DAMAGE LOCALIZATION IN ELASTOPLASTIC PLANE TRUSSES PROMOTED BY THERMOMECHANICAL COUPLING

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Abstract. *Inelastic cyclic deformation promotes heating of metallic structural elements. The temperature rise experimented by a mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. High loading rates and/or high amplitudes of inelastic deformation tends to generate a considerable amount of heat that can dramatically alter the structural response. Nevetheless, traditional design methodologies use isothermal models that do not considers the variation of the material temperature and unreal predictions may be obtained. In this paper, a continuum damage mechanics model is proposed to study the thermomechanical coupling effects on the life prediction of metallic structures subjected to cyclic inelastic loadings. A thermodynamic approach allows a proper identification of the thermomechanical coupling in the mechanical and thermal equations. A numerical procedure is developed based on an operator split technique associated with an iterative numerical scheme in order to deal with the non-linearities in the formulation. With this assumption, coupled governing equations are solved involving three uncoupled problems: thermal, thermoelastic and elastoplastic behaviors. Classical finite element method is employed for spatial discretization in all uncoupled problems. Numerical simulations of steel plane trusses subjected to cyclic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to damage evolution.*

Keywords*: Thermomechanical Coupling, Modeling, Numerical Simulation, Elastoplasticity, Damage.*

1. INTRODUCTION

Thermomechanical coupling is an important phenomenon in different engineering problems. Inelastic cyclic strain promotes heating of metallic structural elements, and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern (Simo and Miehe, 1992; Pacheco, 1994; Barbosa et al., 1995; Pacheco and Mattos, 1997). The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. Nevertheless, traditional low-cycle fatigue models neglect the material temperature variation due to thermomechanical coupling and unreal life predictions may be obtained. Indeed, there are situations where such couplings cannot be neglected and a physically more realistic model must take it into account. Figure 1 shows the feedback phenomenon that can be observed in metallic elements subjected to inelastic cyclic strain loadings.

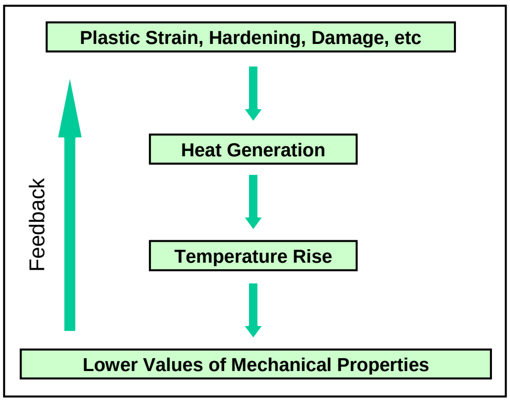


Figure 1. Thermomechanical coupling in metallic elements subjected to inelastic cyclic strain loadings.

Since temperature variation can interfere with the fatigue phenomenon and most classical low-cycle fatigue models only take into account isothermal processes, the ASTM standard for low-cycle fatigue testing (ASTM, 1992) establishes that the gradient of temperature during a testing program must not exceed the range of ±2°C. For situations with high inelastic amplitudes, the standard recommends the use of cooling devices and low frequency loadings to maintain the specimen temperature on the established range. However, this can be a difficult condition to achieve in operational real mechanical components.

In this paper, a continuum damage mechanics model with internal variables is proposed to study the thermomechanical coupling effects on the life prediction of metallic truss structures subjected to inelastic cyclic loadings (Pacheco, 1994; Lemaitre and Chaboche, 1990). A thermodynamic approach permits a rational identification of the thermomechanical coupling in mechanical and thermal equations. A numerical procedure is developed based on the operator split technique associated with an iterative numerical scheme in order to deal with the nonlinearities in the formulation. With this assumption, coupled governing equations are solved considering three uncoupled problems: thermal, thermoelastic and elastoplastic. Classical finite element method is employed for spatial discretization in the uncoupled problems. Numerical simulations considering an austenitic stainless steel (AISI 316L) truss structure subjected to cyclic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to damage evolution.

2. CONSTITUTIVE MODEL

Constitutive equations may be formulated within the framework of continuum mechanics and the thermodynamics of irreversible processes, by considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ (Lemaitre and Chaboche, 1990; Pacheco, 1994).

With this aim, a Helmholtz free energy is proposed as a function of observable variables, total strain, ε_{ij}, and temperature, T. Moreover, the following internal variables are considered: plastic strain, ε_{ij}^p, kinematic hardening, c_{ij}, isotropic hardening, p, and damage, D. The macroscopic quantity D ($0 \le D \le 1$) represents the material local degradation. When $D = 0$ the material is in a virgin state and when $D = 1$ the material is completely damaged. Therefore, the following free energy is proposed, employing indicial notation where summation convention ($i = 1,2,3$) is evoked (Eringen, 1967), except when indicated:

$$\rho\psi(\varepsilon_{ij}, \varepsilon_{ij}^p, c_{ij}, p, D, T) = (1-D)\left[W_e(\varepsilon_{ij} - \varepsilon_{ij}^p, T) + W_a(c_{ij}, p, T)\right] - W_T(T) \tag{1}$$

where ρ is the material density, W_e is the elastic energy density, W_a is the energy density associated to the hardening and W_T is the energy density associated with the temperature, defined as:

