicinity of the quasi-periodic orbit for the low thrust dynamical systems approach method was done with the use of the subroutines and methods presented by Gomez et al., 2001. The reference frame used was the sidereal and the low-thrust model was adapted to it.

3. RESULTS AND DISCUSSION

3.1 Simple CRTBP Model

The first part of the study consisted in fixing the value of thrust magnitude and halo amplitude and test a large set of halo orbit starting phase and coast time values. In order to express the results, we opted for a two-dimensional map with each point corresponds to a set of values to these two parameters. This point is colored according to the eccentricity of the departure orbit archived. All the range of starting phase was tested and a practical interval of coasting time was used.

Figure 3 is an example of the result of this study. The thrust for this test is set as $F_T = 1.00 \cdot 10^{-4}\ ms^{-2}$, and the target halo orbit is of amplitude $\alpha_4 = 0.08$.

Some zones of this map are not colored, but filled with two different point like patterns. The first one, on the upper side of the figure and less dense, represents the orbits that does not reach the reference altitude in a limit time which is also a parameter of the simulations. This time was usually set to $t_{lim} = 15$ CRTBP units, which corresponds approximately to 2.4 years or 870 days.

The other region of figure 3 has a more complex explanation. In this regions, during the numerical integration, the spacecraft velocity with respect to the Earth in the sidereal system vanishes. The consequence is that the direction of the thrust vector becomes undetermined since it is defined by this vector.

Observing the colored regions of the figure 3 it is notable the big blue areas corresponding to high eccentricities and

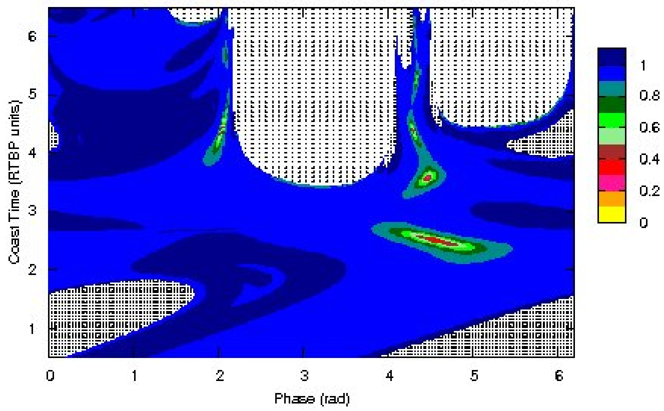

Figure 3. Eccentricities of the departure orbit for $F_T = 1.00 \cdot 10^{-4}[ms^{-2}]$ and $\alpha_4 = 0.08$.

the small regions were the parking orbits present low eccentricity values. This last regions are the ones we are interested in and seem to have a kind of convex and simple shape. In order to facilitate reading, from now on we refer as low eccentricity zones or regions these areas of the halo orbit phase - coast time surface where the parking orbit has low eccentricity.

Once these regions were observed for this fixed set of thrust magnitude and halo orbit amplitude, we repeated the test with other values of these parameters. These tests have shown that the zones change in size and form, but not dramatically in all the range of parameters used, although for some sets of parameters no very-low eccentricity parking orbit was found in some zones.

The two input parameters also affect the shape and size of the two other types of zones described before, however, these other zones were never close to the low eccentricity zones so were not of our interest in this work.

Therefore, we can identify the interesting zones in order to better analyze them. Their number and basic characteristics are presented below

Zone 1 The biggest zone located approximately in the ranges determined by $4.0 \leq \phi \leq 5.0$ and $2.0 \leq t_{coast} \leq 3.0$ CRTBP units. For higher thrust magnitudes, the region splits in two ones, named from now on as Z1A and Z1B, respectively.

Zone 2 The second bigger zone is located above the Zone 1. This is, for the same values of phase ϕ, but bigger coast times. It has a very round shape.

Zone 3 Located at the lower values of ϕ, near the value of 2 radians and coast time about 4 CRTBP units. It has a very different shape compared with the previous two. Being very stretched. With the increase in the thrust magnitude it moves towards lower t_{coast} and phase values.

Zone 4 Also relative to the same values of phase as zones 1 and 2, but having higher coast time.

