APPLICATION OF METHOD MULTIPLICATIVE AHP IN THE MACHINE TOOL CHOICE – CASE STUDY

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Abstract. *The different industrial sectors, both domestic and abroad, are looking for solutions to bring up resources in order to improve productive chains, assembly plant and quality. Machine tool update is a way to meet production goals and achieve products with more quality; therefore, what machine to buy or what technology to use is an important decision case. In this context, decision support tools must be used, since such investments demand large amounts of money with high risk rate. In this work, two multi-criteria decision tools are employed: the AHP - Analytic Hierarchy Process and the MAHP - Multiplicative Analytic Hierarchy Process. The results of these methods have been compared for the choice of 5-axis machine tools. For each method, nine important characteristics present in three types of 5-axis machine tools were applied for the best choice to buy this kind of equipment. The importance of the best and correct choice validate the effort and show the increasing necessity of decision tools based on practical and efficient methodologies.*

Keywords*: multi-criteria analysis, AHP, MAHP, 5-axis machine tool.*

1. INTRODUCTION

The daily life is fully of situations in which some type of decision always have to be made. Either the choice of the flavor of an ice cream, a color for clothes or the choice of one movie, all and many other routine examples need some of decision making process. Old civilizations, like classical Greece or ancient Egypt, they were believed that only oracles and kings could find which was the best solution for a given problem Triantaphyllou (2000). However, on the professional life, the decision making process must be based on rigorous postulates, capable to allow a discernment that presents lucrative or optimized advantages. In this way, many theories and methods are inserted to aim professionals of different areas to find the best alternatives.

Inside of the industrial context, many other methodologies of decision support exist, most of them by military contribution however, only with the advance of the computational resources of the last decade, decision support analysis spread and gained practical and functional base, according to Clemen & Reilly (2001). Many industrial engineering applications need the evaluation of a number of alternatives with different criteria. At this point, a multiple criteria decision analysis become necessary to support scientific choice, instead subjective or self-decision balances.

Amongst the methods of multiple criteria decision making, the Analytic Hierarchy Process – AHP is a tool that has been used in almost all the applications related with decision-making, according to Vaidya & Kumar (2006). In this method, the problem must be divided in hierarchic levels and these compared among themselves through a scale of values. This scale classifies numerically the relation between each pair of levels, allowing a numerical result that, when ordered, classifies the importance of the alternatives. The AHP belongs originally to a family of methods of the American School of Decision Analysis, being developed by the professor *Thomas L. Saaty* in 1980. In accordance with Triantaphyllou (2000), the AHP is the most used method because is easy to use and well understood.

The challenge that always occurs when trying to compare multi-criteria decision methods and chose the best one is that a paradox is reached, Triantaphyllou (2000). The AHP method is enough, in the majority of the cases, to find the best solution among a discrete set of alternatives. However, when compared with another multi-criteria decision method and theirs results converges into the same alternative, both improves the accuracy and confiability of the decision, Guglielmetti *et al.* (2003).

The AHP have many variant methods that complete its functionality and applicability. One of these variants is called MAHP - Multiplicative Analytic Hierarchy Process, developed by *Freerk A. Lootsma* in 1990, being the focus of this work.

2. MULTIPLICATIVE AHP

The MAHP is based on the classic AHP, as shown by Triantaphyllou (2001). All the methods derived from the AHP are systematic methods and easy of implementation. An important characteristic of these methods is the quantification of the uncertainty of the results. This procedure allows the measure of data consistency of the priority decision vector.

Commercial packages – software – for AHP and multiple criteria decision analysis, present a friendly graphical interface capable to reduce time and made it easy to build the comparison matrices, according to Guglielmetti *et al.* (2003). However, the decision maker is confronted with new decision problems, since it involves the choice of which commercial package to acquire, purchasing of equipments – hardware - and training. In most of cases, a simple electronic spread sheet, for example MS Excell ®, is enough for resolution however, this software request an intermediary skill to handle the dada entry.