$$\begin{aligned}
&W_e(\varepsilon_{ij} - \varepsilon_{ij}^p, T) = \frac{E}{2(1+\nu)}\left[(\varepsilon_{ij} - \varepsilon_{ij}^p)(\varepsilon_{ij} - \varepsilon_{ij}^p) + \frac{\nu}{1-2\nu}(\varepsilon_{jj} - \varepsilon_{jj}^p)^2\right] - \frac{\alpha E}{1-2\nu}(\varepsilon_{jj} - \varepsilon_{jj}^p) \\
&W_a(c_{ij}, p, T) = \tfrac{1}{2} a\, c_{ij} c_{ij} + b\left[p + (1/d)\, e^{-dp}\right] \\
&\mathrm{W}_T(T) = \rho \int_{T_0}^{T} C_1 \log(\xi)\, \mathrm{d}\xi + \frac{\rho}{2} C_2 T^2
\end{aligned} \tag{2}$$

where T_0 is a reference temperature, E is the Young modulus, ν is the Poisson ratio, a is a material parameter associated with kinematic hardening, while b and d are material parameters associated with isotropic hardening. C_1 and C_2 are positive constants. The increment of elastic strain is defined as follows:

$$d\varepsilon_{ij}^e = d\varepsilon_{ij} - d\varepsilon_{ij}^p - \alpha_T dT \delta_{ij} \tag{3}$$

The last term is associated with thermal expansion and the parameter α_T is the coefficient of linear thermal expansion.

The general formulation of this model was developed and previously applied to the study of various related problems (Pacheco, 1994; Pacheco and Mattos, 1997; Pacheco *et al.*, 2001; Oliveira *et al.*, 2003; Oliveira, 2004; Silva *et al.*, 2004). A detailed description of this constitutive model may be obtained in the cited references.

This contribution considers life prediction of metallic plane truss structures subjected to cyclic inelastic loadings. From the mechanical point of view it can be assumed that the truss elements experiments a uniaxial stress-sate, as the loadings are applied at the elements joints. On the other hand, concerning thermal characteristics, it is assumed that the truss elements experiments a uniaxial heat flow conduction through the element length, as the truss elements cross-section dimensions are considerable smaller than its length. Under these assumptions, a one-dimensional model is formulated and tensor quantities presented in the general formulation may be replaced by scalar quantities. For this situation the thermodynamics forces (σ, P, B^c, B^p, B^D, s), respectively associated with state variables $(\varepsilon, \varepsilon^p, c, p, D, T)$, are defined as follows:

$$\sigma = \rho \frac{\partial \psi}{\partial \varepsilon} = (1-D)\left[\mathrm{E}(\varepsilon - \varepsilon^{p}) - E\alpha_T (T - T_0)\right] \quad ; \quad P = -\rho \frac{\partial \psi}{\partial \varepsilon^{p}} = \sigma$$
$$B^{c} = -\rho \frac{\partial \psi}{\partial c} = -(2/3)\,X = -(1-D)\,a\,c \quad ; \quad B^{p} = -\rho \frac{\partial \psi}{\partial p} = -R = -(1-D)b\left[1 - e^{-dp}\right] \tag{4}$$
$$B^{D} = -\rho \frac{\partial \psi}{\partial D} = W_e(\varepsilon_{ij} - \varepsilon_{ij}^{p}, T) + W_a(c_{ij}, p, T) \quad ; \quad \mathrm{s} = -\rho \frac{\partial \psi}{\partial T}$$

where X and R are auxiliary variables directly related to kinematic and isotropic hardenings, respectively. In order to describe dissipation processes, it is necessary to introduce a potential of dissipation $\phi(\dot{\varepsilon}^{p}, \dot{c}, \dot{p}, \dot{D}, q)$, which can be split into two parts: $\phi(\dot{\varepsilon}^{p}, \dot{c}, \dot{p}, \dot{D}, q) = \phi_I(\dot{\varepsilon}^{p}, \dot{c}, \dot{p}, \dot{D}) + \phi_T(q)$. This potential can be written through its dual $\phi^{*}(P, X, R, B^{D}, g) = \phi_I^{*}(P, X, R, B^{D}) + \phi_T^{*}(g)$, as follows:

$$\phi_I^{*} = I_f^{*}(P, X, R, B^{D}) \quad ; \quad \phi_T^{*} = \frac{T}{2}\Lambda\, g^{2} \tag{5}$$

where $g = (1/T)\ \partial T/\partial x$ and Λ is the coefficient of thermal conductivity; $I_f^{*}(P, X, R, B^{D})$ is the indicator function associated with elastic domain (Lemaitre and Chaboche, 1990),

$$f(\sigma, X, R) = \left|\sigma - X\right| - (S_Y + R) \le 0 \tag{6}$$

where S_Y is the material yield stress. A set of evolution laws obtained from ϕ^* characterizes dissipative processes,

$$\dot{\varepsilon}^{p} = \frac{\partial \phi^{*}}{\partial P} = \lambda\,\mathrm{sign}(\sigma - X) \ ; \ \dot{c} = \frac{\partial \phi^{*}}{\partial B^{c}} = \dot{\varepsilon}^{p} + \frac{\varphi}{a} B^{c}\dot{p} \quad ; \quad \dot{p} = \frac{\partial \phi^{*}}{\partial B^{p}} = \lambda \ ; \ \dot{D} = \frac{\partial \phi^{*}}{\partial B^{D}} = \frac{B^{D}}{S_0}\dot{p}$$
$$q = -\frac{\partial \phi^{*}}{\partial g} = -\Lambda T\, g = -\Lambda \frac{\partial T}{\partial x} \tag{7}$$

