DYNAMICS AND CONTROL OF THREE-AXIS SATELLITES BY THRUSTER ACTUATORS USING A LINEAR QUADRATIC REGULATOR

Gilberto Arantes Jr., gilberto@spacetec.zarm.uni-bremen.de
Center of Applied Space Technology and Microgravity - ZARM, Bremen, Germany

Adrielle C. Santana, drisanufop@yahoo.com.br
Luiz S. Martins-Filho, luizm@iceb.ufop.br
Federal University of Ouro Preto - UFOP, Computer Science Department - DECOM/ICEB, Ouro Preto, Brazil

Abstract. *The purpose of this paper is to deal with the attitude dynamics and control of a three-axis stabilized satellite which is subjected to different kinds of perturbation, including gravity gradient. In this study the satellite is represented as a rigid body and the attitude motion of the satellite is described by using Euler parametrization in the kinematics equations. A set of thrusters is used for attitude control; each thruster has been modeled as a limited impulsive function. This work analyzes the feasibility of satisfying the attitude control requirements (for example: point accuracy) for three-axis stabilized spacecrafts on Low Earth Orbits (LEO). The simulations have been carried out using some of the parameters of the Brazilian satellite PMM (Multi-Mission Platform). The Linear Quadratic Regulator (LQR) approach is the control strategy which is adopted. In order to control the attitude of the satellite, we've tested maneuvers as well as stabilization in various nominal attitudes. Some uncertainties have been included in order to analyze if the LQR approach would work in the presence of noise. The projected control law is applied in order to complete the dynamics model, i.e. the non-linear model. In summary, this paper presents a detail study of attitude dynamics and control as well as a virtual environment that may be used for a visualization of the system.*

Keywords: *Satellite Attitude Dynamics, Satellite Attitude Control, Three-Axis Attitude Stabilization, Thrusters, Linear Quadratic Regulator*

1. INTRODUCTION

The attitude of an artificial satellite can be defined as the space orientation of this spacecraft. For the attitude representation, different parameters can be used. One of the most important parameters set are the Euler's angles and the quaternions. The control of the satellite attitude is crucial for the adequate performance of the payload functions, as to say remote sensing, meteorology, communication, and other applications (Wertz, 1978).

The problem of the satellite attitude control has already been studied extensively. Different approaches has been proposed and tested on specific satellite configurations, including the hazardous situation of the control of a satellite on failure mode (one pair of attitude actuators doesn't work properly). For instance, in Show et al. (2002), a robust PID controller is proposed. Nonlinear control approaches using only two control gas-jets are shown in Morin et al. (1995) and in Tsiotras et al. (1995). In Yang and Kung (2000), the satellite attitude is controlled using an applied nonlinear H_∞ controller; while in Wu and Chen (1999) a mixed H_2 / H_∞ approach is investigated.

On this study, we suppose that the problem of attitude determination, i.e. the process of computing the orientation of the spacecraft with respect to a known orbital frame or to some object of interest, is satisfactorily solved. This procedure usually involves several types of sensors in each spacecraft and sophisticated data processing (Wertz, 1978).

In this paper, we consider a specific satellite that has been developed in the context of Brazilian Space Program (AEB, 1998), the PMM satellite (Multi-Mission Platform), and we use some of its physical characteristics and dimension data. We give the study focus to the spacecraft stabilization using as actuators three pairs of gas-jets and we adopt a Linear Quadratic Regulator (LQR) control strategy. The main purpose is to contribute to the researches concerning this satellite, conceived for repeated use in the Brazilian Space Program.

The validation of the theoretical and analytic studies of the satellite attitude control problem was accomplished using numerical simulations of the system dynamics.

First, the problem formulation is presented in Section 2, with the aspects of attitude representation and the kinematic and dynamic models. Besides that, the adopted control strategy is discussed in Section 3. Then, a description of the numerical simulations, as well the presentation and analysis of their results, are shown in Section 4. The final remarks and the perspective of future works conclude the paper (Section 5).

2. SATELLITE KINEMATICS AND DYNAMICS

The satellite attitude will be defined in this work by the position of the three principal axes of inertia with respect an orbital reference frame. Naming R the direction cosine matrix between the orbital frame and the reference frame fastened to the satellite body, and using Euler angles in an asymmetric sequence 3-2-1 for describing a rotation (Kaplan, 1976; Wertz, 1978), one finds:

$$R = R_{321} = R_1(\phi)R_2(\theta)R_3(\psi) \tag{1}$$

$$R_3(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{3}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{4}$$

$$R_{321} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \tag{5}$$

The kinematics equation for the satellite attitude is given by:

$$\dot{R}(t) = S(\omega(t))R(t) \tag{6}$$

where S is the skew-symmetric operator with respect to angular velocity ω.

