FINITE ELEMENT MODEL UPDATING OF A ROTOR-GENERATOR UNIT

A. Eliké H. Kloutsey, hermione_kloutsey@hotmail.com
Alberto C.G.C. Diniz, adiniz@unb.br
Cristiano Viana Serra Villa, cvsv@unb.br
Universidade de Brasília – UnB
Departamento de Engenharia Mecânica
Gleba A – Faculdade de Tecnologia
70910-900 Brasília – DF – Brazil

Abstract. *The correction of numerical models using experimental results is a very important current field of research and development for the industries in the area of Mechanical Engineering. Indeed numerical predictions and test results are often called into question when they are not coherent. Strategic sectors of the national economy, as the generation of electricity, demand predictive models of its systems for more trustworthy equipments. The quality of these models can be improved or validated comparing numerical simulations with experimental results. In modern structural design, the Finite Element Method has been established as the universally accepted method of analysis and there are several techniques for Finite Element model updating by processing records of experimental dynamic response from test structures. In structural dynamics, these techniques involve modal analysis.*
In this paper, the theory of Finite Element model updating is treated and one technique is applied to the case of a rotor-generator test rig. Modal analysis tests were carried and the modal parameters identified for application of the updating technique. The results obtained, as well as a description of the methodologies used are presented together with conclusions.

Keywords*: Finite Element models, turbo machinery model updating, hydrogenerator, identification methods*

1. INTRODUCTION

The Finite Element (FE) method seems to be the most efficient method for numerical modeling in engineering design. But the results obtained still need to be refined so that the quality of the predicted data relates well to the prototype model. An approach for this is model updating, which corrects invalid and uncertain assumptions by processing vibration test results. This is especially true in design, construction and maintenance of mechanical systems and civil structures. Indeed, modeling implies assumptions, simplifications, approximations which, although necessary, move the model away from the prototype. In the correction of the model, the modeling parameters are adjusted in order to improve the results of the numerical modal analysis by comparing them with those of experimental modal analysis. As the FE method is based generally on the properties of the material (Young's modulus, Poisson'ratio, mass density...) and the physical dimensions of the structure to be modeled, one chooses the parameters to be corrected among the last ones to guarantee that a variation of the parameters leads also to a variation of the numerical modal data. Thus, assuming the experimental measurement data as exact, the modeling parameters are updated by minimizing an object function between model and experimental data.

Friswell & Mottershread (1995) classified several existing updating methods as direct, iterative and methods using frequency data.

Direct methods have the advantage that they replicate exactly the test data using Lagrange multipliers. We have two possibilities for direct methods. In the first case, the measured eigenvectors are corrected by forcing orthogonality with respect to the numerical mass matrix (in this case considered exact). Then, using the updated eigenvector and the numerical mass matrix in the equation of motion, the stiffness matrix is corrected. Another way to use direct updating methods is using the measured data as a reference. The mass matrix is corrected to verify the orthogonality constraint and the stiffness matrix is corrected to verify the symmetry constraint.

In the direct methods, it is necessary, for the calculations to adjust the size of the measured and numerical vectors. This leads to the introduction of spurious modes in the frequency range of the test. Another difficulty is that, it does not provide the opportunity for the user to select updating parameters. This leads to the loss of the initial physical meaning of the FE model.

Iterative methods on the other hand, allow for a wide choice of the parameters to be updated. In these methods, the correlation between the measured data and the analytical data is improved by minimizing a penalty function involving the mode shape and eigenvalue data. The penalty function is generally the sum of the squares of the difference between the measured and the estimated eigenvalues. It is a non-linear function of the parameter, and so, in the iterative procedure, the analytical model is evaluated at each iteration. The updated parameter is the one which leads to the minimum of the penalty function. When the Frequency Response Functions (FRFs) are used, the damping must be included in the FE model to obtain a good consistency between measured and predicted FRFs. Various updating techniques using FRF data were developed (Imregun et al, 1994; Lin and Zhu, 2006). With modal data, it is possible to use undamped models because the measured natural frequencies and damping ratios may be separated. Steenackers and Guillaume (2006) extended and adapted this method in order to take into account the uncertainty of the estimated modal parameters. Considering that the assumption of 100% accurate experimental data results is not true, they propose a FE updating process that takes into account the standard deviation of the measurements. This is used as a weighting factor in the penalty function. Modak, Kundra and Nakra (2002) made a comparative study of these methods in order to choose the best for a particular application.

