

Satellite Attitude Control using the Generalized Extremal Optimization with a Multi-Objective Approach

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Abstract. *The goal of this work is to evaluate the efficiency of the Generalized Extremal Optimization (GEO) algorithm in the design of an artificial satellite attitude control system. Specifically, the procedure aims at determining the gains of a proportional derivative (PD) control law type, to command a reaction wheel used for the satellite attitude control. A multi-objective approach is employed with the task of minimizing, simultaneously, the time to control the satellite and the energy spent on it. GEO is a new evolutionary algorithm inspired in a simplified model of evolution, developed to show that natural ecosystems have a self-organized criticality behavior. The use of a multi-objective approach allows that a set of optimized trade-off solutions (nondominated solutions) be determined and made available to the control designer for posterior choice of an individual solution to be implemented. The nondominated solutions set in the design space (Pareto optimal set) and in the objective functions space (Pareto front) were obtained through a multi-objective version of GEO, developed recently. Finally, a Kalman filter is used to estimate the satellite attitude states to be feedback by the control law designed previously. Solutions present on the Pareto set are chosen as input to the filter for this application.*

Keywords*: artificial satellite, attitude control, Kalman filter, evolutionary algorithm, multi-objective optimization.*

1. INTRODUCTION

Satellites need to be positioned with a certain attitude in order to fulfill their mission requirements.. Many of them are intended to be Earth oriented, others need to point to the sun or some star of interest, while others need to move its orientation from an object to another (Hughes, 1986). Special devices are used to perform the necessary attitude control of a satellite, such as thrusters, nutation damper and reaction wheels. They can be used alone or in combination, depending on the required accuracy and type of attitude required for a given mission. A typical problem faced by the attitude control system (ACS) of a satellite is to carry out attitude maneuvers, having to keep the level of pointing accuracy and the residual vibration after that operation. Therefore, the ACS must be capable to keep the stability and good performance in order to satisfy the mission's requirements (De Souza, 2006). Recently, stochastic algorithms have been shown interesting results in this area of study (Ge and Chen, 2004).

Stochastic algorithms inspired by nature have been successfully used for tackling optimization problems in engineering and science (Pardalos and Romeijin, 2002; Glover and Kochenberg, 2001; Davis *et al*., 1999). The main advantage of using such kinds of methods is that, differently from traditional gradient ones, they perform a global search in the design space. That is, they use mechanisms that allow them to escape from local optima. In fact, they are usually very robust to non-linearities in the objective function and their constraints, and also deal well with problems that have different kinds of design variables. One kind of such methods are the Evolutionary Algorithms (EAs) (Davis *et al*., 1999; Bäck and Schwefel, 1993). EAs are based on the principles of natural evolution and genetics. They employ a population of candidate solutions that is "evolved" during the search as better individuals (new solutions) are generated from previous ones in the sense that they are closer to the global minimum.

Recently, a new EA called Generalized Extremal Optimization (GEO) was proposed (De Sousa *et al*., 2003). GEO is a generalization of the Extremal Optimization (EO) (Boettcher and Percus, 2001) algorithm. Both algorithms are based on the simplified evolutionary model of Bak-Sneppen (Bak and Sneppen, 1991), which was developed to show the emergence of Self-Organized Criticality (SOC) in ecosystems. GEO is easily implemented to optimization problems, does not make use of derivatives, can be applied to unconstrained or constrained problems and nonconvex or even disjoint design spaces, in the presence of any combination of continuous, discrete or integer design variables. It has been successfully applied to complex design problems (De Sousa *et al*., 2003, 2004), including multi-objective ones (Galski, 2006; Galski et al., 2005).

As a brand new algorithm, research has been performed on the canonical GEO in order to improve its efficiency and range of applications (Galski, 2006). Recently, a new version of GEO algorithm capable of dealing with multi-objectives problems was developed (Galski, 2006).