For the rotation sequence 3-2-1, the satellite attitude kinematics can be described as follows (Wertz, 1978; Wie and Arapostathis, 1989):

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{\cos\theta} \begin{bmatrix} \cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \omega_{ib}^b + \frac{\omega_0}{\cos\theta} \begin{bmatrix} \sin\psi \\ \cos\theta\cos\psi \\ \sin\theta\sin\psi \end{bmatrix} \tag{7}$$

where ω_0 is the medium orbital speed and ω_{ib}^b is the satellite angular velocity vector with respect to an orbital reference frame, expressed in the reference frame fastened to the satellite body.

The linearized kinematic equation is given by:

$$\omega_{ib}^b = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \omega_0 \begin{bmatrix} -\psi \\ -1 \\ \phi \end{bmatrix} \tag{8}$$

The dynamic of a satellite attitude, equipped with six jets, can be modeled using the Euler equations and can be represented starting from the angular momentum rate with respect to the reference frame fastened to the satellite body. Consequently:

$$\tau_{ext} = \left[\frac{dh}{dt}\right]_b + \omega_{ib}^b \times h_b \tag{9}$$

where $h_b = J\omega_{ib}^b$, $\dot{h}_b = \left[\frac{dh}{dt}\right]_b = J\dot{\omega}_{ib}^b$, J is the satellite inertia matrix and τ_{ext} are the external torques that act in the system.

Rewriting the Eq. (9), we have:

$$J\dot{\omega}_{ib}^b + S(\omega_{ib}^b)J\omega_{ib}^b = \tau_d^b + \tau_p^b \tag{10}$$

where τ_d^b represents all the disturbance torques and τ_p^b represents the jet control torques.

The visualization of the considered satellite is given by a graphical interface of the numerical simulations developed in Arantes Jr (2005). The Fig. 1 illustrates the satellite model using this virtual environment.

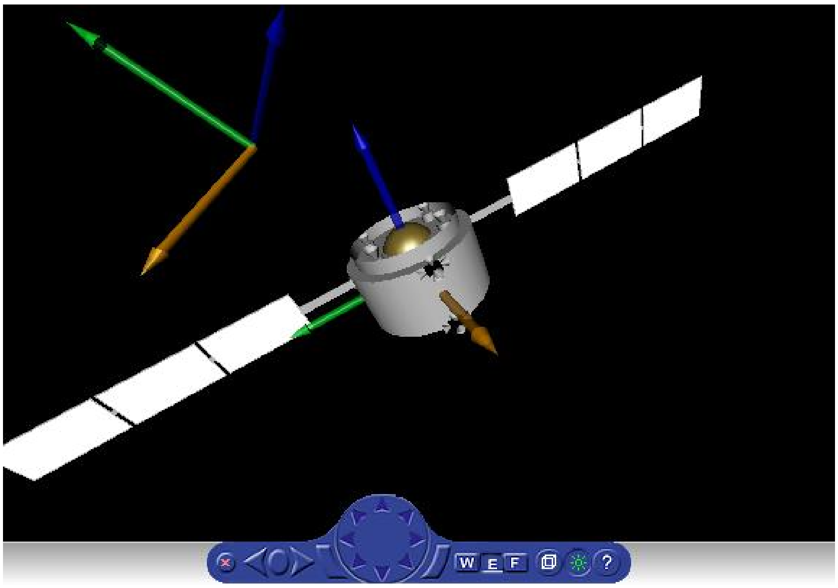

Figure 1. The graphic interface of the simulation tool showing the satellite model.

The jet control torques on the three principal axes of inertia, are given by:

$$\tau_p^b(roll) = du_1 + du_2 \tag{11}$$

$$\tau_p^b(pitch) = du_3 + du_4 \tag{12}$$

$$\tau_p^b(yaw) = du_5 + du_6 \tag{13}$$

where u is the control vector and d is the distance from the mass center to each jet.We consider three pairs of jets, one pair acting on each principal axis of inertia of the satellite body. The value of the jet thrust, considering the model discussed in Salles et al. (2005), is $5N$.

