DYNAMIC MODELING OF AN HYDRAULIC POWER PLANT USING SIGNAL APPROACH

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Abstract. *The aim of this work is to introduce to the study of the dynamic modeling of power plants presenting the development of two generic power plant models, both implemented completely by signal approach. They have different focuses and degree of detail. The first one is a lumped parameter non-linear model assuming an inelastic water column. It simulates the typical no-minimum phase behavior of a power plant (Kundur, 1994). On the other hand, the second is a linearized distributive parameter model assuming an elastic water colum. It models traveling wave phenomena and so enables to simulate the water hammer effects (Casella, 2006). Furthermore it is able to simulate the overspeed due to an electrical load rejection. This models become really useful in the preliminary design phase of new power plants. For instance they enable to evaluate the optimum tradeoff between increasing the thickness of the penstock walls (in order to prop up water hammer overpressures) or augmenting the electrical generator inertia (in order to reduce overspeed). They are also useful for developing new speed control systems for an existing power plant and to improve its efficiency and capabilities (Working Group on Prime Mover and Energy Supply Models for System Dynamic Performance Studies, 1992). The application software used for modeling is AMESim®.*

Keywords*: Power Plant, Hydraulic Turbine Dynamic Model, Over-speed, Water Hammer*

1. INTRODUCTION

The construction of new mathematical models of power plant is necessary, due to the widespread use of electric-hydraulic speed control (not loner purely mechanic), both in construction and in the upgrade of older power plants. It is better to use models describing the reality of the equipment rather than make approximations to fit existing mechanical governor models. In addition to that, the tremendous increase in computer power eliminates the need for less detailed models. These conditions aided to insert the non-linear models where they were required, that is, in the situations in which speed and power changes are large, such as in load rejection or studies of the startup of the system.

The purpose of this paper presents two models of a power plant, implemented mainly by signal approach. However they contain some components that involve power flow approach. The first model is a lumped parameter non-linear model assuming an inelastic water column; the second is a linearized distributive parameter model assuming an elastic water column, where the traveling wave phenomena occur, and so represents the water hammer effects.

2. NON-LINEAR MODEL ASSUMING INELASTIC WATER COLUMN

The first model is a set of equations representing: the turbine (modeled as a variable orifice), the hydraulic power and finally the differential equation of the dynamic of the penstock (modeled as a pressure pipe) (Working Group on Prime Mover and Energy Supply Models for System Dynamic Performance Studies, 1992):

$$\begin{cases} Q = KG\sqrt{H} \\ P_m = A_t HQ \\ (1/g)(L/A)\,dQ/dt = H_{gr} - H - H_l \end{cases} \quad (1)$$

Where:

Q : Turbine flow rate [m^3/s]

K : Orifice gain factor at the rated operating point [m$^{5/2}$/s]
A_t : Turbine gain [null]
G : Gate opening [null]
L : Length of the penstock [m]
A : Area of the penstock [m^2]

H_{gr} : Gross head of the water column (difference of height between reservoir and tail race) [m]
H : Net head [m] (available energy at the turbine inlet)
H_l : Head loss due to friction in the conduit [m]
g : Gravity acceleration [m/s^2]
P_m : Mechanical power [W]

Linearizing Eq. (1), by considering small perturbation about an operating point and neglecting friction losses in the penstock, we can obtain the transfer function between the gate opening and the mechanical power expressed in per unit (Kundur, 1993) :

$$\delta P_m = \frac{1 - sT_w}{1 + \frac{1}{2} sT_w} \delta G \tag{2}$$

Where $T_w = \left(L/A\right)\left(Q_0/H_{gr}\right)$ is the water starting time (that depends by the operating point (Q_0,H_{gr})).