In this context, a recent application of the algorithm was in the optimization of the gains of a control law for a reaction wheel to be used in a satellite. In this work, it is used a simple linear rigid satellite model that has a reaction wheel as actuator and an angular sensor. The satellite model is put in matrix state space form in order to design a proportional plus derivative control law. The main objective was to design a simplified satellite ACS using the multi-objective GEO (M-GEO) algorithm optimizing two conflicting objective functions. The first is the time to control the satellite, and the other is the energy spent to do that job. The Kalman filter algorithm was included with the purpose of verifying the robustness of the control law. Kalman filter theory (Anderson and Moore, 1979) is very well known and it was developed to solve a specific spacecraft navigation problem, and since then, it has been applied in diverse areas. In mathematical terms, the Kalman filter (Sorenson, 1985) estimates the states of a linear system, often embedded in control systems in order to obtain an accurate estimation of some states, which are not always measured directly by the sensors. In the present paper, noise was simulated and introduced in the angular measurement. Additionally, the angular velocity mesurement was omitted and the Kalman filter was used to estimate this satellite attitude state.

The simulations have shown that the control system behaved as expected, controlling the satellite in no longer than 21 seconds, and demonstrating a good response with the Kalman filter. Even if the angular velocity is omitted the satellite is still kept under control. As a result of the multi-objective problem a Pareto frontier is presented for the two conflicting objective functions.

The paper has the following organization. Section 2 presents the derivations of the equations of motion of the rigid satellite, where a reaction wheel produces the control torque that is the input and angles and angular velocities are the outputs. In Section 3 the control law, the satellites parameters and the model manipulation to implement the simulations are introduced. The simulations results and discussions are presented in Section 4. Section 5 concludes the paper.

2. SATELLITE MODEL

An attitude control system for satellites includes a sensor for generating output signals in response to variations in the attitude of the satellite. These signals are applied to a Kalman filter, which models the dynamic state of the satellite and instructs an attitude controller to generate control signals for controlling the actuators. The system is responsive to the actual physical performance of the actuators to provide feedback to the modeling circuit. In this section the satellite model will be presented as well as how the Kalman filter is applied to it.

2.1.Equations of motion

The satellite model consists of a rigid central body and a reaction wheel lined up with the center of the rigid body, which is the origin of the coordinated system (X, Y, Z), coincident with its principal axes. The reaction wheel is the actuator responsible for the rotation control of the satellite and has its center of mass coinciding with the satellite's origin, as shown in Fig 1.

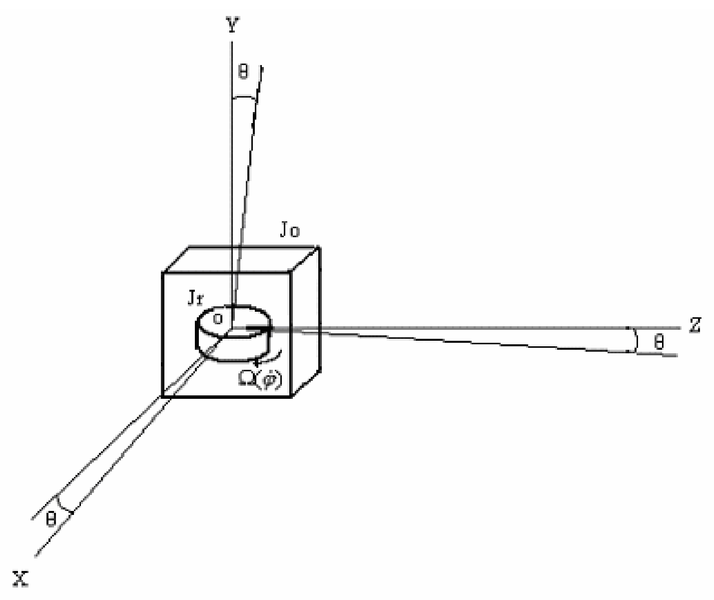

Figure 1 – Satellite model with a reaction wheel

In Figure 1 J_o and J_R are the moments of inertia of the rigid body and reaction wheel in relation to the mass center.