This paper describes the FE model of a rotor-generator used in an experimental test rig which simulates a hydropower generator unit. An iterative updating procedure is used to obtain a good modeling of the generator winding parameters. A modal test with an impact hammer has been performed, and the resonance frequencies and modal shape estimated. The mass and moment of inertia, which are the updating parameters, are evaluated to improve the model data relating to the measured one.

2. UPDATING USING EXPERIMENTAL RESULTS

FE model updating using experimental results involves the identification of the structural parameters and the adjustment of the model using global or local techniques. Currently different techniques have been used for the validation and correction of FE models through experimental results (Friswel & Mottershead, 1995). In this section, a brief review about identification and adjustment of models and the state of the art of the correction methods using experimental results were made.

2.1 Identification

In modal analysis, the identification of the structural parameters can be made in the time domain, through the equations of motion, or in the frequency domain, through the Frequency Response Functions (FRFs) at different points of the structure. In the two cases one looks for the determination of the modal parameters: modal frequency, modal damping and modal shape. The identification is a numerical process of adjustment of experimental data to one or several functions coming from physical models. The result of this process is a set of coefficients or parameters of the functions used to represent the structure analytically. The identification methods are generally classified in two groups: Single Degree-Of-Freedom (SDOF) methods and Multiple Degree-Of-Freedom (MDOF) methods (Ewins, 1984). Some identification methods have been developed and validated, with very small errors of approximation. R. J. Allemang (2002) presents basic methods of identification in the time domain and the frequency domain. For the SDOF method, the author presents the methods of the amplitude peak and the least squares method (curve fitting); for the MDOF methods, some algorithms exist as Least Squares Complex Exponential, the method Ibrahim Time Domain and Rational Fraction Polynomial. New techniques have been applied to facilitate the use of data in the time domain, as Alvin, Robertson, Reich, Park (2003) who use wavelet expansion for structural identification from the Impulse Response Functions (IRF).

2.2. Model updating

The predictions of the numerical or analytical calculations are frequently questioned when they are not in accordance with the experimental results. The model updating purpose is to correct the numerical model using test model as reference. Friswell and Mottershead (1993) consider three types of errors in the modeling:

- Errors in modeling of the structure;
- Errors in the modeling of the parameters, what include the application of unsuitable boundary conditions and simplified hypotheses;
- Errors in the choice of the discretization order for complex systems.

To compare the results of the calculations with the experimental ones, it is necessary to define a criterion. In structural dynamics, among the criteria of comparison of modal data most used we have:

- the modal effective parameters
- the distributions of the modal energies
- the modal deformed frequencies and
- the MAC, modal assurance criterion

The effectiveness of the criterion depends on the quantity and the quality of the available experimental data. For example, in the determination of the modal shapes, it is necessary that the number of measurement points is sufficient to characterize the complexity of the structure and the number of modes to be measured.

3. MODEL UPDATING OF A ROTOR-GENERATOR GROUP

In order to study the dynamics of a turbine-generator unit, used in hydroelectric plants, the Laboratory of Vibrations and Structural Dynamics at UnB constructed an experimental test rig, reproducing, in a reduced scale model, a real hydrogenerator used by ELETRONORTE in the Coaracy Nunes hydroelectric. This test rig allows the simulation of common phenomena taking place in a real turbine-generator unit.

The test rig (figure 1) is a vertical rotor unit composed of a three-phase synchronous generator, with 6 salient poles independently excited. The real turbine used in hydroelectric plants is simulated by a rotor with dynamical properties similar to the full scale prototype. The unit is powered by an electrical motor and supported by two roller bearings (figure 2).