where λ is the plastic multiplier (Lemaitre and Chaboche, 1990) from the classical theory of plasticity, sign(x) = x / |x|, φ is a material parameter associated with kinematic hardening and q is the heat flow. By assuming that the specific heat is $c_p = -(T/\rho)\,\partial^2 W/\partial T^2$ and also considering the set of constitutive Eqs. (4) and (7), the energy equation can be written as (Pacheco, 1994):

$$\frac{\partial}{\partial x}\left(\Lambda \frac{\partial T}{\partial x}\right) - h\frac{Per}{A}(T - T_\infty) - \rho c_p \dot{T} = -a_I - a_T \qquad \text{where} \quad \begin{cases} a_I = \sigma\dot{\varepsilon}^{p} - X\dot{c} - R\dot{p} + B^{D}\dot{D} \\ a_T = T\left(\dfrac{\partial \sigma}{\partial T}\left(\dot{\varepsilon} - \dot{\varepsilon}^{p}\right) + \dfrac{\partial X}{\partial T}\dot{c} + \dfrac{\partial R}{\partial T}\dot{p} - \dfrac{\partial B^{D}}{\partial T}\dot{D}\right) \end{cases} \tag{8}$$

where h is the convection coefficient, T_∞ is the surrounding temperature, Per is the perimeter and A is the cross section area. Terms a_I and a_T are, respectively, internal and thermal coupling. The first one appears in the right hand side of the energy equation and is called internal coupling. It is always positive and has a role in the energy equation similar to a heat source in the classical heat equation for rigid bodies. The last term in the right hand side of the energy equation can be positive or negative and is called the thermal coupling.

3. NUMERICAL PROCEDURE

The numerical procedure here proposed is based on the operator split technique (Ortiz *et al.*, 1983; Pacheco, 1994) associated with an iterative numerical scheme in order to deal with nonlinearities in the formulation. With this assumption, coupled governing equations are solved from three uncoupled problems: thermal, thermo-elastic and elastoplastic. In this article, finite element method is employed to perform spatial discretization of governing equations. Therefore, the following moduli are considered:

Thermal Problem - Comprises a one-dimensional conduction problem with surface convection. Material properties depend on temperature and, therefore, the problem is governed by nonlinear parabolic equations. Classical finite element method is employed for spatial discretization while *Crank-Nicolson* method is used for time discretization (Lewis *et al.*, 1996; Segerlind, 1984).

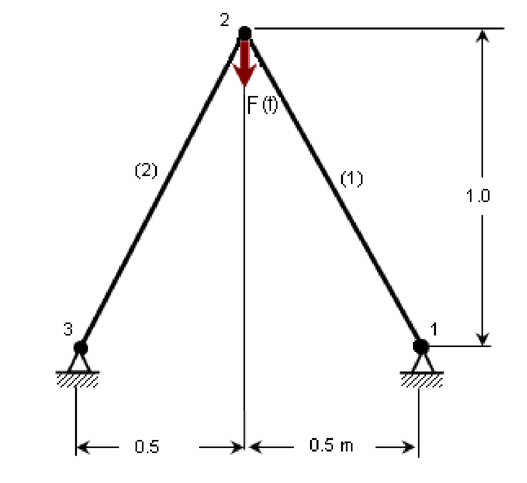
Thermo-elastic Problem - Stress and displacement fields are evaluated from temperature distribution. Classical finite element method is employed for spatial discretization (Segerlind, 1984).

Elastoplastic Problem - Stress and strain fields are determined considering the plastic strain evolution in the process. Numerical solution is based on the classical return mapping algorithm (Simo and Miehe, 1992; Simo and Hughes, 1998).

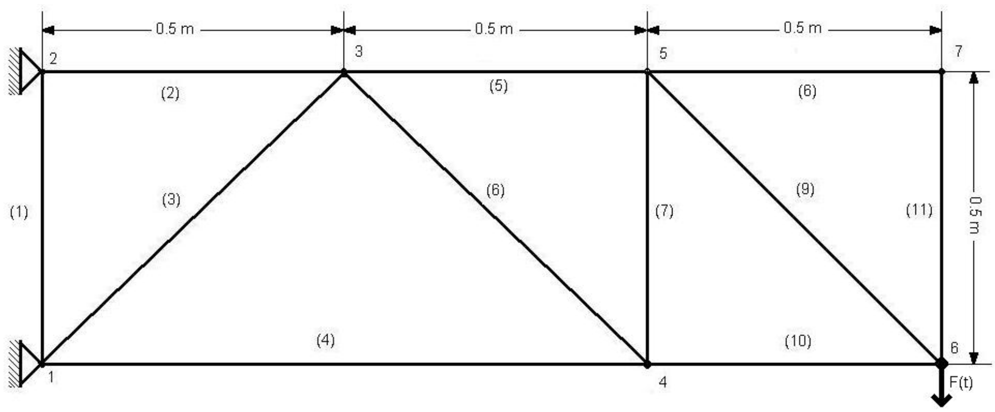
As an application of the general procedure technique, plane FEM truss elements are considered. Linear shape functions are adopted for all finite element moduli (Segerlind, 1984).