The gravity gradient torque is caused for the difference in the intensity and direction of the gravitational force with that satellite different parts are attract for the Earth (Sene et al., 2006). Due to the fisical characteristic of PMM, the gravity gradient torque is very small and for a final desired atitude, it acts as a "colaborator". For small angle maneuvers, we can use the model of the gravity gradient torque given by (Wie and Arapostathis, 1989; Kaplan, 1976):

$$\tau_g^b = 3\omega_0^2 \begin{bmatrix} (J_z - J_y)\phi \\ (J_x - J_z)\theta \\ 0 \end{bmatrix} \tag{14}$$

Using the linearized kinematics Eq. (8), replacing in the Eq. (10), and including the jet control torques and the gravity gradient torque, we obtain the following dynamic equations:

$$J_x\ddot{\phi} = \phi\left[4\omega_0^2(J_z - J_y) - \omega_0\dot{\theta}(J_z - J_y)\right] + \dot{\theta}\dot{\psi}(J_y - J_z) + \dot{\psi}\omega_0(J_x - J_y + J_z) + du_1 + du_2 + \tau_d^b \tag{15}$$

$$J_y\ddot{\theta} = 3\omega_0^2(J_x - J_z)\theta + \phi\left[\psi\omega_0^2(J_x - J_z) + \dot{\phi}\omega_0(J_z - J_x)\right] + \dot{\psi}\psi\omega_0(J_x - J_z) + \dot{\psi}\dot{\phi}(J_z - J_x) + du_3 + du_4 + \tau_d^b \tag{16}$$

$$J_z\ddot{\psi} = \psi\left[\omega_0^2(J_x - J_y) + \dot{\theta}\omega_0(J_y - J_x)\right] + \dot{\phi}\left[\omega_0(J_y - J_x - J_z) + \dot{\theta}(J_x - J_y)\right] + du_5 + du_6 + \tau_d^b \tag{17}$$

The state equation for this system, i.e. the expression given by:

$$\dot{x} = Ax + Bu \tag{18}$$

is obtained from the Eq. (15), (16) and (17), where the matrices A and B are given by:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{4\omega_0^2(J_z - J_y)}{J_x} & 0 & 0 & 0 & 0 & \frac{\omega_0(J_x - J_y + J_z)}{J_x} \\ 0 & \frac{3\omega_0^2(J_x - J_z)}{J_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\omega_0^2(J_x - J_y)}{J_z} & \frac{\omega_0(J_y - J_x - J_z)}{J_z} & 0 & 0 \end{bmatrix} \tag{19}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{d}{J_x} & \frac{d}{J_x} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{d}{J_y} & \frac{d}{J_y} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{d}{J_z} & \frac{d}{J_z} \end{bmatrix} \tag{20}$$

Finally, the state equation becomes:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{4\omega_0^2(J_z-J_y)}{J_x} & 0 & 0 & 0 & 0 & \frac{\omega_0(J_x-J_y+J_z)}{J_x} \\ 0 & \frac{3\omega_0^2(J_x-J_z)}{J_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\omega_0^2(J_x-J_y)}{J_z} & \frac{\omega_0(J_y-J_x-J_z)}{J_z} & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{d}{J_x} & \frac{d}{J_x} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{d}{J_y} & \frac{d}{J_y} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{d}{J_z} & \frac{d}{J_z} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} \tag{21}$$

3. THE LQR CONTROL APPROACH

The LQR (Linear Quadratic Regulator) method is based on the linearization of the dynamic system, since the methodology is formulated for linear systems. The linearized and time invariant system in Eq. (18) is used in the control design.

The optimization problem consists of finding a linear control law of the type:

$$u = -K_c(t)x \tag{22}$$

that minimizes the quadratic performance index given by:

$$J_p = \int_0^T \left[x^T Q_c x + u^T R_c u\right] dt \tag{23}$$

The matrices Q_c and R_c are defined to ponderate the relative importance of the term concerning the state vector and the control vector, respectively.

For the existence and stability of the LQR problem solution, the necessary and enough condition is that the system is completely controllable (Dorato and Cerone, 1995; Maciejowski, 1989). The controllability analysis made for the Eq. (18) guarantees the LQR problem solution.