The transfer function (Eq. (2)) represents a "non-minimum phase" system. Indeed it has a zero on the right plane. The special characteristic of the transfer function may be illustrated in Figure 1 by considering the response to a positive step change in gate position that points out an initial inverse response:

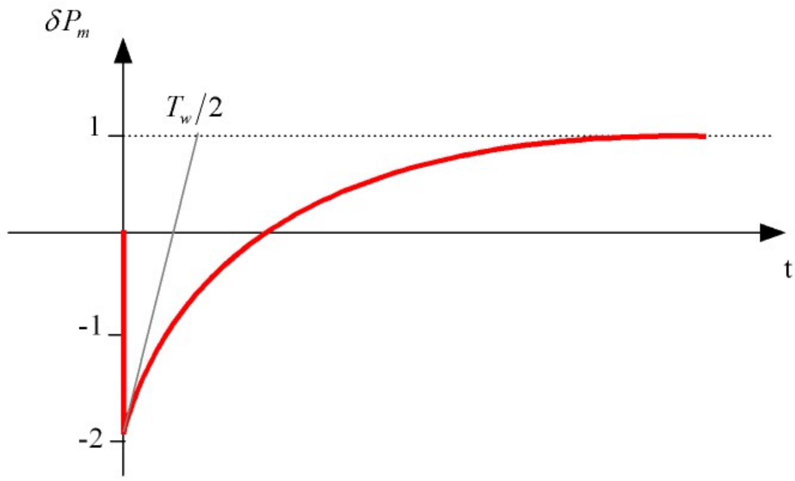

Figure 1 Power response to a gate step variation (Kundur, 1993)

We will implement subsequently the above-mentioned equations in AMESim that is one of the wide-spread simulation tools that employ power flow approach. AMESim is a virtual environment composed of a set of computational tools to create, analyze, exchange and customize dynamic models with a support of different libraries. These libraries embrace models from different domains, such as hydraulic, pneumatic, mechanical, signal, and so on. The AMESim implementation of the Eq. (1) together with a rotary mass that represents the inertia of the rotor is presented in the figure below:

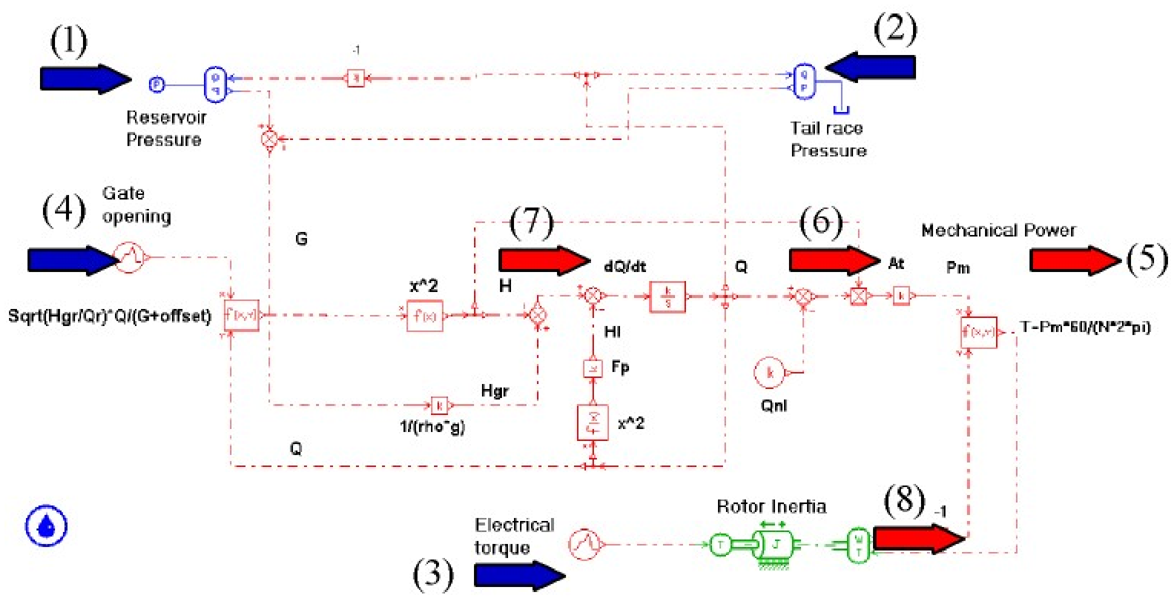

Figure 2 AMESim diagram of the non-linear model assuming inelastic water column

As showed in Figure 2 the inputs are: the pressure in the reservoir (1) and in the tail race (2), the electrical power demanded by electrical loads (3) and the position of gate opening (4). We can read as outputs of this model: the mechanical power of the system (5), the volumetric flow rate of the turbine (6), the net head (7) and the turbine shaft speed (8), that is directed linked with the frequency of the electrical power supplied to the electrical net. Here is used an hybrid approach since equations are implemented by signal approach while the rotary mass is connected by a physical mechanic interface (power flow approach).

If we give a small perturbation on the gate:

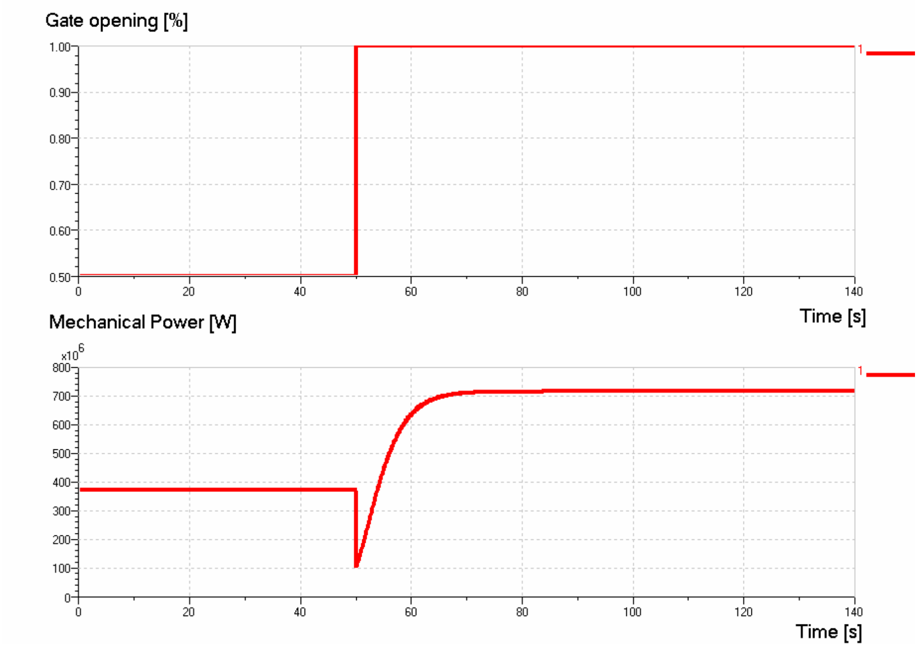

Figure 3 Power response to a 50% positive gate step variation

The result of the simulation of the mechanical power presents an inverse response that fits perfectly the theoretical consideration drawn out in Figure 1. We can also give a ramp variation and see an analogous behavior in Figure 4:

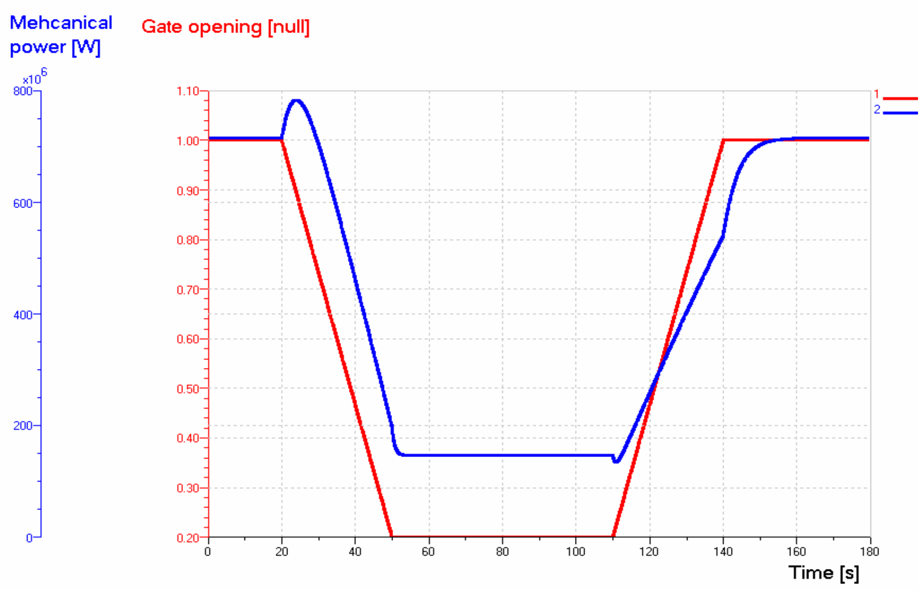

Figure 4 Gate ramp variation between 100% and 20%, first to close, and after to open

The difference of the power overshoot amplitudes is due to the fact that a non-linear system is working in two different operating points. We need to highlight that the rated point for the power depends by the value of the gross head that can change in consequence of seasonal variations of the level of the reservoir.

3. DISTRIBUTIVE PARAMETER LINEAR MODEL ASSUMING ELASTIC WATER COLUMN

This model is more detailed since it includes the effects of the water inertia, water compressibility, and pipe wall elasticity in the penstock. The effect of the water inertia, after a changing in the turbine gate opening, leads a delay in the changing of the flow. The effect of elasticity of the walls and of the compressibility of the water causes traveling waves of pressure and flow in the pipe, this phenomenon is commonly referred as "water hammer".

As exposed in (Kundur, 1993), relaxing the initial assumption of incompressibility of the water, we obtain a set of partial differential-algebraic equation. The solution of the correspondent linearized equations gives the typical traveling wave solution for flow and pressure. This enables to obtain the transfer functions between the gate opening area A_v and, respectively, the mass flow (w_{out}) and the pressure (p_{out}) at the penstock end:

$$\Delta w_{out} = \frac{\mu_v}{1+\beta\tanh(s\tau)}\Delta A_v = G_w(s)\Delta A_v \tag{3}$$

$$\Delta p_{out} = -Z\frac{\mu_v \sinh(s\tau)}{\cosh(s\tau)+\beta\sinh(s\tau)}\Delta A_v = -R_v\frac{\mu_v}{1+\beta^{-1}\tanh^{-1}(s\tau)}\Delta A_v = G_p(s)\Delta A_v \tag{4}$$

Working out the parameters:

$$\mu_v = \overline{w}\big/\overline{A}_v \quad R_v = 2\left(\overline{p}_{in}-\overline{p}_{out}\right)\big/\overline{w} \quad \tau = L/c \quad Z = c\big/A \quad \beta = Z\big/R_v \tag{5}$$

Where, at the chosen operating point:

β : A*llievi*'s parameter

μ_v : Orifice gain [kg.s^{-1}.m^{-2}]

R_v : Orifice resistance [Pa.s.kg^{-1}]

τ : Elastic time [s]

L : Penstock length [m]

A : Penstock area [m^2]

Z : Hydraulic surge impedance [m^{-1}s^{-1}]

$\overline{A}_v$: Orifice (turbine inlet) area [m^2]

$\overline{w}$: Mass flow [kg/s]

$\overline{p}_{in}$ and $\overline{p}_{out}$ are respectively the static pressure at the turbine inlet and outlet [Pa]

c : Speed of sound [m/s]

By equation (3) and (4), normalizing for per unit values, it is easy to find out the transfer function of the hydraulic power P_{hyd} in function of the normalized gate opening G:

$$\delta P_{hyd} = G_{hyd}(s)\delta G = \frac{1-2\beta\tanh(s\tau)}{1+\beta\tanh(s\tau)}\delta G \tag{6}$$