4. NUMERICAL SIMULATIONS

The proposed model is applied to the life prediction of two metallic (austenitic stainless steel AISI 316L) plane truss structures subjected to cyclic inelastic loadings. The two plane truss structures are shown in Fig. 2 and both have 10 mm diameter round truss elements. Constant displacement boundary conditions ($u_x = u_y = 0$) are applied to nodes 1 and 3 for *Structure I* and 1 and 2 for *Structure II*. A harmonic load (F) with a period of 10 s is applied to both structures (F = 55 kN for *Structure I* and F = 10 kN for *Structure II*).



(*a*)



(*b*)

Figure 2. Plane truss structures. *Structure I* (*a*) and *Structure II* (*b*).

Thermal and mechanical properties are temperature dependent and are fitted by linear equations from experimental data shown in Tab. 1 (Peckner and Bernstein, 1977; Pacheco, 1994). A linear kinematic hardening is considered ($\varphi = 0$). Moreover, the following constant parameters are used: convection coefficient (h) of 30 W/m^2, surroundings and initial structure temperature of 20°C, density (ρ) of 7800 kg/m^3 and a critical damage (D_{cr}) of 0.85. The heat flow removed by convection is calculated considering an average temperature obtained from the element nodal temperatures.

Table 1. Material parameters for AISI 316L (Peckner and Bernstein, 1977; Pacheco,1994).

	Temperature	
Properties	20°C	600°C
E (GPa)	196	150
S_Y (MPa)	225	108
b (MPa)	60	80
d (-)	8	10
a (GPa)	108.3	17.5
α ($1\text{x}10^{-6}$/K)	15.4	18.0
c_p (J/Kg K)	454	584
Λ (W/m K)	13	21

Numerical simulations are performed with the aid of computational software developed in *C* programming language. In order to allow the evaluation of the thermomechanical effects and the thermal boundary conditions in the damage localization of the truss, three models are considered:

Model 1: neglects the thermomechanical coupling terms present in Eq. (8) and therefore, the thermal problem is solved as a rigid body;

Model 2: considers the thermomechanical coupling terms and fixed thermal boundary conditions;

Model 3: considers the thermomechanical coupling terms and free thermal boundary conditions.

Real structures present thermal boundary conditions between the idealized fixed and free conditions. For example, an idealized fixed condition can be used to represent a structure whose supports are connected to a large mass with high thermal inertia.

The low-cycle fatigue parameter (S_0) that appears in the Eq. (7) is adjusted through a process involving a direct comparison of the life predictions obtained with the *uncoupled* modelwith experimental data (Bathias and Bailon, 1980; Pacheco and Mattos, 1997). This procedure is, therefore, in accordance with the ASTM recommendations for low-cycle fatigue tests (ASTM, 1992). Figure 3 shows a comparison between a ε-N curve (Stephens, *et al.*, 2000) obtained from experimental data from a low-cycle fatigue test (Bathias and Bailon, 1980) and the model prediction, considering *Model 3*, after the parameter adjustment for a 316L stainless steel bar at room temperature (20 °C). The results indicate a good agreement between the experimental data with those predicted by fatigue life for a strain amplitude range from 2% to 5%. The coefficient S_0 of Eq. (7) presents a dependency with plastic deformation amplitude, and can be adequately represented by the following equation (in Pa) (Pacheco, 1994; Pacheco and Mattos, 1997):

$$S_0 = (546.3\,\text{x}\,10^9)\,\text{e}^{-103.1\,\Delta\varepsilon^p} \tag{9}$$

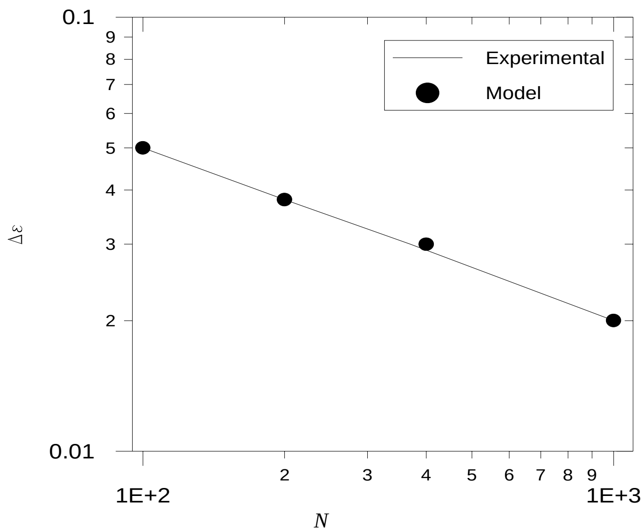
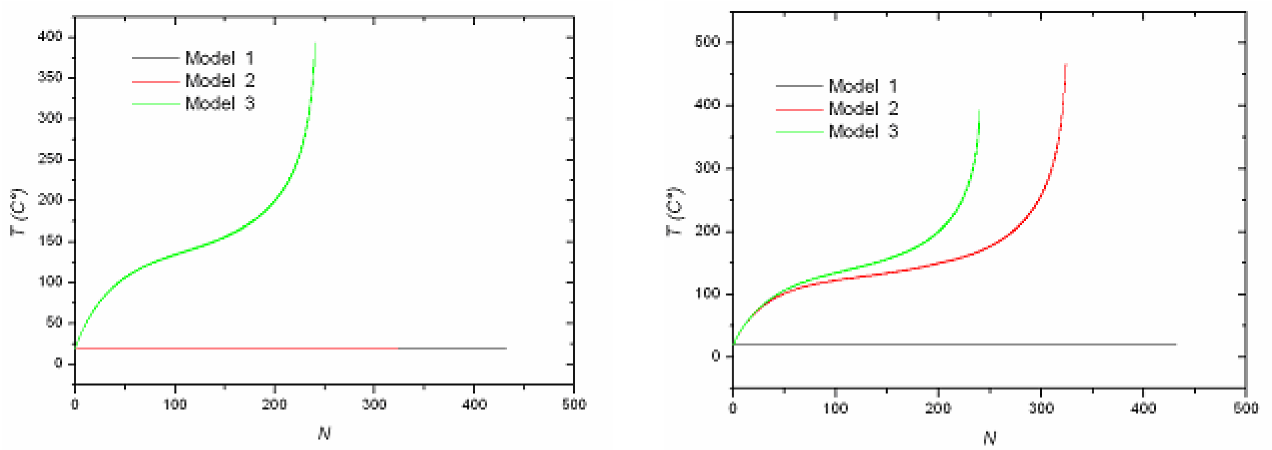