Figure 1: Rotor-Generator Test Rig.

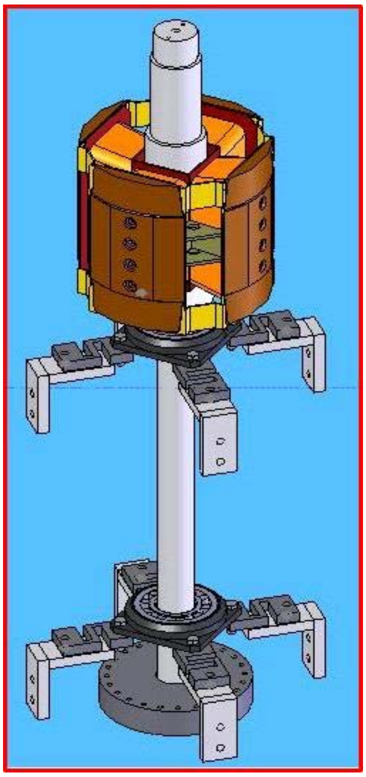

Figure 2: Internal view

Figure 3: Rotor-Generator Unit

A FE model of the rotor-generator unit of the test rig (Fig. 3) was built, but the modeling of the generator winding was only tentative. A correction of FE model using experimental modal data was performed to have a better physical representation of this part.

An iterative method using modal data was used to update the mass M and the moment of inertia I (in directions perpendicular to the rotor axis) of the generator winding. Global mass and stiffness matrices of the rotor-generator unit are sensitive to these two parameters and these can be considered good parameters for updating the rotor-generator model.

3.1. Finite Element Model of the Rotor-Generator

This section describes the FE model of the rotor-generator unit developed in Matlab, using the method proposed by Gmür (1997).

The rotor-generator used in the test rig was built recycling an old motor-generator group. Therefore the dimensions of the generator winding and the rotor shaft are unknown. So, the rotor shaft geometry showed in Fig. 4 was admitted. All uncertainties in FE model are also assumed to be localized in the generator winding region. This assumption was necessary to start the updating process.

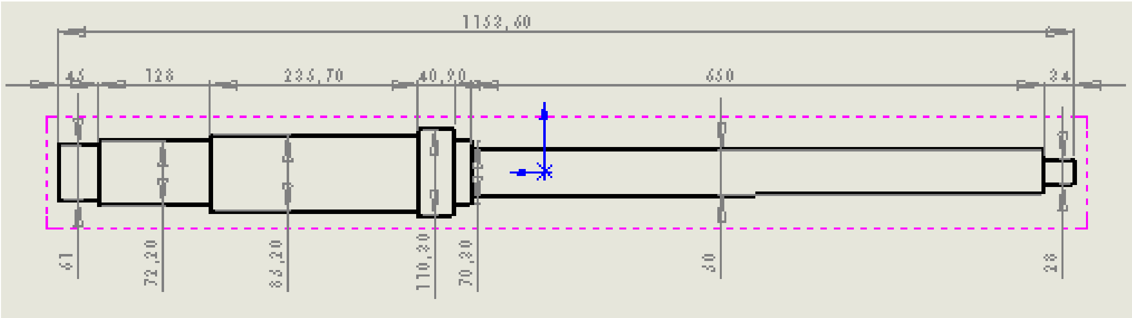

Figure 4: Dimensions of rotor shaft (without the generator).

The rotor is defined as a cylindrical beam with 7 sections (Figure 4) and the generator is modeled as a concentrated mass placed at a node in the third section of the shaft (from the left end).

The linear equation of motion for the "i-th" mode for the rotor-generator unit is:

$$\left([K]_{r-g} - \omega_i^2 [M]_{r-g}\right) X_i = 0 \tag{1}$$

where $[K]_{r-g}$ is the global stiffness matrix, $[M]_{r-g}$ is the global mass matrix, and ω_i^2 and X_i are the eigenvalue and the eigenvector, respectively, for the "i-th" rotor-generator unit mode. The global matrices are obtained assembling the elementary matrices of the generator and the characteristic matrices of the shaft rotor.