The LQR problem solution, that is, the control gain calculation is gotten deciding the equation of Ricatti. As the system described by Eq. (18) is time invariant and considering the optimization interval as infinite, the Ricatti matrix differential equation becomes an algebraic matrix equation:

$$0 = -P_c A - A^T P_c + P_c B R_c^{-1} B^T P_c + Q_c \tag{24}$$

Taking the solution for Ricatti equation, we obtain a control law for the system given by Eq. (18):

$$u = -R_c^{-1} B P_c x \tag{25}$$

where the matrices A and B are given in Eq. (19) and Eq. (20), respectively, and the control gain is defined by:

$$K_c = R_c^{-1} B P_c \tag{26}$$

The Fig. 2 shows the block diagram of the feedback system using the LQR strategy.

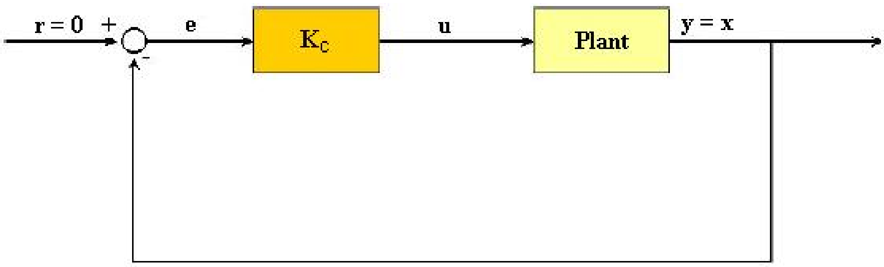

Figure 2. The block diagram of the LQR controller.

4. NUMERICAL SIMULATIONS AND RESULTS

The parameters and specifications considered in this work are shown in Tab. 1. Some of this data corresponds to the satellite PMM (Multi-Mission Platform), developed by the Brazilian Institute for Space Research - INPE (AEB, 1998). This table also shows the initial conditions used in the simulations.

The numerical simulations for the proposed control validation use two different PMM configurations. The principal momentum of inertia, for each configuration, is given in Tab. 2.

We apply the linear control law presented in Section 3, to the nonlinear model for the two configurations of the PMM. We present here some of the obtained results. The time variation of attitude angles on the realized simulation is shown in Fig. 3. The Fig. 4 illustrates the Euler rates for the two configurations of the PMM. The time variation of the control variables that act in the nonlinear system is shown in Fig. 5. The Fig. 6 shows the simulated noise acting in the attitude angles and another one acting in the Euler rates. These figures show the stabilization of the satellite attitude in an appropriate time, i.e. the attitude parameters go to the desired values, including the null angular velocities. The control torques reach their upper limit in the beginning of the maneuver but promptly fall to low values when the attitude moves to the commanded pointing direction.

Table 1. Simulation parameters

Parameters	Values
$d(m)$	0.5
Medium Orbital Speed (rad/s)	$\omega_0 = 0.001$
Mass (Kg)	578.05239
Height (Km)	750
Maximum Torque (Nm)	2.5
Eccentricity	$\cong 0$
Ascending Node Ascension (degrees)	30
Inclination (degrees)	20
Orbital Period	$\cong$ 1h40min
Initial Attitude (degrees)	(ϕ,θ,ψ)=(10,10,10)
Initial Angular Rate (degrees/s)	$\omega_{ib}^b = [1, 1, 1]^T$

Table 2. The values of the principal momentum of inertia.

Configuration	Values (Kgm^2)
1	$J_x = 305.89126$ $J_y = 314.06488$ $J_z = 167.33919$
2	$J_x = 296.16226$ $J_y = 505.52026$ $J_z = 361.12732$