Figure 3. Proposed model fatigue life prediction and ε-N curve obtained from experimental data for stainless steel 316L bars at room temperature (Pacheco and Mattos, 1997).

The forthcoming analysis considers results obtained for the three models applied for the trusses shown in Fig.2. For the truss with two elements (Fig. 2*a*) the symmetry of the geometry, loading and boundary conditions guarantee that both elements present the same behavior. Therefore only one of the two elements is considered in the presented results. For the truss with 11 elements (Fig. 2*b*) results for the three models indicate a damage localization process occurring in element 2, which is the first to reach the critical damage (D = 0.85). This element is the critical one since it experiments the higher levels of stress and temperature and the presented results are related to this element.

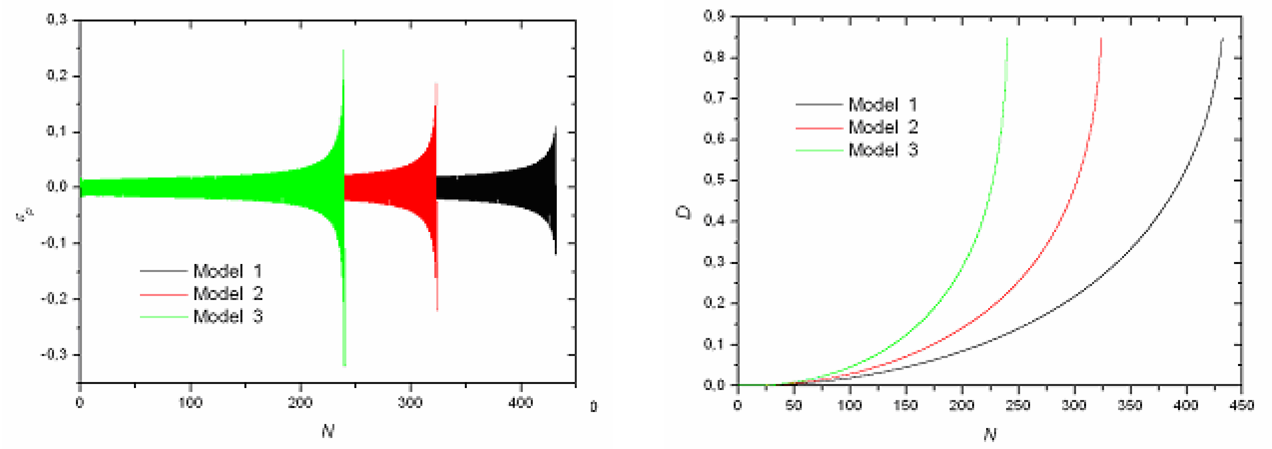
Figure 4 presents the temperature evolution for nodes 1 and 2 for *Structure I*. It is important to note that node 1 is localized on the structure support and therefore it temperature remains constant for *Models 1* and *2*. The temperature evolution behavior for *Model 2* (node 2) and *Model 3* (nodes 1 and 2) may be understood as a two stage process. The first one occurs as a result of two opposite but "balanced" forces in the energy equation (8): the thermomechanical coupling terms (a_I and a_T) acts as a heat source and the conduction/convection terms remove the heat. In this stage, the structure experiments a controlled temperature rise. In the last stage, as the thermomechanical coupling terms dominates the energy equation, the temperature rises abruptly until the critical damage is reached.