Other zones Very small zones can be noted above zones 3 and 4, and at other values of ϕ and t_{coast} for specific values of thrust, none of them proved to be of much interest.

It is interesting to present examples of transfer trajectories associated with each one of these zones. Figure 4 presents this for the thrust magnitude $F_T = 3.00 \cdot 10^{-4} ms^{-2}$. The filled circles in each transfer trajectory mark the end of the low-thrust arc and the beginning of the coast trajectory.

Besides the eccentricity, the inclination of the parking orbit was also studied, since it is a key factor in the launchers performance. The other orbital elements are not so important for this first analysis: the semi-major axis is already defined from the 1000 km altitude and low eccentricity limits; the other three (longitude of the right ascension of the ascending node, longitude of the perigee and true anomaly) are not so important, since we can choose launch windows which satisfies them.

The results had shown that the inclination of the starting orbit for all the low-eccentricity zone vary from 25^o to 40^o with respect to the Equator, which are acceptable values for parking orbits. The halo orbit amplitude plays a big role in this value while the thrust magnitude almost do not influence it.

With respect to the propellant used and total transfer time the situation is the opposite, i.e., both parameters are totally dependent of the thrust magnitude while the final halo orbit amplitude almost do not counts on it.

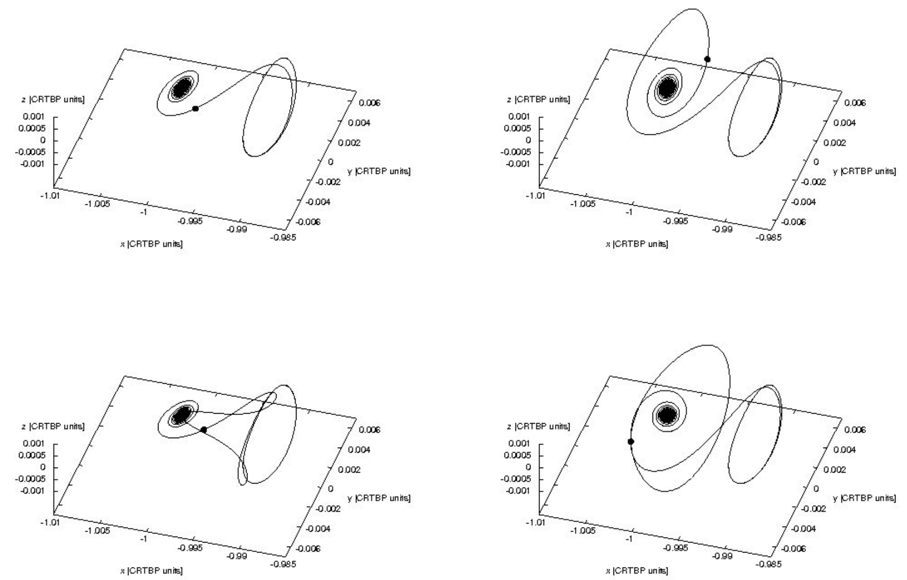

Figure 4. Examples of trajectories with low-eccentricity parking orbits. All of them calculated with $F_T = 3.00 \cdot 10^{-4}[ms^{-2}]$ and $\alpha_4 = 0.08$. From left to right and from top to bottom: Z1 $\phi = 4.0626$ $t_{coast} = 2.7050$ [CRTBP units], Z2 $\phi = 4.6190$ $t_{coast} = 3.6500$ [CRTBP units], Z3 $\phi = 1.8910$ $t_{coast} = 4.0700$ [CRTBP units], Z4 $\phi = 4.2470$ $t_{coast} = 4.5650$ [CRTBP units].

The results shown until now were all for libration point 1 transfers. Basically, the same tests were repeated for the SEL2 ones. The results of both, however, are very similar. This fact is not completely unexpected. When the value of $\mu \to 0$ the behavior of the libration points 1 and 2 tends to be the same. The complete symmetry is obtained in the limit, known as Hill's case.

3.2 Effect of the Near-Earth Perturbations

Following our study, the transfers of interest found with the simple CRTBP model were tested in a model that considered the near-Earth perturbations. The results have shown that only the solar radiation pressure can affect in a practical way our selected transfers.