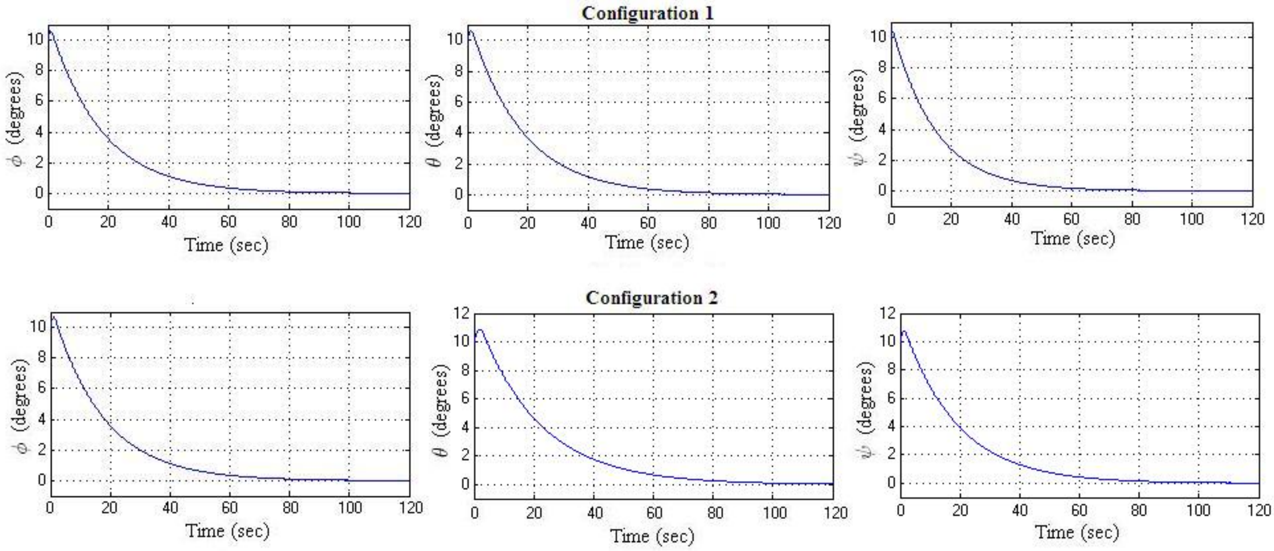

Figure 3. The attitude angles evolution during the simulated maneuver.

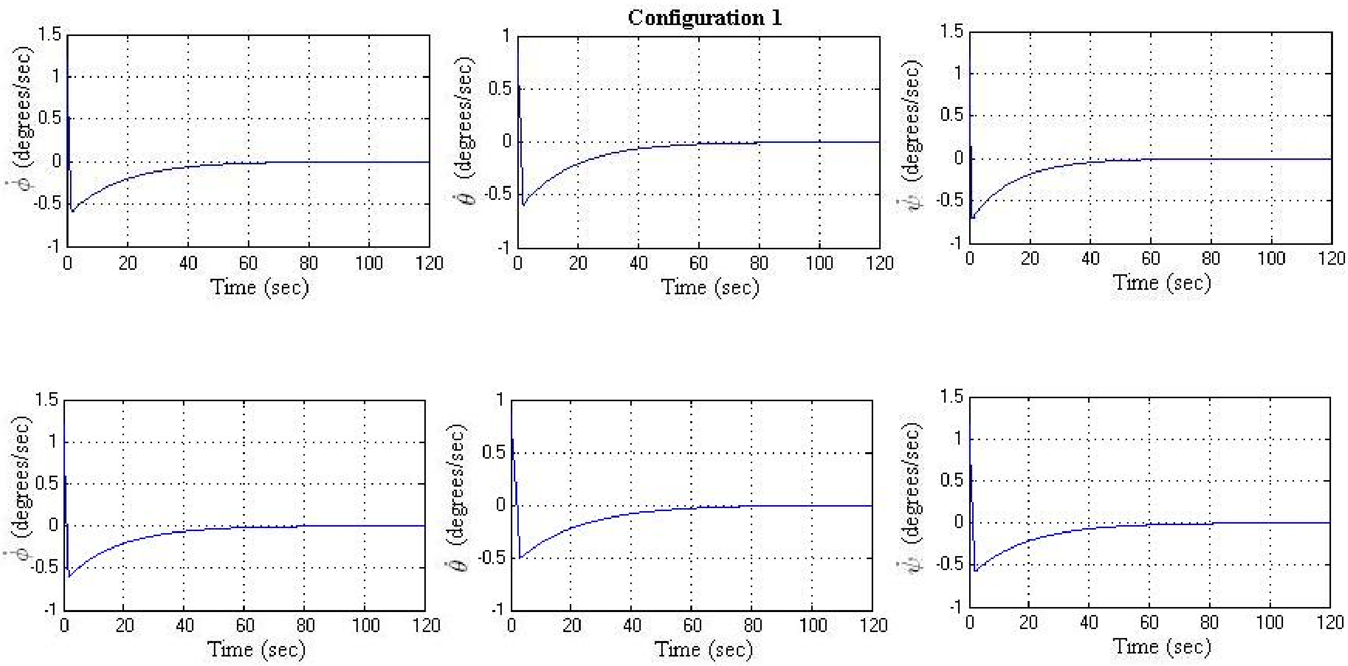

Figure 4. The Euler angle rates during the simulated maneuver.