(*a*) (*b*)

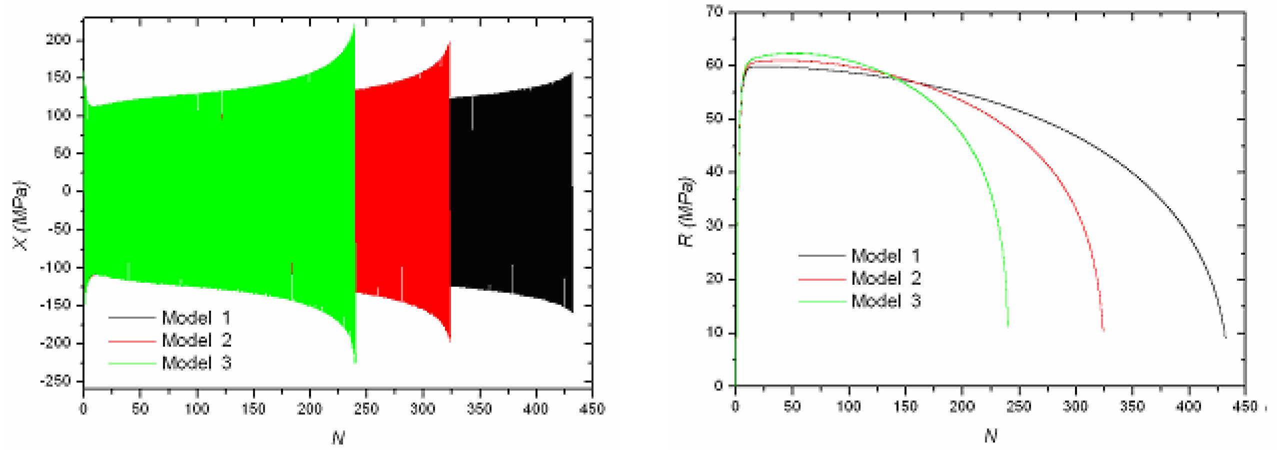
Figure 4. Temperature evolution for nodes 1 (*a*) and 2 (*b*). *Structure I*.

Figure 5 shows the plastic strain and damage evolution whereas Fig. 6 shows the kinematic and isotropic hardening for the three models. *Models 1*, *2* and *3* present the following fatigue life predictions, respectively: 432, 324 and 240 cycles. Using *Model 1* as reference, *Model 2* predicts a reduction of fatigue life of 25% whereas *Model 3* predicts a reduction of fatigue life of 44%. The thermomechanical coupling can be seen as a feedback phenomenon: the heat generated by the mechanical process causes an increase of temperature that promotes a decrease in the mechanical strength. As a consequence, the plastic strain amplitude tends to increase causing a greater temperature rise and so on. It is important to observe that the temperature boundary conditions influence the temperature evolution and therefore the mechanical behavior.



(*a*) (*b*)

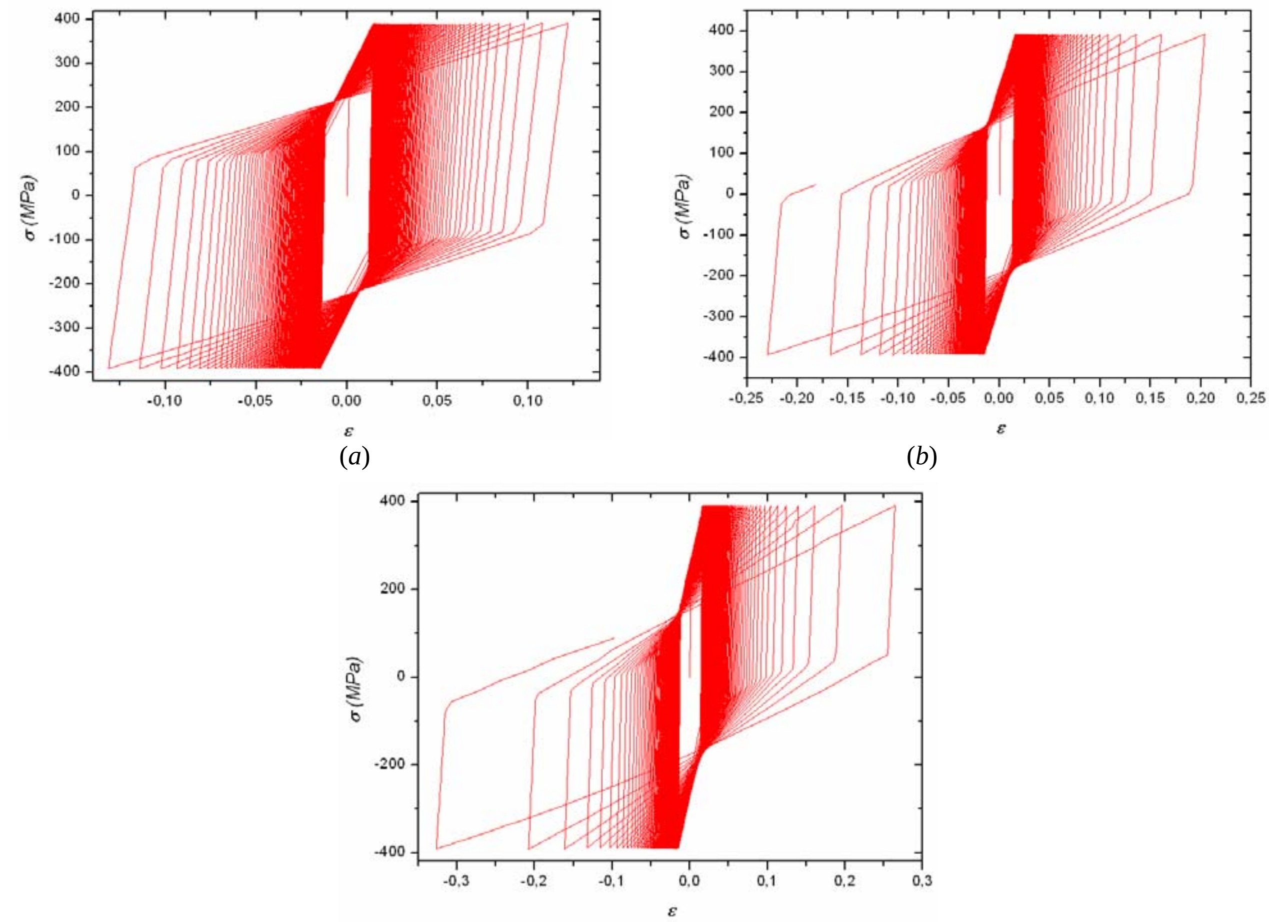
Figure 5. Plastic strain (*a*) and damage (*b*) evolution. *Structure I*.