As already mentioned, MAHP is a systematic method that process a sequence of procedures. The work of Triantaphyllou & Mann (1995), presents practical examples of application of the AHP to the solution of real engineering problems, with a step-by-step methodology to assure minimal errors. This same sequence of procedure was presented in the work of Vaidya & Kumar (2006), also showing an extensive review about 150 articles on AHP. The Table 1 presents a procedure for the application of MAHP method, adapted from the work of Vaidya & Kumar (2006) for AHP implementation. The procedures listed in Table 1 could also be applied in classic AHP method anyway, except for stage 7 in which, for the AHP, they consist of the consistency analysis of the values.

Table 1. Sequence to execution of MAHP.

Stage	Execution	Description
1	Problem definition	Is necessary to define what the main objective to reach is. A way to do this is identify decision situations with *value-focused thinking* technique according to Keeney (1994).
2	Definition of the alternatives to problem solution.	The MAHP allow at last nine alternatives. This limit is called of psychology limit once, up to this value, the number of comparing **is** very high.
3	Definitions of criteria and sub-criteria.	The definition of the criteria and sub-criteria of decision form a hierarchy of solutions of the problem.
4	Build the hierarchy decision tree.	This stage is optional, but could be useful to visualize the entire decision problem.
5	Build the alternative matrix and evaluate a pair-wise comparing with a semantic scale.	The semantic scale for MAHP implementation follow the geometric scale proposed by Lootsma (1996), show in column 2 of Table 2. The semantic scale for Classic AHP follow the fundamental scale proposed by Saaty (1977), shown in column 3 of Table 2.
6	Build the criteria matrices and evaluate a pair-wise comparing with a semantic scale.	
7	Normalize and evaluate the weighted comparing with scale parameter.	Each evaluated matrix is turn to a vector. These vectors are used to build a decision matrix.
8	Evaluate the decision matrix to find the priority vector.	The priority vector magnitude shows the best alternative of the decision problem.

The objective (goal) can be defined and structured with the methodology presented in Keeney (1994). This technique improves productivity and provides guidelines to separate the relevant and less important point of the problem at first stage.

Stages 2 and 3 can be best understood through the assembly of the hierarchical structure in stage 4. Figure 1 illustrates a typical example of a decision to buy a good car with three criteria, two sub-criteria and two alternatives. The criteria are price, performance and comfort. The performance criterion requests two sub-criteria; engine power and fuel consumption.

Each alternative and criteria matrices of stage 5 and 6, respectively, follow the semantic judgment of Table 2, also present in Olson (1995). For Multiplicative AHP implementation, this semantic judgment needs to reflect the gradations of decision proposed by Loostma (1996), showed in column 2 of Table 3. For Classic AHP implementation, the numeric value of judgments follows the values of column 3 of Table 3. The semantic relationship between S_i and S_j is an artifice to translate the qualitative preference of a criteria, or alternative, with another, into a numeric value or quantitative measure. This data are personal judgments between criteria and alternatives and only reflect the experience of the decision maker.

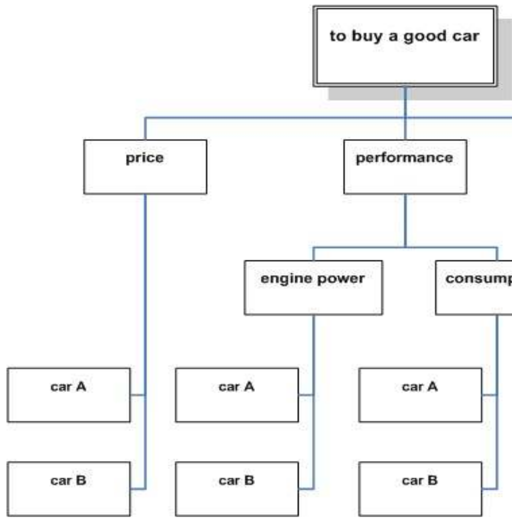


Figure 1. Example of a hierarchical structure

Table 2. Comparative judgment and numerical value for MAHP and Classic AHP.