In the model, the rotor shaft is considered symmetric without taking into account the uncertainties in the mass distribution. In fact, the experimental test rig has a pole more salient than the others. This hypothesis allows only two DOFs per node.

The shaft rotor elementary matrices are defined using a Bernoulli beam element as:

$$[M]_S = \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \qquad [K]_S = \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \tag{2}$$

The generator winding is a concentrated mass, defined by its mass M and its moment of inertia I, and so it does not influence the global stiffness. The elementary generator matrices are:

$$[M]_G = \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \qquad [K]_G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

A first approximation for the generator mass was obtained by the difference between the rotor-generator unit mass and the mass of the rotor shaft calculated by using the FE routine in Matlab. The rotor-generator unit weighs 88.6 kg, the calculated mass of rotor shaft for a density of 7800 kg/m3 is 29.38 kg, and so the estimated mass of the generator winding is 59.22 kg.

The influence of mesh size on the convergence of results was evaluated and the obtained results are shown in the table. 1.

Table 1. Mesh influence on the rotor shaft without the generator winding.

Frequencies [Hz]	Number of elements used in rotor shaft model				
	18	44	86	134	266
Mode 1	181.9964	181.2885	181.2869	181.2869	181.2867
Mode 2	798.9843	593.5714	593.5286	593.5261	593.5257

The finite element number can be limited to 134 for the shaft model without loss of quality, because beyond this value the natural frequencies have no noticeable change.

Another important parameter for construction of FE model is the position where the generator properties are concentrated. To evaluate the best position, five different positions in the third stage of the shaft (from the left end) were considered.

The results are shown in Table 2 and compared with experimental ones (see Table 3). Using the table 2 results, the generator winding mass and moment of inertia were concentrated at 251.5 mm from the shaft left end.

Table 2: Evaluation of the best position to concentrate the generator on the rotor shaft

Position of the generator on the rotor shaft [m]:	**0.2123**	**0.2515**	**0.29085**	**0.3301**	**0.3694**
1^{st} frequency [Hz]	139.1017	139.7055	138.7518	136.1642	132.1415
2^{nd} frequency [Hz]	475.4922	488.5078	499.5005	508.0450	513.6952

After the modeling process, we present, in the next section, the updating method used in this paper.

3.2. Updating Methodology

In this section, the iterative updating method used to update the generator mass M and moment of inertia I (in directions perpendicular to the rotor axis) is presented. These updating parameters can be considered acceptable for the rotor-generator model updating, because they are physical parameters describing the generator and they can be easily included in the global matrices of the rotor-generator unit. Global mass and stiffness matrices of the rotor-generator unit are sensitive to these two parameters.

The model updating method using modal data is based on minimizing an objective function representing an error between the measured and the analytical results (Friswell & Mottershread, 1995). This objective function is minimized using linear optimization.

In this work, it is proposed to update a FE model using two criteria, the frequencies and the modal shapes.

In the first case, the updated parameters (mass and moment of inertia) are the ones that minimize the difference between the experimental frequencies and those obtained by the FE model. In this case, the optimization problem was formulated as:

$$p_f = \min\left(\frac{f_{meas} - f_{num}}{f_{meas}}\right) \tag{4}$$

where f_{meas} and f_{num} are the measured and calculated natural frequencies corresponding to vibration modes and p_f is the parameter associated to the minimum of the relative difference between numerical and measured frequency.

For the optimization using the modal shapes, the Modal Assurance Criterion (MAC) was used to establish the best correspondence between measured and analytical modes. The MAC is a matrix of correlation coefficients between two eigenvectors, defined by:

$$MAC(p,x) = \frac{\left|\sum_{i=1}^{n} \psi_{xi}\psi_{pi}^{*}\right|^2}{\left(\sum_{i=1}^{n} \psi_{xi}\psi_{xi}^{*}\right)\left(\sum_{i=1}^{n} \psi_{pi}\psi_{pi}^{*}\right)} \tag{5}$$

where: ψ_{xi} and ψ_{pi} are the experimental and calculated eigenvectors (* indicates the complex conjugate).