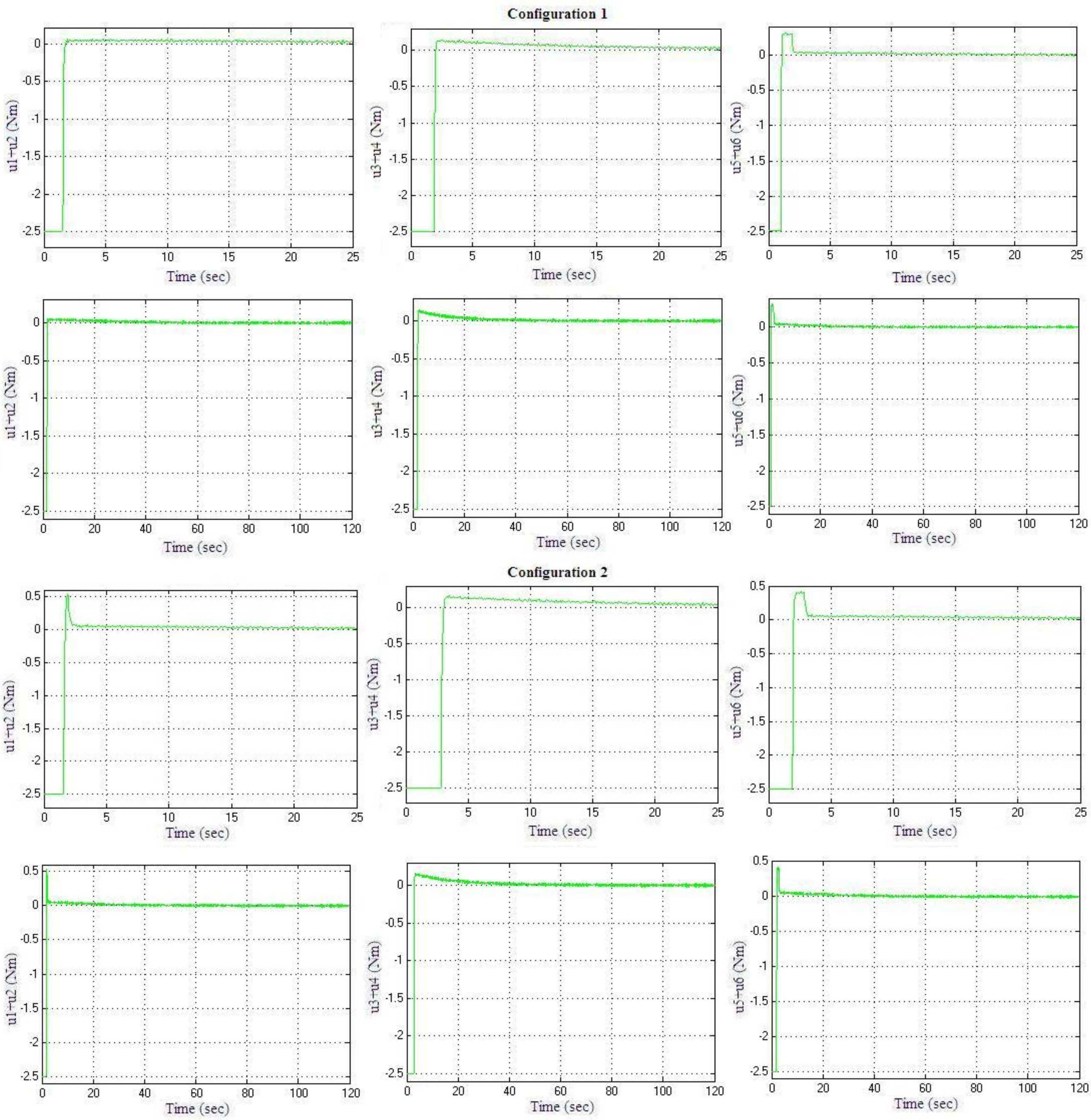

Figure 5. The control variables evolution during the simulated maneuver.