The equations of motion are derived using the Lagrange methodology (Meirovitch, 1970), where only the satellite rotation and the reaction wheel angular velocity around the Y are considered, the translation of the satellite is disregarded. Therefore, the expression of the kinetic energy of the satellite plus the reaction wheel is given by

$$T = \frac{1}{2} J_o \dot{\theta}^2 + \frac{1}{2} J_R f^2 \tag{1}$$

The angular velocity of the reaction wheel is given by

$$f = \dot{\theta} + \dot{\varphi} \tag{2}$$

where the angular rotation of the satellite around Y is θ, and the angular velocity of the reaction wheel also around Y is $\Omega = \dot{\varphi}$. As a result, the total kinetic energy is given by

$$T = \frac{1}{2} J_o \dot{\theta}^2 + \frac{1}{2} J_R \dot{\theta}^2 + J_R \dot{\theta}\dot{\varphi} + \frac{1}{2} J_R \dot{\varphi}^2 \tag{3}$$

Considering that there is no external torques acting over the satellite and there is no potential energy, the lagrangian ($L = T - V$) assumes the form

$$L = \frac{1}{2} J_o \dot{\theta}^2 + \frac{1}{2} J_R \dot{\theta}^2 + J_R \dot{\theta}\dot{\varphi} + \frac{1}{2} J_R \dot{\varphi}^2 \tag{4}$$

Lagrange Equations are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{5}$$

where Q_i represents the generalized forces of the system. The equations of motion are derived for two generalized coordinates q_i (i=1,2), that is: the angular rotation of the satellite θ(t) and the angular velocity of the reaction wheel Ω.

The equation of motion that describes the satellite rotation is obtained substituting θ in the Lagrange equations which after derivations is given by:

$$(J_o + J_R)\ddot{\theta} + J_R \dot{\Omega} = 0 \tag{6}$$

The equation of motion of the reaction wheel is obtained substituting the generalized coordinates $\Omega = \dot{\varphi}$ in the Lagrange equation, which after derivations is given by:

$$J_R\left(\dot{\Omega} + \ddot{\theta}\right) = -\xi \tag{7}$$

where ξ represents the torque applied in the reaction wheel from an engine, for example a DC motor, which dynamics is not considered here.

The Eq. (6) and Eq. (7) are a set of linear coupled equations that represent the total dynamics of the satellite, that is: rotation of the rigid body and the rotation of the wheel of reaction all around the Y-axis. Therefore, the equations of motion of the rigid satellite are given by:

$$\begin{aligned} &(J_o + J_R)\ddot{\theta} + J_R \dot{\Omega} = 0 \\ &J_R\left(\dot{\Omega} + \ddot{\theta}\right) = -\xi \end{aligned} \tag{8}$$

In order to use the fourth-order Runge-Kutta method and a Kalman filter algorithm one has to manipulate and put Eq. (8) in matrix state space form of 1st. order. Suppressing the angular velocity of the reaction wheel the Eq. (8) is given by:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{\xi}{J_0} \tag{9}$$

where $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $u = \dfrac{\tau}{J_0}$, $x_1 = \theta$ and $x_2 = \dot{\theta}$.

The control law, which will command the reaction wheel motion, is a simple proportional derivative (PD) type, where the gains K_1 and K_2 are determined through GEO algorithm. The PD control law is given by

$$\xi = -K_1\theta - K_2\dot{\theta} \tag{10}$$

Substituting the torque ξ in Eq. (9) one has

$$\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= -\frac{K_2}{J_o} X_2 - \frac{K_1}{J_o} X_1 \end{aligned} \tag{11}$$

The optimum gains K_1 and K_2 will be obtained by the use of GEO algorithm. This optimization will be discussed in section 3.

2.2. Kalman filter algorithm

In order to use a Kalman filter to estimate the states that are not measured, the physical process must be described by a linear system, which can be described by the following two equations:

State equation:

$$\dot{x} = Ax + Bu + w \tag{12}$$

Output equation:

$$y = Cx + z \tag{13}$$

where ***A***, ***B***, and *C* are the matrices, and the input ***u*** and output *y* are as defined before. Now the system takes into account the process noise ***w*** and the measurement noise *z*. The vector ***x*** contains all of the information about the present state of the system, but one cannot measure ***x*** directly, instead one measures *y*, which is a function of ***x***. As a result, the Kalman filter uses *y* to obtain an estimate of ***x***.

In the Kalman filter methodology it is assumed that the average value of the state estimate is equal to the average value of the true state. The algorithm minimizes the expected value of the square of the estimation error, which means that on average, the algorithm gives the smallest possible estimation error. One also assumes that the average value of the process noise ***w*** and the measurement noise *z* are zero and that there is no correlation between them.