Semantic relationship	Value (δ_{ij})	
	MAHP	AHP
Very strong preference for S_i versus S_j	-8	1/9
Strong preference for S_i versus S_j	-6	1/7
Definite preference for S_i versus S_j	-4	1/5
Weak preference for S_i versus S_j	-2	1/3
Indifference between S_i and S_j	1	1
Weak preference for S_j versus S_i	+2	3
Definite preference for S_j versus S_i	+4	5
Strong preference for S_j versus S_i	+6	7
Very strong preference for S_j versus S_i	+8	9

If the problem has m alternatives and n criteria, then it is required to construct n judgment matrices (one for each criterion) of order $m \times m$ and one judgment matrix of order $n \times n$ (for the n criteria) Triantaphyllou & Mann (1995). Each criteria and alternatives must be compared between themselves and each alternative must be compared with each criteria ate stage 5. The numbers of comparisons in each comparison matrix are defined in Eq.(1),

$$\frac{n(n-1)}{2} \tag{1}$$

where n is the number of criteria or alternatives in the comparison matrix.

The matrices normalization of stage 7 is performed first by the use of Eq.(2) for MAHP. Each value is transformed into a new numerical value, both in criteria and alternative matrix, as follow;

$$a_{ij} = e^{\gamma\,\delta\,ij} \tag{2}$$

where δ_{ij} is the semantic value of the matrix of comparisons and γ, the appropriated scale parameter. This scale defines the ratio of the geometric scale of column 2 of Table 3. In Loostma (1996), the value suggested is ln(2), that implies in a progression factor about 2. A second normalization divides each value of a cell by the sum of all value in the same column.

From the data of the alternative matrix and the criteria matrix, its is used to build the decision matrix at stage 8. The decision matrix has order *n* x *m* and its data are applied in Eq.(3) for MAHP evaluation. This procedure generates a final score priorities vector that composes the ranking of alternatives,

$$P_i^{MAHP} = \prod_{j=1}^{n} \left(a_{ij}\right)^{W_j} \tag{3}$$

the weights W_j of Eq. (3) are extracting from the criteria matrix through Eq. (4). These weights data, together with the performance values a_{ij}, are used in the last step of the MAHP, according to Triantaphyllou (2001).

$$W_j = \frac{1}{n}\sum_{j=1}^{n} a_{ij} \qquad \text{i = 1, 2,... m} \tag{4}$$

The main step of MAHP is the application of the Weighted Production Model – WPM instead the additive function on the data of the decision matrix in AHP. The WPM approach suggested by Lootsma (1996), compare two alternatives at a time and, in this way, the ranking irregularities cannot occur. The WPM is called dimensionless analysis by Stam & Silva (2003) because its structure eliminates the units of measure. The WPM procedure is shown in Eq. (5).

$$R\left(\frac{A_K}{A_L}\right) = \prod_{j=1}^{n} \left(\frac{a_{Kj}}{a_{Lj}}\right)^{W_j} \tag{5}$$

The value *R* allows the ranking between two alternatives of the decision matrix at time. If the alternative A_K is more preferred than alternative A_L, so $A_K > A_L$ and according to the Eq. (5), $R > 1$. In accordance with Triantaphyllou (2001), the transitivity property of Eq. (5) is always satisfied when tree or more comparison are evaluate. This demonstration leads to the Theorem 1 of MAHP, proved by Triantaphyllou (2001), that guaranteed the transitivity property of comparison matrices and the consistency of the result.

The AHP final approaches are evaluated by Eq. (6). For each comparison matrix, the consistency of data must be checked. According to Triantaphyllou & Mann (1995), one of the most practical issues in the AHP methodology is the slightly non-consistent pairwise comparisons.

$$P_i^{AHP} = \sum_{j=1}^{n} a_{ij} W_i \qquad i = 1, 2,... m \tag{6}$$

The measuring of the consistency of a decision matrix is made by the traditional eigenvector method, proposed and justified by Saaty (1977) and Saaty (2003), for estimating weights in the AHP, but there are other alternative methods to measure the data consistency, according to Peláez & Lamata (2003).

Due to multiplicative nature of MAHP, the dimensions of both criteria and alternative does not influence the priorities vector, as describe above, then the normalization is not necessary. Nevertheless, the normalization process is an obligatory procedure when AHP is used, Loostma (1996). In the AHP, this procedure may lead to inconclusive and erroneous results, because the qualitative data in the decision matrix have many and different dimensions and units (e.g. acceleration, cost, monetary units, mass, time, etc.). According to Saaty (2003), the numerical judgment matrix deals with intangibles due to human inconsistent judgment and, in this way, the validity of AHP result needs validity with the priorities of a decision.