Remembering that:

$$\Delta A_v = \overline{A}_v\delta G \tag{7}$$

So, substituting the expression *tanh* by the known expression with exponentials, we can express all the previous transfer functions with Laplace delays:

$$G_w(s) = \frac{\mu_v}{1+\beta\dfrac{e^{s\tau}-e^{-s\tau}}{e^{s\tau}+e^{-s\tau}}} = \frac{1+e^{-2s\tau}}{(1+\beta)+(1-\beta)e^{-2s\tau}} = \frac{\mu_v}{1+\beta}\frac{1+e^{-2s\tau}}{1+\alpha e^{-2s\tau}} \tag{8}$$

$$G_p(s) = -R_v\frac{\mu_v}{1+\dfrac{1}{\beta}\dfrac{e^{s\tau}+e^{-s\tau}}{e^{s\tau}-e^{-s\tau}}} = -R_v\mu_v\beta\frac{1-e^{-2s\tau}}{(1+\beta)+(1-\beta)e^{-2s\tau}} = -R_v\mu_v\frac{\beta}{1+\beta}\frac{1-e^{-2s\tau}}{1+\alpha e^{-2s\tau}} \tag{9}$$

$$G_{hyd}(s) = \frac{1-2\beta\tanh(s\tau)}{1+\beta\tanh(s\tau)} = \frac{e^{s\tau}(1-2\beta)+e^{-s\tau}(1+2\beta)}{e^{s\tau}(1+\beta)e^{-s\tau}(1-\beta)} = \frac{1}{1+\beta}\frac{(1-2\beta)+(1+2\beta)e^{-2s\tau}}{1+\alpha e^{-2s\tau}} \qquad (10)$$

These transfer functions can be easily implemented in AMESim creating a super component. This will be inserted in the implementation of the second model showed in Figure 5. Rated values of pressure, flow and power must be added to the respective variations that come out from the super component. Finally it is added the rotary mass that represents the rotor inertia.