The process noise covariance matrix S_w and the measurement noise covariance matrix S_z are defined as

$$S_w = E(w_k w_k^T) \tag{14}$$

$$S_z = E(z_k z_k^T) \tag{15}$$

where $(.)^T$ represents matrix transposition and $E(\cdot)$ the expected value.

There are many forms to express the Kalman filter algorithm equations. One of the formulations is given by equations:

$$K_k = AP_k C^T (CP_k C^T + S_z)^{-1} \tag{16}$$

$$\hat{x}_{k+1} = (A\hat{x}_k + Bu_k) + K_k(y_{k+1} - C\hat{x}_k) \tag{17}$$

$$P_{k+1} = AP_kA^T + S_w - AP_kC^TS_z^{-1}CP_kA^T \tag{18}$$

where $(.)^{-1}$ indicates matrix inversion. The K matrix is called the Kalman gain, and the P matrix is called the estimation error covariance.

In the state estimate Eq.(17) the first term is the state estimate at time ($k+1$), which is just A times the state estimate at time k, plus B times the known input at time k. The second term is called the correction term and it represents the amount by which to correct the propagated state estimate due to our measurement.

Inspection of the K equation shows that if the measurement noise is large, S_z will be large, so K will be small and one will not give much credibility to the measurement y when computing the next one. On the other hand, if the measurement noise is small, S_z will be small, so K will be large and one will give a lot of credibility to the measurement when computing the estimate of x.

3. OPTIMIZATION ALGORITHM

In multi-objective problems, one says that a vector of decision variables $\boldsymbol{x}^* \in \aleph$ is *Pareto optimal*, i.e., it is *non-dominated*, if there does not exist another $\boldsymbol{x} \in \aleph$ such that $f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{x}^*)$ for all i=1,...,M and $f_j(\boldsymbol{x}) < f_j(\boldsymbol{x}^*)$ for at least one j. In words, this definition says that $\boldsymbol{x}^*$ is *Pareto optimal* if there exists no feasible vector of decision variables $\boldsymbol{x} \in \aleph$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Unfortunately, this concept almost always gives not a single solution, but rather a set of solutions called *Pareto optimal set* or even *Pareto set*. The vectors $\boldsymbol{x}^*$ corresponding to the solutions included in the Pareto optimal set are called *non-dominated*. The plot of the objective functions whose *non-dominated* vectors are in the *Pareto optimal* set is called the *Pareto Front*.

A multi-objective version of GEO, called M-GEO, was developed (De Sousa et al., 2003), in order to generate the Pareto front of such problems. This version was created to solve multi-objective problems maintaining the GEO essence (universality of application and as few algorithm parameters to adjust as possible). A flowchart of the M-GEO algorithm is presented in Fig. 2.

M-GEO has the same basic functioning as GEO. The differences are that i) in M-GEO the bits are ranked based on one of the objective functions that is randomly chosen at each algorithm iteration. M-GEO does not work with sub-populations, hence, at a given iteration, only one of the objective functions is used for the fitness assignment of all species (bits); ii) each new solution created during the search is compared to the ones in the set of non-dominated solutions and incorporated to it if it is also a new non-dominated solution. If it dominates previous solutions contained in the set, these are deleted from the set; iii) M-GEO can be re-started during an execution. As in the canonical GEO, in M-GEO the initial population is created from a single point in the design space. This could lead to an initial population close to one of the edges of the Pareto front in the objective space, then delaying the spread of the population over the entire frontier. To avoid this, the algorithm is restarted some times during a run. The restarting time (rt) is an additional adjustable parameter and represents the number of times the algorithm is re-initiated during a single execution. It is important to note that the population of bits can be re-initiated during a run, but the set of non-dominated solutions is kept in a separated file and preserved during the complete run of M-GEO. In an early version, with no restarts, M-GEO was successfully used for the optimal design of a remote sensing satellite constellation (Galski et al., 2005), but further tests have shown that restarting the population during the run increases its efficiency in finding the Pareto front.