3. FIVE-AXIS MACHINE TOOL

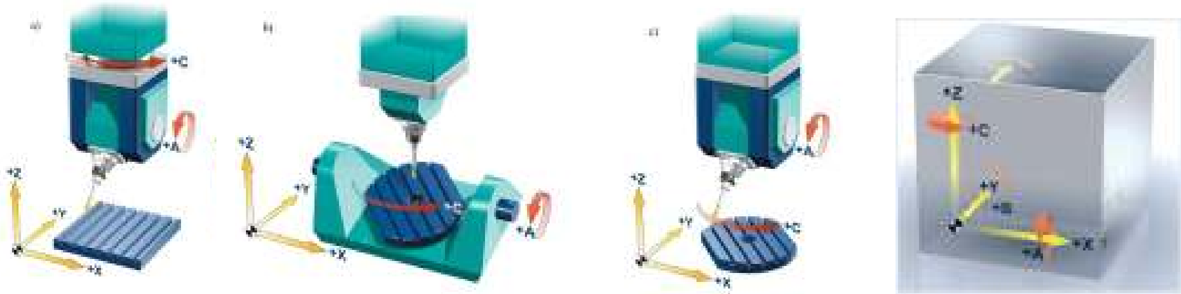
In a competitive industrial environment, each time more companies and service provider need to be in tune with the necessities of global product policy and upmarket. An alternative very attractive to stay ahead with innovations is the use of high-end machines tool with 5-axis and high operation speeds, mainly in the main axle, called the spindle. The machine tool technology, according to Silva (2005), are equipments of high performance, high technology and, consequently, demand voluminous of money. These kinds of investments are classified as high risk since machine tools are job-order product with no return to manufacturer. Milling machine with tree coordinate system are very usual and the 5-axis technology is not recent, however, its use was restricted to a few narrow industrial market and specialized groups such as aerospace and military companies. A 5-axis machine-tool – 5AM with high milling speed, or simply High Speed Milling - SHM, represents a considerable investment by the company, once for all, this machines need a complete modification of production style, administrative manager and engineering philosophy of work according to Silva (2005). The considerable investment to acquisition of 5-axis HSM machine could bring large investment income and investment revenue since the product life cycle becomes best guesstimate, however the decision problem is not to buy one machine, but a set of ten or even hundreds.

The 5-axis machine technology has some basic concepts and important characteristics that must be decided before the procurement. An organizational purchasing need to select the strategy feature of the machine in order to distinguish which machine to buy. This decision is not restricted to the brand, although what set of technology to acquire. Table 3 show a set of features used to select 5-axis machine tools, extracted from internal report of CCM/ITA team.

Table 3. Set of features for 5-axis machine selection.

Item	Description
1	Machine geometry – position of axis and how they moves (cinematic)
2	CNC command – the type of control of tools and axis.
3	Structure stiffness – ability to absorb vibrations and external noise.
4	Axis interpolation – ability to move the tool in line or curve pathway.
5	Accuracy – overall precision of the machine.
6	Spindle torque – rotational strength of motor spindle.
7	Motor acceleration – ability to speed up axis and positioning the tool.
8	Collision protection – automatic system to protect axis to collide.
9	Spindle maintenance – facility to make preventive or repair maintenance.

The first and an important feature of any 5AM is the machine geometry that could provide for machining complex and free-form surface. Usually, 5AM present three orthogonal axles of translation and two perpendicular axles of rotation. The arrangement of the work axis, which can be head rotation type (Figure 2a), table rotation type (Figure 2b) or both (Figure 2c). The orthogonal axles use the notation X, Y, Z, and rotational axles use the notation A, B or C, as show in Figure 2d. The selection of the best alternative is a decision choice of the machine geometry in item 1 of Table 3.



a) **b)** **c)** **d)**

Figure 2. Arrangement of axis for milling machine tool.

The control of the axis is performed by a specialized computer called Computer Numerical Control – CNC. This system integrates the operational control functions of the axis, movement, speed and interface support between the operator and the machine. These possibilities of choice are referring to item 2 of Table 3.