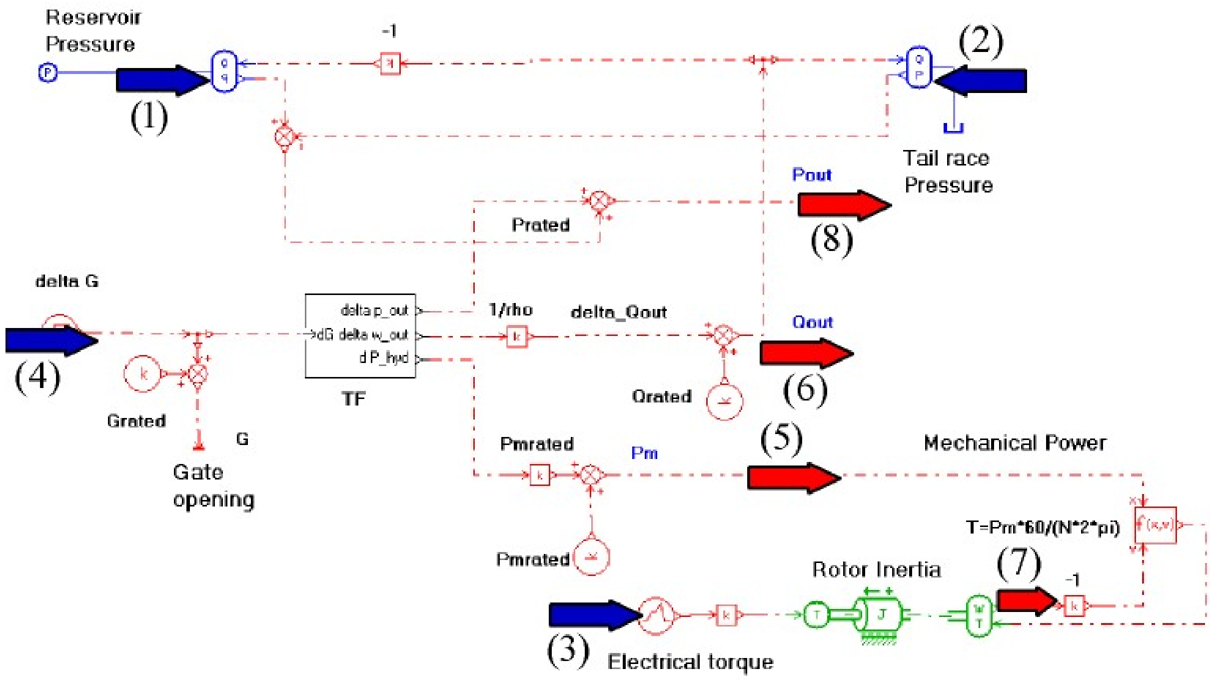

Figure 5 AMESim diagram of the distributive parameter linear model assuming elastic water column

Observing the step response it is possible to evaluate the effects of the traveling wave over pressure, flow and power:

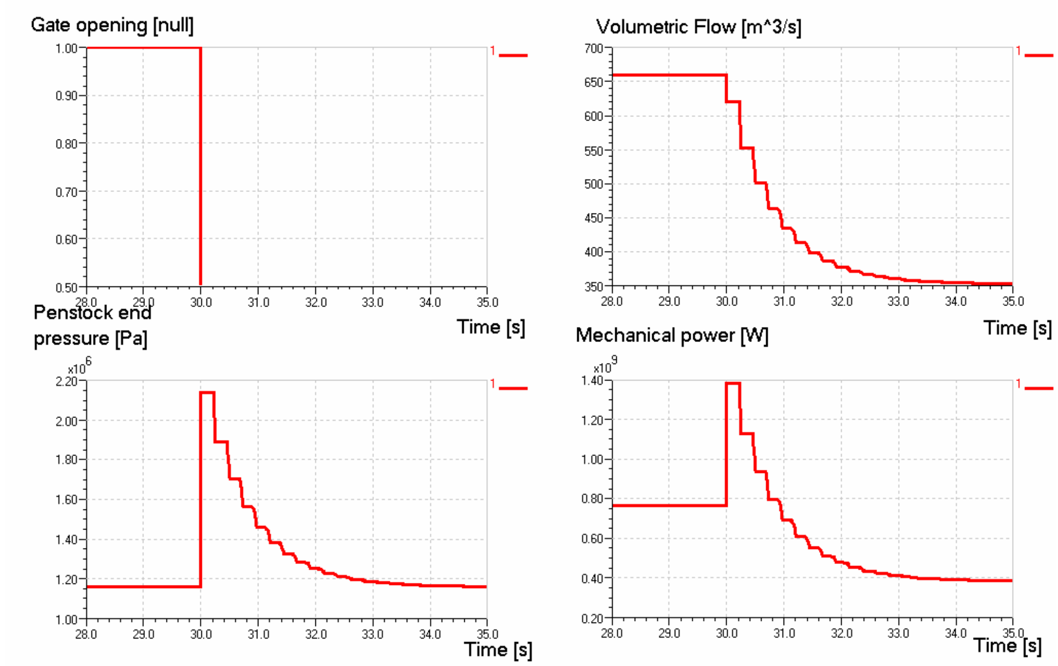

Figure 6 Power, flow and head response to a 50% gate closing respect to the gate at the full power

Overpressure waves reflect up and down blowing over after a while due to the losses in the valve (turbine) at the penstock outlet. This causes the water hammer to be damped with the time. However, since penstock friction is neglected in this mathematical model of the pipe, pressure keeps constant during the wave traveling time. Conversely, simulations made with a physical model, show that pressure should vary continuously during traveling time.