It was used two objective functions in this problem. The first is the time to control the satellite, one wants to control in minimum time. The other is the energy spent in the control process. These two objective functions are conflicting, so M-GEO will return the Pareto frontier and the Pareto set. The time and the energy needed to the control process were obtained by stipulating a value to the angular shift and angular velocity that one wants to reach. Once this value was reached by the Runge-Kutta integration, the process was stopped and the time interval was returned

$$\text{Minimize } F_1 = T \tag{19}$$

and the energy. The energy was calculated by

$$\text{Minimize } F_2 = \sum_{i=0}^{nStep-1} \left(K_1X_{1i} + K_2X_{2i}\right)^2 \tag{20}$$

where X_{1i} and X_{2i} are, respectively, the i-th angular shift and angular velocity value given by the Runge-Kutta integration and nStep is the total number of step. The gains K_1 and K_2 are subject to the follow side constraints

$$0 \le K_1 \le 20000 \tag{21}$$

$$0 \le K_2 \le 20000 \tag{22}$$

The results of this study will be presented in section 4.

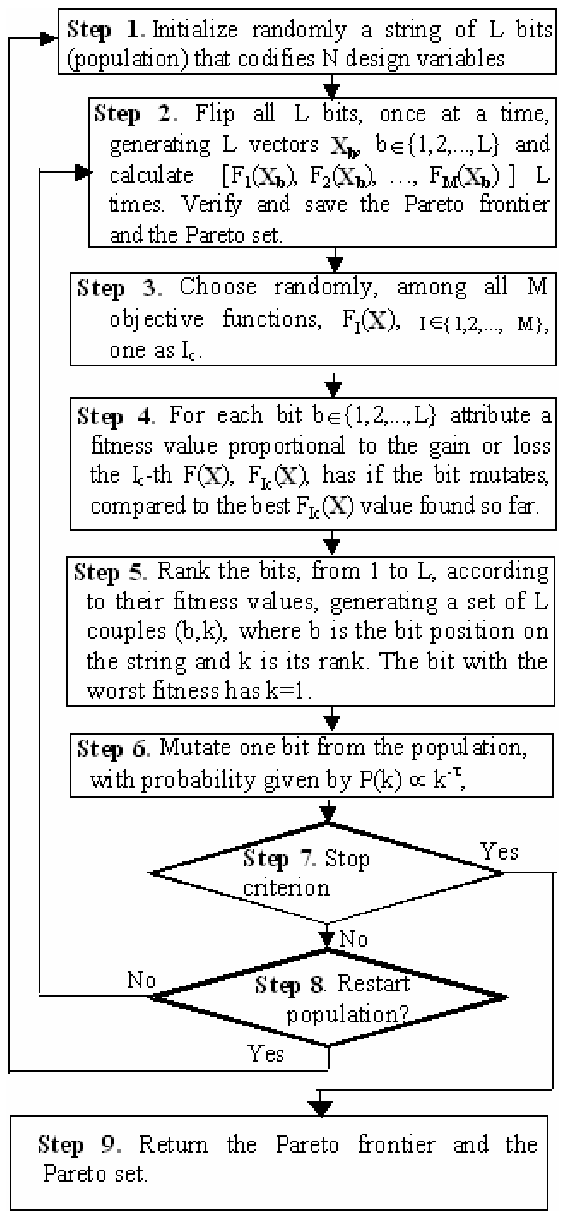

Figure 2. M-GEO algorithm.

4. SIMULATION RESULTS

A valuable result achieved in this work is the solution of the multi-objective optimization problem proposed in the section 3.3. As a result, one obtains a Pareto Frontier, that is, a set of optimal solutions of the problem. Later, it's possible to choose one of these solutions according to the mission objective. The M-GEO parameters used were L = 8, N = 2 (K_1 and K_2), $\tau = 0.5$, and rt = 50. The Runge-Kutta step size used was 0.01 and satellite was considered controlled when $|X_1| < 1.75 \times 10^{-3}$ rad and $|X_2| < 5.23 \times 10^{-4}$ rad/s. The satellite's moment of inertia J_o was 720 kg.m^2. The Pareto Frontier is presented in Fig. 3.

In order to test the gains obtained by the M-GEO algorithm, one studies the dynamic behavior of the system applying the Kalman filter algorithm. This study was realized by choosing the solution that spends less energy, because in this case the attitude control system has more limitation of response. Therefore, if the Kalman filter presents good performance with low energy, it will respond well with more energy available. For this case, the time to control the satellite is 20.63 seconds and the gains K_1 and K_2 for this case are 78.43 and 470.59, respectively. The parameters used in the Kalman algorithm are presented in table 1.