When high speeds of rotation or translation movement are applied to an axis, e.g. the main cutting spindle, the physical structure of the machine could vibrate. A very important characteristic of good quality 5AM is its capability to absorb vibrations, since these oscillations could damage the tools and reduce the quality of the milling part or product. These characteristics of structural stiffness (rigidity) are related to the choice of item 3 of Table 3.

The axis interpolation rule is the ability to move the tool in a complex workpiece contours and features. The choice of a good axis interpolator computer allow finished workpieces in just a single machine cycle and is related with the decision item 4 of Table 3.

Companies, which make products and/or parts of high precision, need machines with high or ultra high precision as well, Silva (2005). The choice of machine precision is the option of item 5 of Table 3.

The torque of the spindle is related with the type of material to be milling in the machine. If the intention is to work with titanium alloys, stainless steel or special alloy steel, the spindle torque is an important decision criterion. Other alloys or easily malleable material like aluminum, cooper or iron, the torque spindle is not critical. These features reflex in acquisition cost and could represent important criteria to be analyzed in item 6 of Table 3.

The motor acceleration constitutes a characteristic related with the delayed time for tool positioning or spindle harm-up. Mass production machine needs special improvements in speed in order to promote high net reproduction rate.

A 5AM machine has a significant patrimonial value and cannot become damaged in case of a collision between its axis or parts. In this way, the presence of mechanisms or technologies for axis preservation constitutes an important criterion of selection, as item 8 of Table 3.

Any industrial machine presents a limited life time or useful life. The main spindle of a 5AM working at high speed (usually 15 000 rpm) and its maintenance presents a considerable operational cost in relation to other parts of the machine. Any technological advantages that also improve the spindle maintenance have important decision criteria for 5AM choice.

4. MULTIPLE CRITERIA ANALYSIS

This work presents a best choice of tree brand of 5 axis machine tool with high spindle speed. Two multi-criteria decision-making are used in this choice; a Classic AHP and the Multiplicative AHP. The Figure 3 shows a decision tree with the nine criteria of Table 3 as follows.

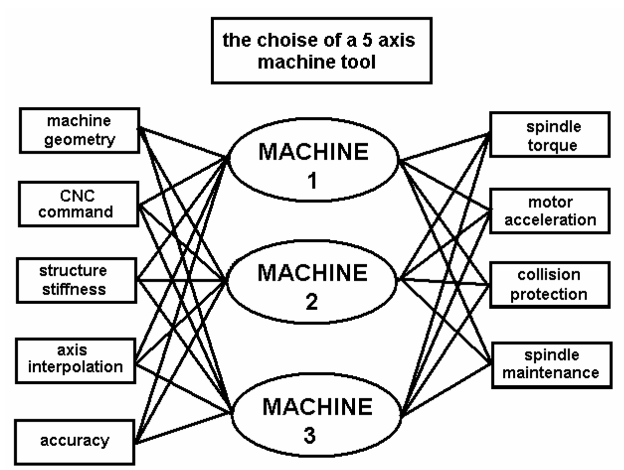


Figure 3. Decision tree to choice a 5axles machine tool.

The pair wise comparison matrixes of the nine selected criteria are showed at Table 4, both for AHP and MAHP method. Each semantic comparison is adjusted by internal reports of CCM/ITA team expertise. The judgment values for pair wise comparison are extracted from semantic relationship present at Table 2. The matrix normalization and the criteria weight vector of AHP method is show in Table 5. For MAHP method, the normalization and the criteria weight vector are obtained from Eq.(2) and Eq.(4), in conformity with the description of section 2, shown in Table 6.

Table 4. Pair wise comparison matrix of criteria by AHP and MAHP semantic judgment.