A MAC coefficient is a scalar (even if the eigenvectors are complex). When the MAC value is close to 1.0, this indicates a good proximity between the two modes of vibration. On the other hand, if the value is close to zero, it can be concluded that the modes of vibrations are different.

In this case, we search the mass and moment of inertia that minimize the optimization problem formulated as:

$$p_m = \min(diagonal(MAC) - I) \tag{6}$$

where *I* is the identity matrix.

To summarize, the updating algorithm consists in four steps:

(1) Modal parameter identification

The real modes were estimated from the measured complexes modes. The natural frequencies and damping ratio were estimated from the experimental FRF data. The measured resonance frequencies were estimated by a least square routine applying a SDOF method. For solving the least squares problem, it is necessary to specify a starting value for the natural frequencies. The identifying algorithm stops when the difference betwen measured and estimated eigenfrequencies becomes smaller than a user-defined threshold.

(2) Interpolation of numerical eigenvectors
The DOF number of FE model was reduced to that of the experimental data. Linear interpolation was performed to obtain the same localization for the experimental and FE nodes. The mass distribution of the FE model and the real structure may be different, and so to match the mode shapes, a modal scaling factor was used.

(3) Correlation with the MAC
To verify the correlation between experimental and numerical modes we calculated the corresponding MAC matrix. This step assures that we compare similar modes.

(4) Parameter optimization
Using the Eq. (4) and (6), the parameters were corrected.

4. IDENTIFICATION AND UPDATING RESULTS

In this section, the results from the numerical simulation and experimental measurements are presented and commented, as well as the updating results for the two first vibration modes of the rotor-generator unit. An Experimental Modal Analysis was performed to identify the frequency, the damping rate and modal shape for the two first vibration modes. Afterwards the FE model updating was performed by the optimization of the mass and moment of inertia of the generator winding.

4.1. Experimental Modal Test Results

The rotor was excited with an impact hammer in free–free boundary conditions. Frequency Response Functions (FRFs) have been measured on nine points distributed on the axis of the rotor by using the average of 20 measurements for each point. The measured resonance frequencies were estimated by a least square routine applying a SDOF method.

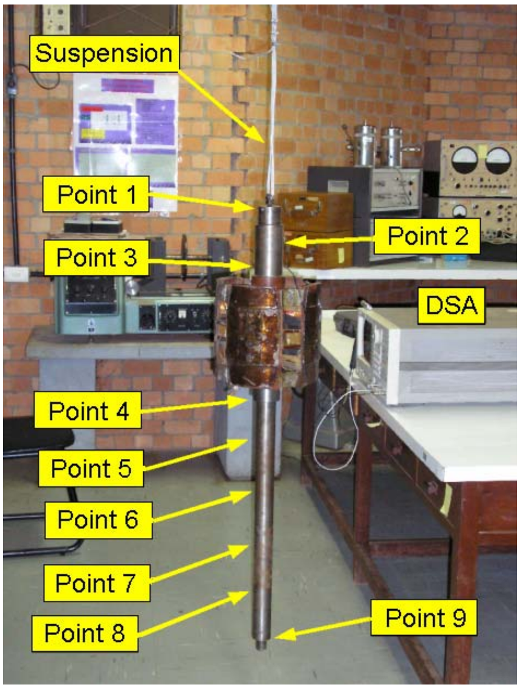

Figure 5. Modal test in free-free boundary conditions

Table 3 shows the modal data (modal frequencies and damping rates) estimated from the experimental FRFs of the two first vibration modes. In figures 6 and 7, the measured, estimated and calculated modal shapes were compared. The figures show that numerical and measured mode shapes are very close.