Table 1. Parameter value used in Kalman filter algorithm, where C is a matrix 1 x 2 that related the measurements with the states, Freq is the frequency that the measurement is obtained, S_w and S_z is defined earlier.

Parameter	Value
C	[1 0]
S_w	0.0001
S_z	0.0025
Freq	10 Hz

The simulation results, presented in Fig. 4, 5 and 6, indicate that the satellite can be controlled even if there is no angular velocity measurement. The Fig. 4 (a) and (b) shows the dynamic behavior with no measurements corruptions, so the Kalman filter does not need to be used. The Fig. 5 (a) and (b) shows how would be if the angular measurements have some corruptions and there is no angular velocity measurements. In this case, the use of the Kalman filter is needed. The Fig. 6 (a) and (b) is the same of Fig. 5 (a) and (b), but shows the dynamic behavior during 100 minutes instead of 60 seconds. It indicates that even during a long time the system can be controlled and the Kalman filter is efficient to estimate the angular velocity.

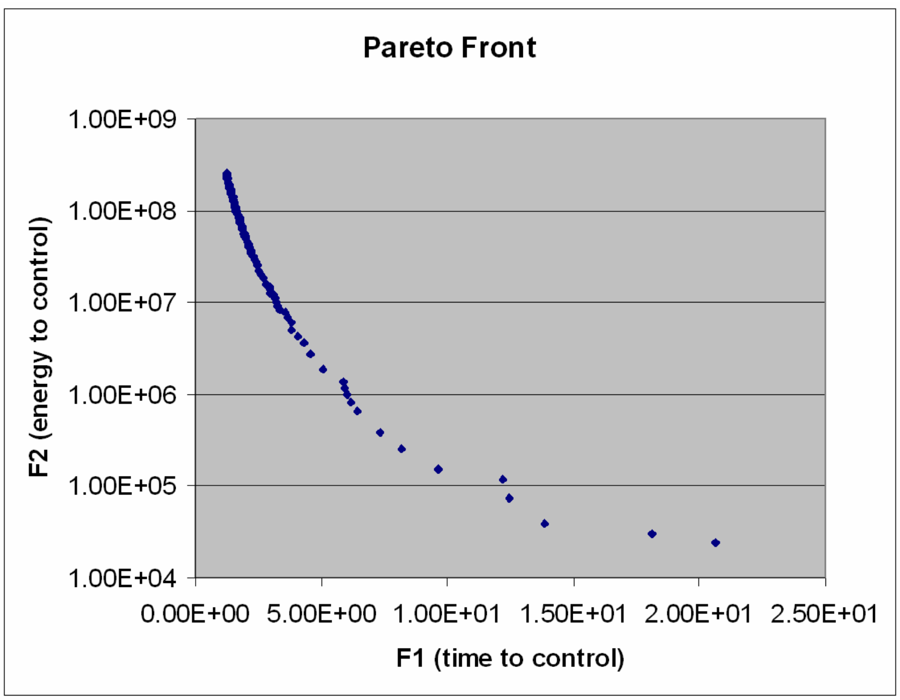

Figure 3. Pareto Frontier, where F_1 is the time to control and F_2 represents the energy used to control the satellite.