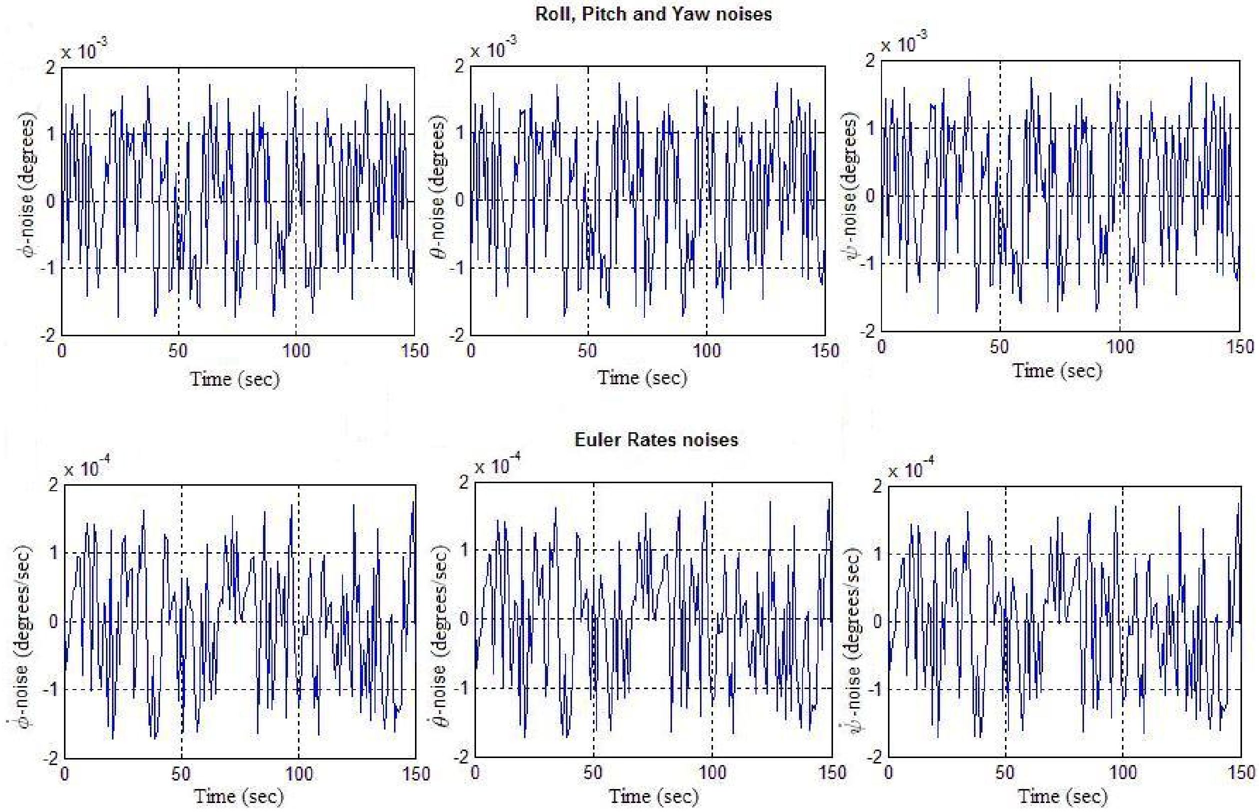

Figure 6. The simulated noise signals disturbing the system.

The obtained results, using some realistic models and physical parameters of the PMM satellite, allow us to conclude positively about the validity of the proposed control strategy based on the Linear Quadratic Regulator Theory. In spite of this approach simplicity applied to a highly nonlinear system, the satellite kinematic and dynamic equations, we have shown that this strategy can be considered to this problem with interesting advantages for the controller implementation.

5. CONCLUSION

The results presented by the numerical simulations lead to the conclusion that the control law based on the LQR control theory stabilizes the system exponentially. This is an interesting result since the problem of satellite attitude stabilization constitutes a nonlinear control system problem and the application of linear approaches represents an attractive issue in terms of experimental and real-time implementation.

This study on satellite attitude control aims to contribute to the PMM technological project, the Brazilian standard satellite conceived to be used on a large number and different types of missions, in the context of an ever-advancing Brazilian space program.

The perspectives of continuation of this study include the attitude control of the satellite on failure mode (i.e. when the satellite can use only two gas-jet pairs), the analysis of the condition of the stabilization for this critical situation, the simulations of the satellite model taking into account other systems perturbations and dynamical effects, and the extension of the LQR approach performance analysis to improve the design of the controller.

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