We can also estimate analytically the value of pressure at the turbine inlet for a step variation of the area of the guide vane cascade ΔA_v using the initial value theorem:

$$\Delta A_v(t) = \Delta A_v step(t)$$

$$\Delta P_{out}(0) = \lim_{s\to+\infty} sA_p(s)\frac{\Delta A_v}{s} = \lim_{s\to+\infty} -R_v \frac{\mu_v}{1+\frac{R_v}{Z}\,\mathrm{c}\tanh(s\tau)} s\frac{\Delta A_v}{s} = -\mu_v Z \| R_v \,\Delta A_v \qquad (11)$$

To present a real case in the model simulated above we used the data of ITAIPU power plant:

L: Penstock length	140 m
A: Penstock area	80 m^2
H_{gr}	118m
$\overline{w}$: mass flow	660000 kg/s
c : speed of sound	1000 m/s
$\overline{A}_v$: Orifice (turbine inlet) area	64.5 m^2

In this case, occurs that $R_v < Z$ (high flow and low head power plant) and so in the parallel it matters the only orifice resistance R_v. Therefore, Eq. (11) becomes:

$$\Delta P_{out}(0) = -\mu_v R_v \Delta A_v \simeq -\frac{\overline{w}}{\overline{A}_v}\frac{2\left(\overline{p}_{in}-\overline{p}_{out}\right)}{\overline{w}}\overline{A}_v \delta G \cong 10^6\, Pa \qquad (12)$$

That means that for closing abruptly half of the gate, we have an over pressure of almost the 200% of the rated pressure, since for a gross head of 118 meters we have a static pressure of about 10^6 Pa. That matches perfectly with the simulation results presented in

Figure 6. (Souza,1983) recommends that the maximum overpressure for a net head up to 150 meters must not be higher than 25% of the static pressure *($\rho g H_{gr}$)*.

For a more realistic case, the fast closing variations behave as ramps instead of steps. The next simulation enables to estimate the maximum closing speed to prevent damage in the penstock due to an excessive increasing of pressure:

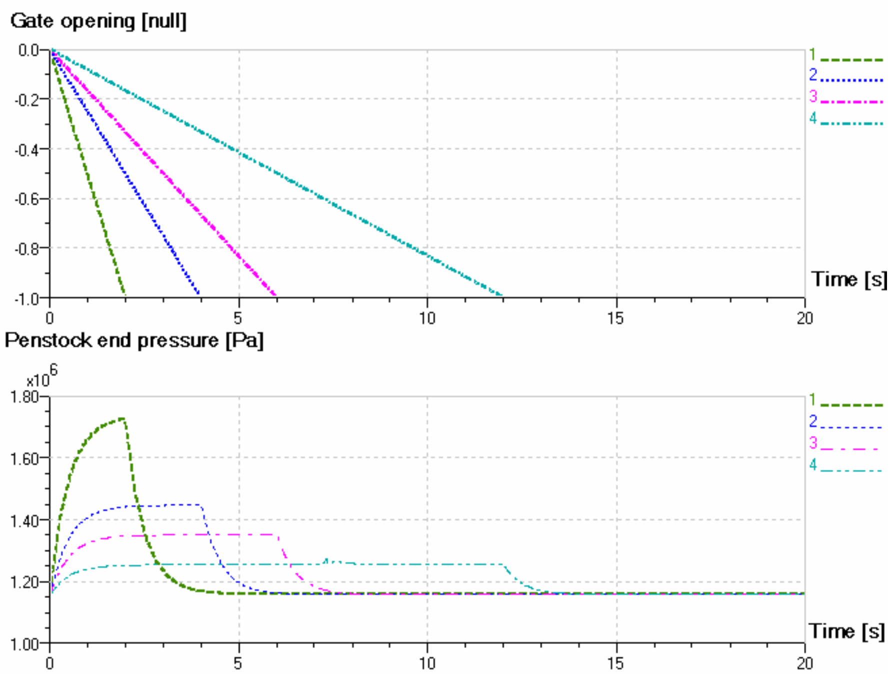

Figure 7 Overpressure at the penstock outlet corresponding to different speed of the gate closing ramp