(*a*) (*b*)

Figure 6. Kinematic (*a*) and isotropic hardening (*b*) evolution. *Structure I*.

Figures 7 present the stress-strain curves predicted by the three models. These figures show different stress-strain hysteresis loops, confirming that the thermomechanical coupling terms and the thermal boundary conditions can affect considerably the material behavior. The models show the damage evolution promoting the enlargement of the loop and the reduction of the stress-strain ratio (apparent Young modulus) in the elastic region (unloading linear region) as the damage directly affects the elastic energy density.

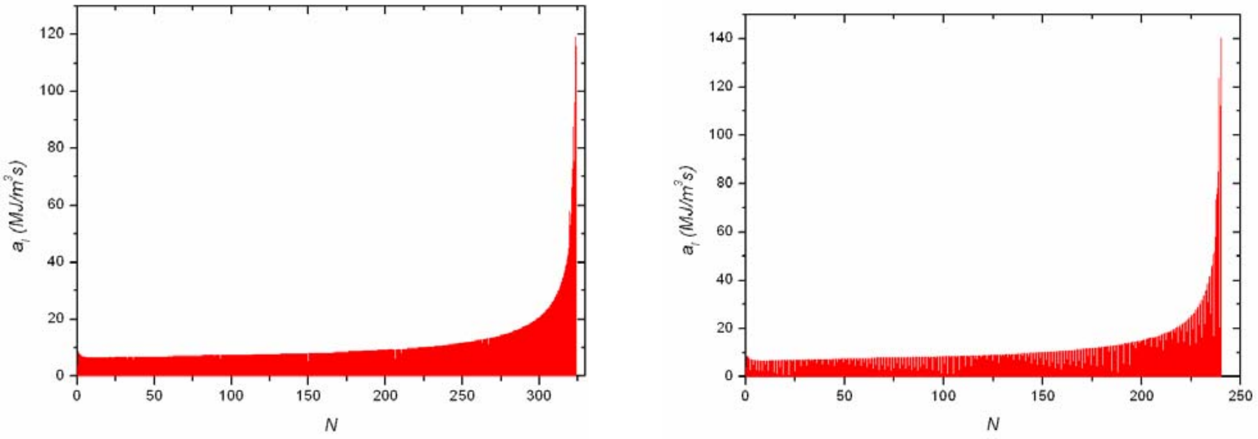


(*a*) (*b*)

(*c*)

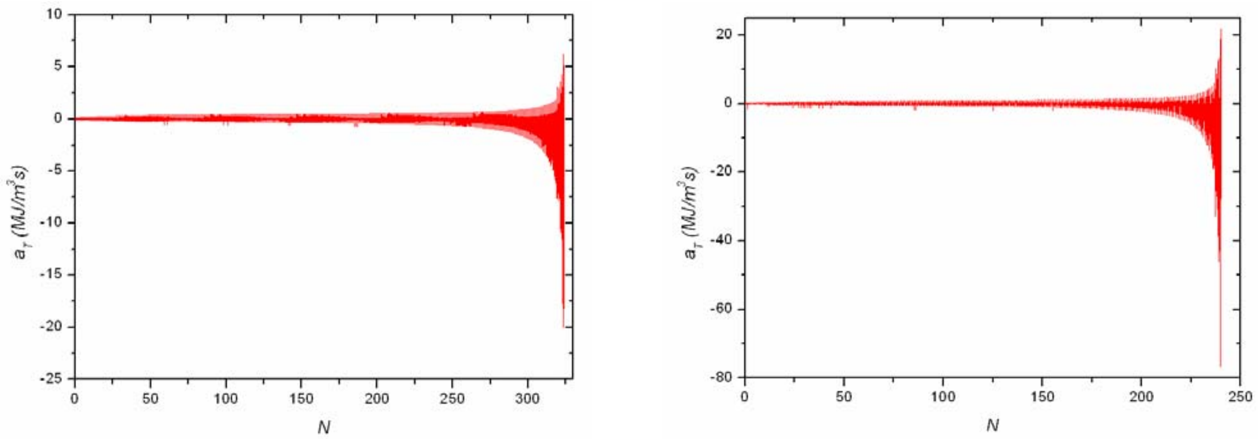
Figure 7. Stress-strain hysteresis loops for *Model 1* (*a*), *Model 2* (*b*), and *Model 3* (*c*). *Structure I*.

Figures 8 and 9 present the evolution of the thermomechanical couplings terms defined in Eq. (8): internal coupling (a_I) and thermal coupling (a_T). A considerable rise can be observed for the two couplings terms in the last cycles. This behavior is associated to the evolution of the variables affected by damage shown in previous figures. Figure 8 shows that the internal coupling is always positive, as predicted by the model. This guarantees that the second law of thermodynamics is not violated (Pacheco, 1994).