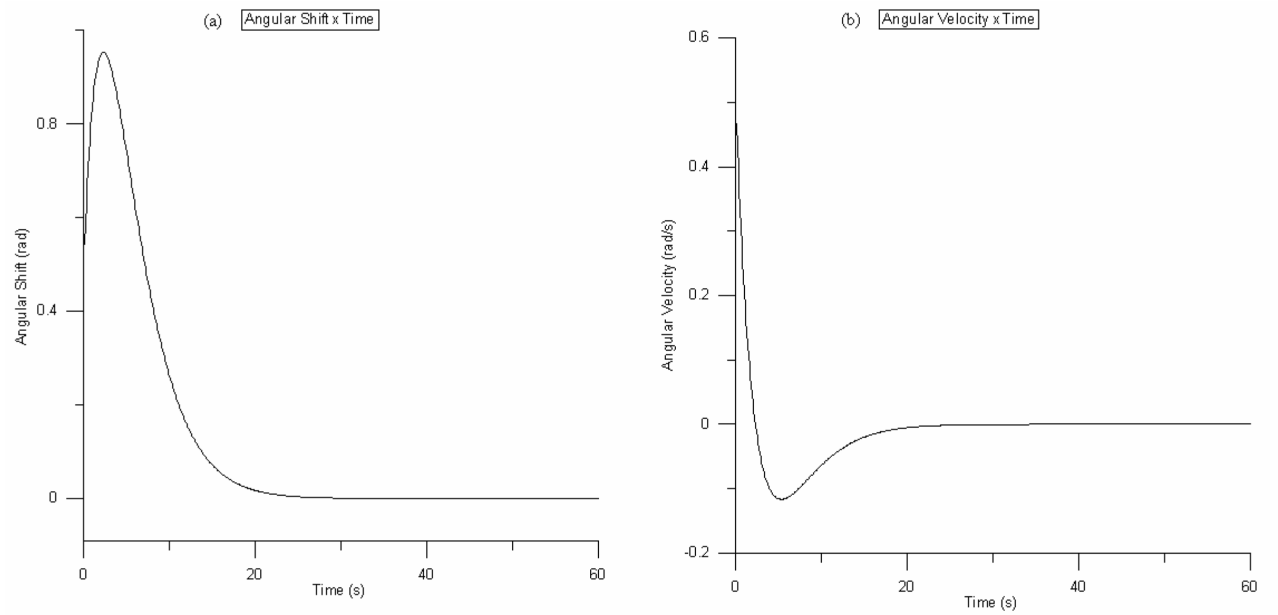

Figure 4. (a) - The satellite’s angular shift versus time without Kalman filter. (b) - The satellite’s angular velocity versus time without Kalman filter.

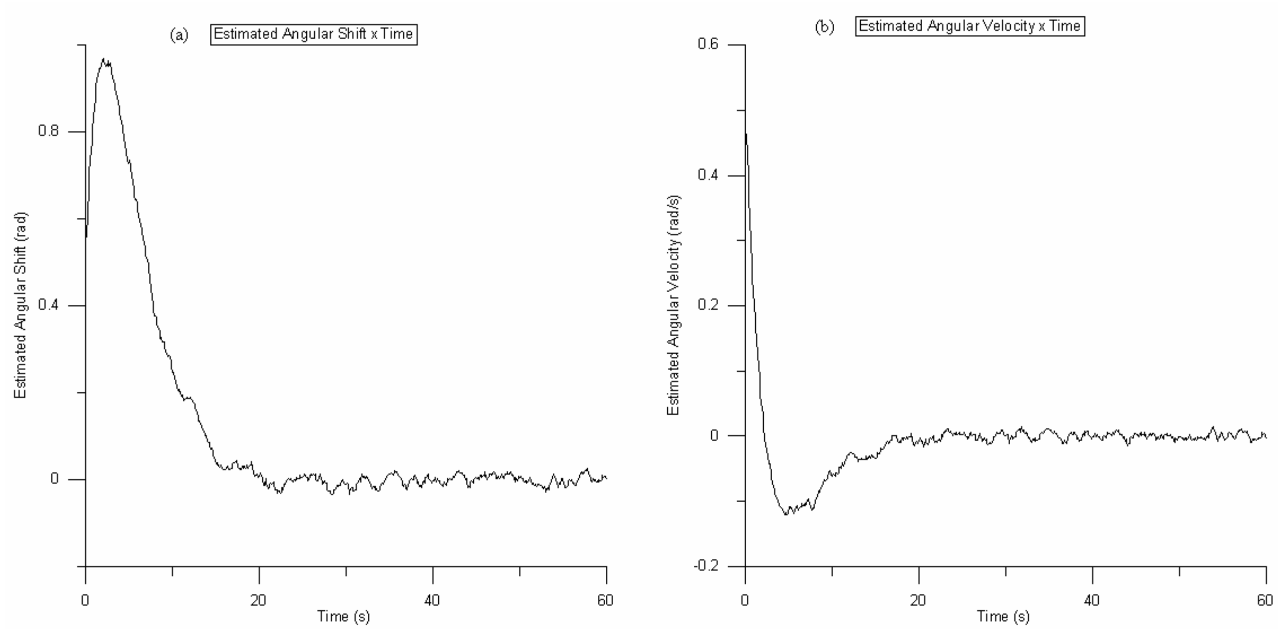

Figure 5. (a) - The satellite’s angular shift versus time with Kalman filter. (b) - The satellite’s angular velocity versus time with Kalman filter.