The gravitational harmonics have not changed significantly the eccentricity or the inclination of the parking orbits of the interesting transfers. This was expected, since its main influence on orbits is the introduction of secular variations in the right ascension of the ascending node and on the argument of the perigee. The use of more then $4x4$ coefficients does not change the transfers significantly.

The "rule of the thumb" for atmospheric drag states that it should only be considered for orbits bellow 1000 km. Thus, we tested the transfers in this case with a 500 km limit rather then the 1000 km as usual in this work. The results have shown that its main influence is in the eccentricity of the parking orbit. The more the atmospheric drag acts, more eccentric is the parking orbit. This behavior was also expected, since the atmospheric drag reduces the eccentricity of orbits and we make the integrations backwards in time. Nevertheless this influence is very low to be considered significant in our study.

Finally, the solar radiation pressure proved to move the low eccentricity zones in the halo orbit phase-coast time plane, i.e., changing the parameters that correspond to the interesting transfers. Therefore, it proved to be the most important near-Earth perturbation to be considered in a mission design.

3.3 The JPL Ephemerides Model Results

The first step in the use of this model was the determination of the relationship between the halo orbit phase of the CRTBP simple model and the manifold index in the JPL Ephemerides one. The result of the use of the method described before, for $\alpha_4 = 0.08$ halo orbit amplitude, was the following

$$n_{JPL} = \frac{500}{620}\left(\phi_{CRTBP} - 5.00\right) + 40. \tag{11}$$

where n_{JPL} is the manifold index (see Gomez et al, 2001).

The use of this method proved to be satisfatory for this work, but not very precise for further applications. Once the manifold was determined, our main objective in using this model was studying the influence of the Moon in interesting transfers. In order to do so, we have tested interesting transfers considering different dates in a month.

For example, using the results of the use of the simpler models, we obtain that the parameters for an interesting transfer would be manifold index of 482 and coast time of 202.2 days, in the JPL ephemeris model. We present the transfers computed with the same parameters, but different reference times around a month in figure 5. Note that only the part of the transfer close to the Earth is presented.

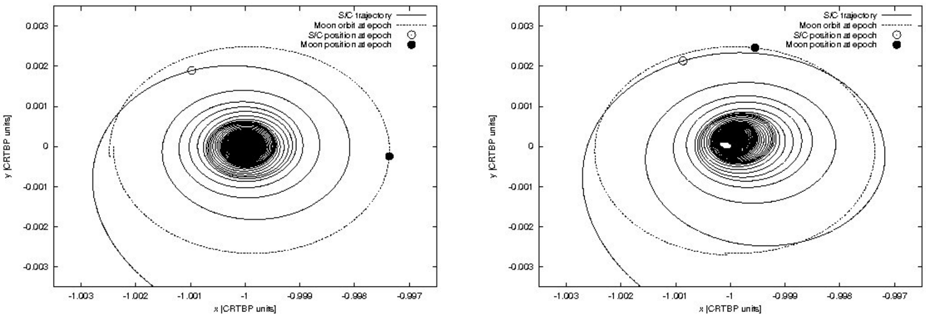

Figure 5. Transfer trajectories computed for manifold index equal to 482 and coast time equal to 202.2 days. Reference time and epoch time (JUL 15 06 ; NOV 20 06) (left) and (JUL 22 06 ; NOV 29 06) (right).

As can be seen in one date the transfer starts in a very low-eccentricity orbit, while 7 days latter the parking orbit obtained is very eccentric. This result illustrates the two main conclusions archived with the use of the JPL ephemerides about the Moon: it plays a strong role in the transfer but, at the same time, its influence can be avoided by the proper choice of launch window.

4. CONCLUSIONS AND FUTURE WORKS

In this project, we were able to determine low-thrust transfers with practical interest using a very simple methodology and few computational work. Moreover these transfers could be determined in the complex JPL Ephemerides model with use of results obtained with the simple CRTBP one. This represents a big potential save of time and work in the design of new libration points missions.

As future works, the authors can cite the development of the method to link the parameters of both models, a deeper study of the Moon influence on the transfers and a comparison with the use of the optimal transfer method in the design of the transfers.

5. ACKNOWLEDGEMENTS

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7. Responsibility notice