Criteria	AHP									MAHP								
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
Machine geometry 1	1	1/7	1	3	1/9	3	3	1	1/9	0	-6	0	2	-8	2	2	0	-8
CNC command 2	7	1	5	3	1/3	5	3	3	1/3	6	0	4	2	-2	4	2	2	-2
Structure stiffness 3	1	1/5	1	1/3	1/5	1/3	1/3	1	1/5	0	-4	0	-2	-4	-2	-2	0	-4
Axis interpolation 4	1/3	1/3	3	1	1/7	3	3	1/3	1/5	-2	-2	2	0	-6	2	2	-2	-4
Accuracy 5	9	3	5	7	1	9	5	5	3	8	2	4	6	0	8	4	4	2
Spindle torque 6	1/3	1/5	3	1/3	1/9	1	1	1/7	1/7	-2	-4	2	-2	-8	0	0	-6	-6
Motor acceleration 7	1/3	1/3	3	1/3	1/5	1	1	1/3	1/7	-2	-2	2	-2	-4	0	0	-2	-6
Collision protection 8	1	1/3	1	3	1/5	7	3	1	1/3	0	-2	0	2	-4	6	2	0	-2
Spindle maintenance 9	9	3	5	5	1/3	7	7	3	1	8	2	4	4	-2	6	6	2	0

Table 5. Normalized AHP and MAHP criteria matrix and criteria weight vector.

	AHP										MAHP									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**criteria weight**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**criteria weight**
1	,034	,017	,037	,130	,042	,083	,114	,068	,020	0,061	,002	,002	,016	,042	,002	,009	,040	,036	,001	0,016
2	,241	,117	,185	,130	,127	,138	,114	,203	,061	0,146	,109	,100	,255	,042	,145	,038	,040	,145	,043	0,102
3	,034	,023	,037	,014	,076	,009	,013	,068	,037	0,035	,002	,006	,016	,003	,036	,001	,002	,036	,011	0,012
4	,011	,039	,111	,043	,054	,083	,114	023	,037	0,057	,000	,025	,063	,010	,009	,009	,040	009	,011	0,020
5	,310	,351	,185	,304	,380	,248	,190	,338	,549	0,317	,442	,406	,255	,687	,589	,628	,162	,586	,711	0,496
6	,011	,023	,111	,014	,042	,028	,038	,010	,026	0,034	,001	,006	,063	,003	,002	,002	,010	,001	,003	0,010
7	,011	,039	,111	,014	,076	,028	,038	,023	,026	0,041	,000	,025	,063	,003	,036	,002	,010	,009	,003	0,017
8	,034	,039	,037	,130	,076	,193	,114	,068	,061	0,084	,002	,025	,016	,042	,036	,155	,040	,036	,043	0,044
9	,310	,351	,185	,217	,127	,193	,266	,203	,183	0,226	,442	,406	,255	,169	,145	,155	,656	,145	,175	0,283

The Table 6 shows the alternatives judgment matrix, by AHP method, for each decision criteria. At this side, the normalized comparing matrix and the resulted alternative weight vector.

Table 6. Pair wise comparison matrix of alternatives by AHP, the normalized matrix and the alternative weight vector.

		AHP			**Normalized value**			**Priority vectors**
		M1	**M2**	**M3**	**M1**	**M2**	**M3**	
C1 – Machine Geometry	**M1**	1	9	5	0,76	0,60	0,81	0,723
	M2	1/9	1	1/5	0,08	0,07	0,03	0,061
	M3	1/5	5	1	0,15	0,33	0,16	0,216
C2 – CNC Command	**M1**	1	9	5	0,76	0,60	0,81	0,723
	M2	1/9	1	1/5	0,08	0,07	0,03	0,061
	M3	1/5	5	1	0,15	0,33	0,16	0,216
C3 – Structure Stiffness	**M1**	1	7	5	0,74	0,64	0,79	0,724
	M2	1/7	1	1/3	0,11	0,09	0,05	0,083
	M3	1/5	3	1	0,15	0,27	0,16	0,193
C4 – Axis Interpolation	**M1**	1	5	3	0,65	0,45	0,71	0,607
	M2	1/5	1	1/5	0,13	0,09	0,05	0,090
	M3	1/3	5	1	0,22	0,45	0,24	0,303
C5 – Accuracy	**M1**	1	1	3	0,43	0,33	0,60	0,454
	M2	1	1	1	0,43	0,33	0,20	0,321
	M3	1/3	1	1	0,14	0,33	0,20	0,225
C6 – Spindle Torque	**M1**	1	3	7	0,68	0,71	0,54	0,643
	M2	1/3	1	5	0,23	0,24	0,38	0,283
	M3	1/7	1/5	1	0,10	0,05	0,08	0,074
C7 – Motor Acceleration	**M1**	1	3	1	0,43	0,60	0,33	0,454
	M2	1/3	1	1	0,14	0,20	0,33	0,225
	M3	1	1	1	0,43	0,20	0,33	0,321
C8 – Collision Protection	**M1**	1	1	3	0,43	0,45	0,33	0,405
	M2	1	1	5	0,43	0,45	0,56	0,480
	M3	1/3	1/5	1	0,14	0,09	0,11	0,115
C9 – Spindle Maintenance	**M1**	1	9	3	0,69	0,53	0,72	0,649
	M2	1/9	1	1/7	0,08	0,06	0,03	0,057
	M3	1/3	7	1	0,23	0,41	0,24	0,295