(a) (b)

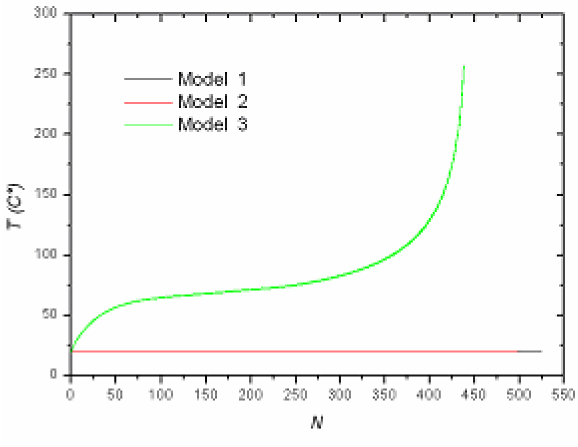
Figure 8. Internal coupling for *Model 2* (*a*) and *Model 3* (*b*). *Structure I*.



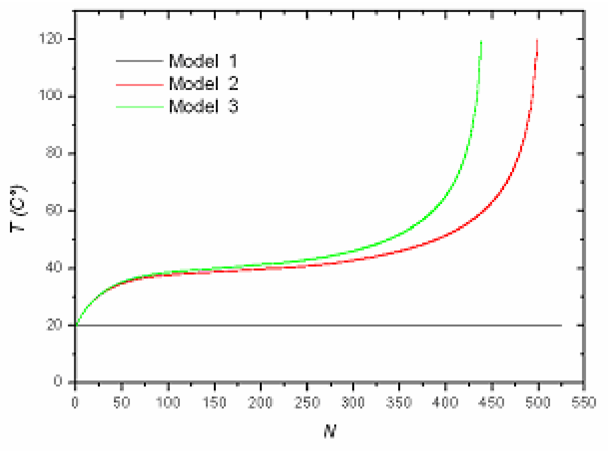
(a) (b)

Figure 9. Thermal coupling for *Model 2* (*a*) and *Model 3* (*b*). *Structure I*.

At this point *Structure II* is considered. Figure 10*a* and 10*b* presents the temperature evolution for nodes 2 and 3. These two nodes are localized at the ends of element 2, the critical element that experiments the higher levels of stress and temperature and the presented results are related to this element. A similar behavior presented for *Structure I* is observed. However *Structure II* presents a less accentuate fatigue life reduction and smaller temperature values for the two models that considers the thermomechanical coupling. *Models 1*, *2* and *3* present the following fatigue life predictions, respectively: 525, 498 and 438 cycles. Using *Model 1* as reference, *Model 2* predicts a reduction of fatigue life of 5% whereas *Model 3* predicts a reduction of fatigue life of 17%. This difference is due to a larger heat removal capacity for *Structure II*. Whereas in *Structure I* the heat is removed by convection and, for *Model 2* also by conduction with the boundary at the supports, in *Structure II* heat conduction can contribute more effectively for the heat removal as part of the structure does not develop plastic strain and consequently is not submitted to the mechanism of temperature rise promoted by plastic strains.



(a)



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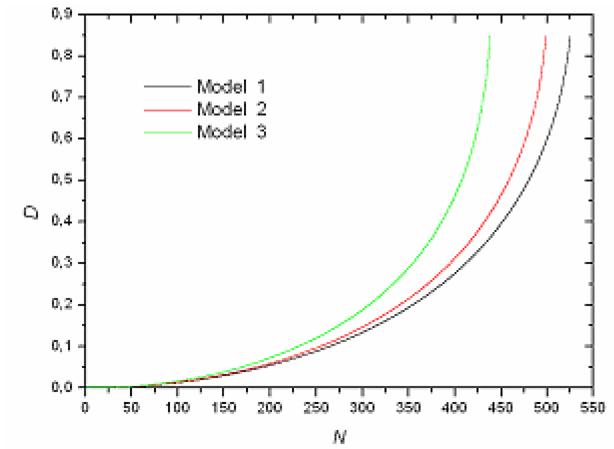

(*b*)

Figure 10. Temperature evolution for nodes 2 (*a*) and 3 (*b*). Damage evolution for element 2 (*c*). *Structure II*.

5. CONCLUSION

In this paper an anisothermal constitutive model with internal variables based on continuum damage mechanics is proposed to study the thermomechanical coupling effects in elastoplastic truss bars subjected to inelastic cyclic mechanical loadings. This formulation provides a rational methodology to study complex phenomena like the amount of heat generated during plastic strain of metals and how it affects its structural integrity. The numerical procedure developed is based on the operator split technique and allows one to deal with the nonlinearities in the formulation using traditional tested classical numerical methods, as the finite element method which is used for spatial discretization. Numerical simulations considering two austenitic stainless steel (AISI 316L) truss structures subjected to cyclic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to damage evolution. It is also shown that it is important to consider the thermomechanical coupling effects and the thermal boundary conditions in low-cycle fatigue design of mechanical components, especially when high loading rates are involved. In these situations, wrong predictions can be obtained and unexpected failures may occur when thermomechanical effect is neglected.

6. ACKNOWLEDGEMENTS

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