We can see in the Figure 7, that a closing time lower than 5 seconds causes certainly a damage in the penstock. Conversely is possible to design the penstock in order to prop up a certain overpressure and so enable to close the gate with the desired speed. For the purpose can be used the Mariotti equation that estimates the minimum thickness s of a steel penstock to prop up a maximum pressure p_{max} (Souza, 1983). The equation is valid only when the ratio D/s>20:

$$s > 0.05 \frac{p_{\max} D}{\sigma_{ta}} \tag{13}$$

where:

s : Thickness

$p_{\max}$: Maximum pressure at the penstock outlet

D: Penstock diameter

σ_{ta} : working tension at the maximum traction (1/3 of the breakout tension)

To estimate the diameter can be considered as a guideline to keep the fluid velocity v_c lower than 8 m/s in order to reduce severe friction losses:

$$D = 1.13\sqrt{Q / v_c} \tag{14}$$

A further useful employment of the model is for simulating the turbine over speeding. This happens normally when an electric falling in the demand occurs (load rejection) and the group turbine-generator speeds-up because there is no longer an electric load. This can cause serious mechanical damage in the flanges and bearings of the turbine.

From

Figure 8 it is possible to observe how speed raises abruptly in consequence of a load rejection:

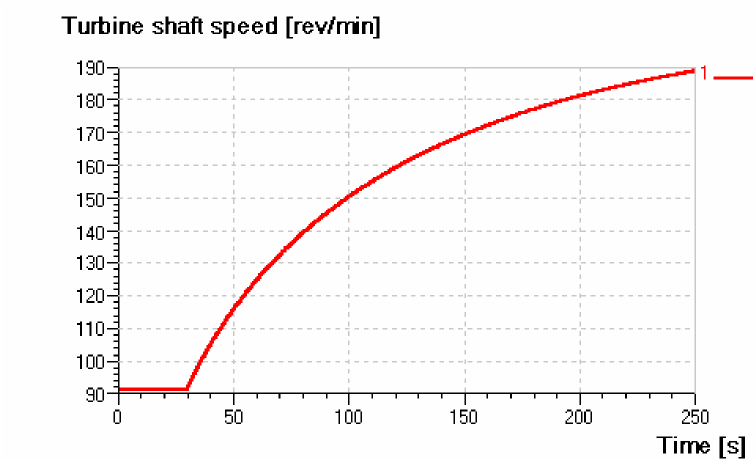

Figure 8 Turbine over speed after an electrical load rejection

In this simulation we perform load rejection setting an abrupt drop (of about 75%) of the electrical power demand. We can notice that the rated value of the turbine speed, that was about 91.6 rpm, get close to 190 rpm, that is about 200 % of the rated value as commonly happens in the reality (Henn, 2001). (Souza, 1983) introduces an useful expression that can be used to show the parameters the over-speed depends on:

$$\frac{\omega_{os}^2}{\omega_n^2} = K \frac{t_s}{T_M} + 1 \tag{15}$$

Where:

ω_{OS} : Over-speed

t_s : Gate closing time

ω_n : Rated speed

$T_M = J\omega_n^2 / \overline{P}_m$: Mechanical starting time

J : Turbine rotor inertia

$\overline{P}_m$: Rated mechanical power

The main way to reduce over speed is reducing the gate closing time, and so raise the closing speed. This increases water hammer effects. A constructive solution can be increasing the inertia J of the rotor-generator group, this will raise the mechanical staring time T_w but implies additional costs in the whole design.

4. CONCLUSIONS

The presented models give an idea of different methods to build a simulator for a power plant, with two different levels of detail. Have been simulated two of the most important phenomena: water hammer and over speed. The developed simulators are able to draw out guidelines to take into account in the preliminary design of a new power plant. Some of these guidelines are can be in contraposition. For example the phenomenon of the water hammer determines a constraint over the thickness of the penstock that must be strong enough to prop up the overpressures due to a fast closing on the gate. On the contrary the phenomenon of the over-speed requires that closing time be the smallest is possible. Alternatively it can be reduced increasing the generator inertia leading to a certain additional cost. These considerations help the designer to determine the optimum economical trade-off between the various structural solutions that he is taking into account. In other words a power plant simulator helps to rationalize the preliminary design phase and to avoid expensive corrective actions on the work.

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