Table 7 shows the alternative judgment matrix, by MAHP method for each decision criteria. At this side, the normalized comparing matrix and the resulted alternative weight vector.

Table 7. Pair wise comparison matrix of alternatives by MAHP, the normalized matrix and the alternative weight vector.

		MAHP			Normalized value			Priority vectors
		M1	M2	M3	M1	M2	M3	
C1 – Machine Geometry	**M1**	0	8	4	0,939	0,939	0,939	0,939
	M2	-8	0	-4	0,003	0,003	0,003	0,003
	M3	-4	4	0	0,057	0,057	0,057	0,057
C2 – CNC Command	**M1**	0	8	4	0,939	0,939	0,939	0,939
	M2	-8	0	-4	0,003	0,003	0,003	0,003
	M3	-4	4	0	0,057	0,057	0,057	0,057
C3 – Structure Stiffness	**M1**	0	6	4	0,930	0,930	0,930	0,930
	M2	-6	0	-2	0,014	0,014	0,014	0,014
	M3	-4	2	0	0,057	0,057	0,057	0,057
C4 – Axis Interpolation	**M1**	0	4	2	0,765	0,485	0,793	0,681
	M2	-4	0	-4	0,047	0,030	0,012	0,029
	M3	-2	4	0	0,189	0,485	0,195	0,290
C5 – Accuracy	**M1**	0	0	2	0,445	0,333	0,670	0,483
	M2	0	0	0	0,445	0,333	0,165	0,315
	M3	-2	0	0	0,110	0,333	0,165	0,203
C6 – Spindle Torque	**M1**	0	2	6	0,793	0,793	0,793	0,793
	M2	-2	0	4	0,195	0,195	0,195	0,195
	M3	-6	-4	0	0,012	0,012	0,012	0,012
C7 – Motor Acceleration	**M1**	0	2	0	0,445	0,670	0,333	0,483
	M2	-2	0	0	0,110	0,165	0,333	0,203
	M3	0	0	0	0,445	0,165	0,333	0,315
C8 – Collision Protection	**M1**	0	0	2	0,445	0,485	0,189	0,373
	M2	0	0	4	0,445	0,485	0,765	0,565
	M3	-2	-4	0	0,110	0,030	0,047	0,062
C9 – Spindle Maintenance	**M1**	0	8	2	0,800	0,800	0,800	0,800
	M2	-8	0	-6	0,003	0,003	0,003	0,003
	M3	-2	6	0	0,197	0,197	0,197	0,197

Table 8 composes the decision matrix of AHP with the criteria weight from Table 5 and alternative weight from Table 6. The same procedures are applied to build the decision matrix of MAHP. The criteria weight from Table 5 and the alternative weight from Table 7 are shown in Table 9.