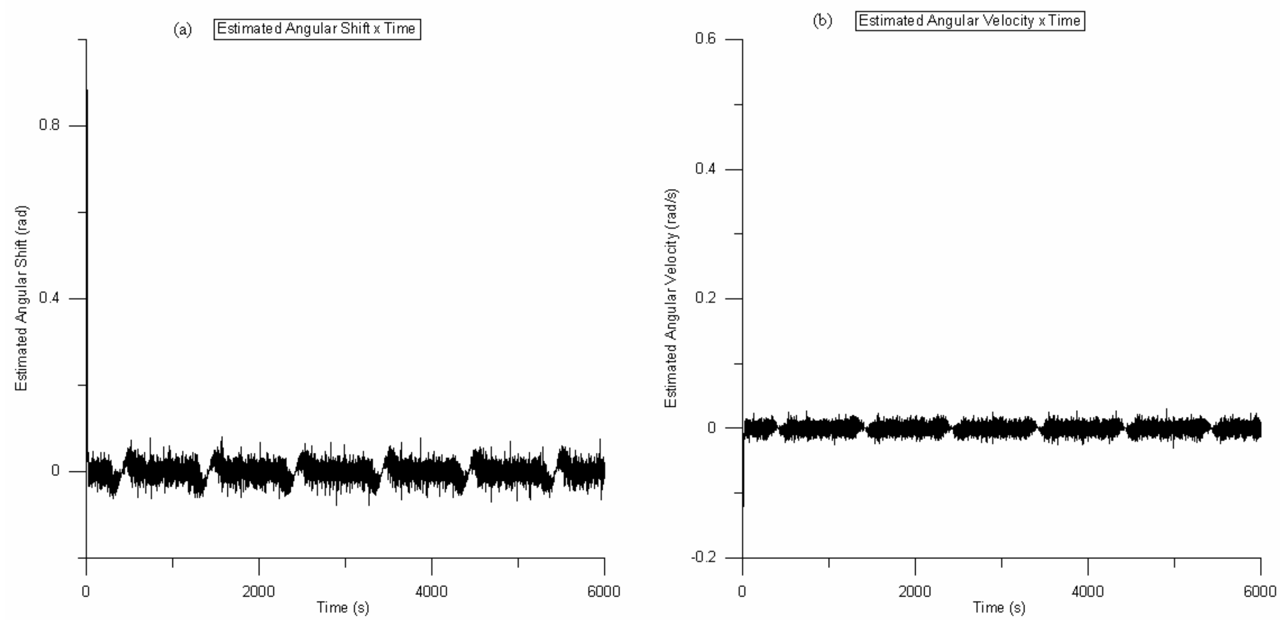

Figure 6. (a) - The satellite's angular shift versus time with Kalman filter and integrated for 100 minute. (b) - The satellite's angular velocity versus time with Kalman filter and integrated for 100 minute.

5. CONCLUSION

In this paper the optimization of a satellite attitude control law with a multi-objective approach was studied. The satellite was modeled as a rigid body with a reaction wheel, which is controlled by a proportional derivative control law. A multi-objective version of the Generalized Extremal Optimization (M-GEO) was used to find a set of optimal gains of the control law. As far as the authors are aware, satellite attitude control law optimization with multi-objective approach using evolutionary algorithm is an original study. Additionally, a Kalman filter was applied to verify the robustness of the control law, considering no angular velocity measurements.

The main goal of this paper was to optimize the time to control the satellite and the energy spent in such job. These objective functions are conflicting, so a Pareto front was obtained as main result. As expected, this result presents a set of optimal solutions for the proposed problem.

The test with the Kalman filter algorithm uses one of the optimal solutions obtained by M-GEO algorithm. The solution chosen was the one that spent less energy, because with low energy available the attitude control system has more limitation. If the Kalman filter presents good response in this case, it is expected to respond better with more energy available. This study showed that the satellite is still under control even if there is no angular velocity measurement available.

Although the obtained results are according to the expectations, it is necessary to point out that the satellite equations are simple and the problem is completely observable. Further work with a more realistic and complex satellite model and considering a system that is not completely observable is needed.

6. ACKNOWLEDGEMENTS

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