Table 8. Decision matrix of AHP.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternative	0,061	0,146	0,035	0,057	0,317	0,034	0,041	0,084	0,226
M1	0,723	0,723	0,724	0,607	0,454	0,643	0,454	0,405	0,649
M2	0,061	0,061	0,083	0,090	0,321	0,283	0,225	0,480	0,057
M3	0,216	0,216	0,193	0,303	0,225	0,074	0,321	0,115	0,295

Table 9. Decision matrix of MAHP.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9
Alternative	0,016	0,102	0,012	0,020	0,496	0,010	0,017	0,044	0,283
M1	0,939	0,939	0,930	0,681	0,483	0,793	0,483	0,373	0,800
M2	0,003	0,003	0,014	0,029	0,315	0,195	0,203	0,565	0,003
M3	0,057	0,057	0,057	0,290	0,203	0,012	0,315	0,062	0,197

5. AHP AND MAHP ANALYSIS

The data of the decision matrix of Table 8 was applied in Eq. (6) for AHP evaluation. Both of the resulted final priorities are show in Table 10 for AHP and MAHP. The overall consistency ratio for all AHP comparing matrix was less then 10% by the use of traditional eigenvector method, not shown.

Table 10. Priority ranking of AHP and MAHP result.

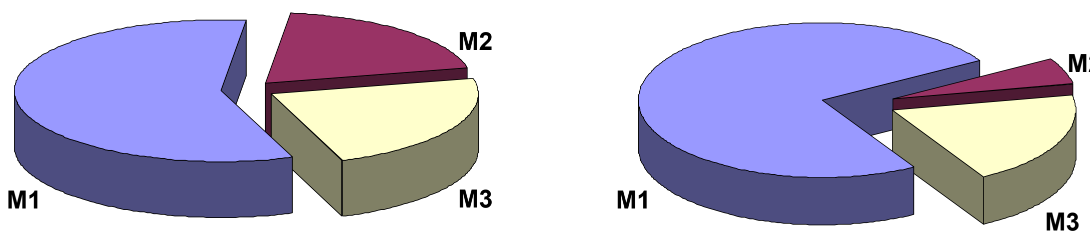
Analysis Results	AHP	MAHP
Machine 1	0,574	0,608
Machine 2	0,194	0,046
Machine 3	0,232	0,160

MAHP allows alternatives comparing two at a time with the WPM, show in Eq. (5). This ranking play a relative performance analysis of the resulted final priorities, showing how much is the alternative A_K more preferred than alternative A_L. Table 11 show the result of the application of WPM in the data of Table 9.

Table 11. Relative priority ranking of MAHP result.

$R(M1/M2)$	13,2631
$R(M1/M3)$	3,8073
$R(M2/M3)$	0,2871

The data of Table 10 was normalized and plotted in Figure 4. The AHP priority ranking is showing in Figure 4a and the MAHP in Figure 4b.



a) AHP b) MAHP

Figure 4. Decision diagrams of AHP and MAHP

6. CONCLUSIONS

The AHP is an efficient method of decision analysis according to Peláez & Lamata (2003), Triantaphyllou (2001), Ramanathan (1997) and Vaidya & Kumar (2006). The MAHP follows the same line with extra features with no ranking irregularities, no ranking reversed and perfectly data consistency. Both methods present commercial package eg. Expert Choise and Rembrandt System, used in the work of Triantaphyllo & Mann (1995) and Olson *at al.* (1993), respectively. This software aid peoples to make decisions with a lot of features like graphical and friendly interface, automatic report generator, on line help and etc., but most of the decision making method could be evaluated in a simple spreadsheet, like done in this work with MS Excel®.

The problem of choosing what type of 5-axis machine tools to buy was compared with the classic AHP method and its multiplicative version. Both results have converged to the choice of Machine 1, as shown in Table 10. The choice of Machine 1 is about 13 times more preferred than Machine 2 and about 4 times more desired than Machine 3, as seen in Table 11. Only the MAHP with WPM evaluation allow the estimation of how much such decision is better than the other. The pizza diagrams of Figure 4 perform a good visualization way of the data of Table 10, exposing the decision magnitude in a fast graphical understanding.

One still does not know why some complex decision making problems evaluated by different methods present different solutions Guglielmetti *et al.* (2003). According to Clemen (1996), each multi-criteria method presents specific advantages and disadvantages and the AHP priority values should not be taken literally. More careful and extensive analysis must be investigated when complex or critical decisions are requested. Triantaphyllou & Mann (1995) reported the occurrence of a decision making paradox when different methods present different solutions, however the positive indication of best decision of two or more decision support method suggests the truly best solution.

The AHP and MAHP present a strict commitment, and both provide a convenient approach for solving valuable problems in many scientific and engineering applications.

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