Table 3. Modal frequencies, damping rate

Mode	f_{id} (Hz)	ξ
1	139.5300	0.006
2	487.2193	0.002

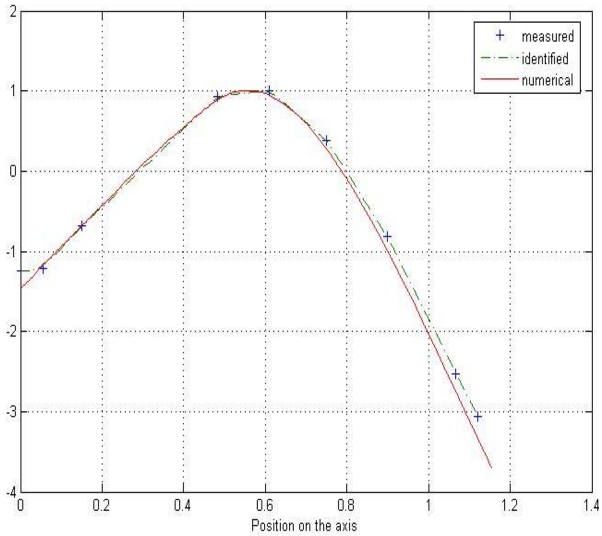

Figure 6: First mode of vibration

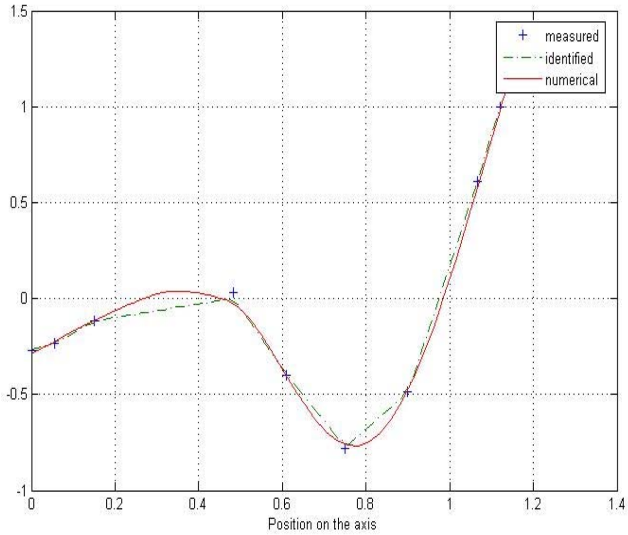

Figure 7: Second mode of vibration

4.2. Updating results

The two criteria presented in section 3.2 were used for updating the FE model. The first criterion optimizes the mass and the moment of inertia, so that numerical and experimental natural frequencies of the rotor-generator unit result as close as possible. The second criterion optimizes the parameters to adjust the mode shapes obtained by the FE model and the measured ones. The optimization was made considering each mode separately first and the two modes simultaneously afterwards.

Table 4 presents the updating results, the calculated frequencies and updated generator winding mass and moment of inertia, using the frequency criterion. Three cases were implemented: optimization of the first natural frequency, optimization of second frequency and optimization of both of them. These three cases were considered to evaluate the influence of each mode on the updating process.

Table 4. Updating results using frequency criterion

	Experimental Frequencies	Frequency criterion optimizing first mode	Frequency criterion optimizing second mode	Frequency criterion optimizing both modes
1^{st}frequency [Hz]	139.5300	139.53	140.2184	139.5201
2^{nd}frequency [Hz]	487.2193	487.6354	487.2001	487.1996
M updated [kg]	- - - - -	65.1904	85.4609	69.9599
I updated [kg.m^2]	- - - - -	0.6543	0.6353	0.6543

The used method leads to a calculated frequency very close to the one chosen in the optimization process. This shows that the routine used leads to a good approximation. The criterion, using the second frequency leads to a good approximation of the frequencies, but the updated mass is very high. This result can be justified by the fact that the second experimental mode shape and numerical one are not correlated exactly in the region on the rotor shaft where the generator winding is located (see Fig. 7). In this region, the rotor-generator winding is supposed to bend, but the experimental data does not show this bending because there are no measurement points on the generator winding region (Fig. 5).

The updated parameters obtained by optimization using the first frequency are better than those obtained from the second, and also from the third mode optimization, because of the influence of the second mode.

Figures 8 and 9 show the optimization functions for the first mode and both modes.

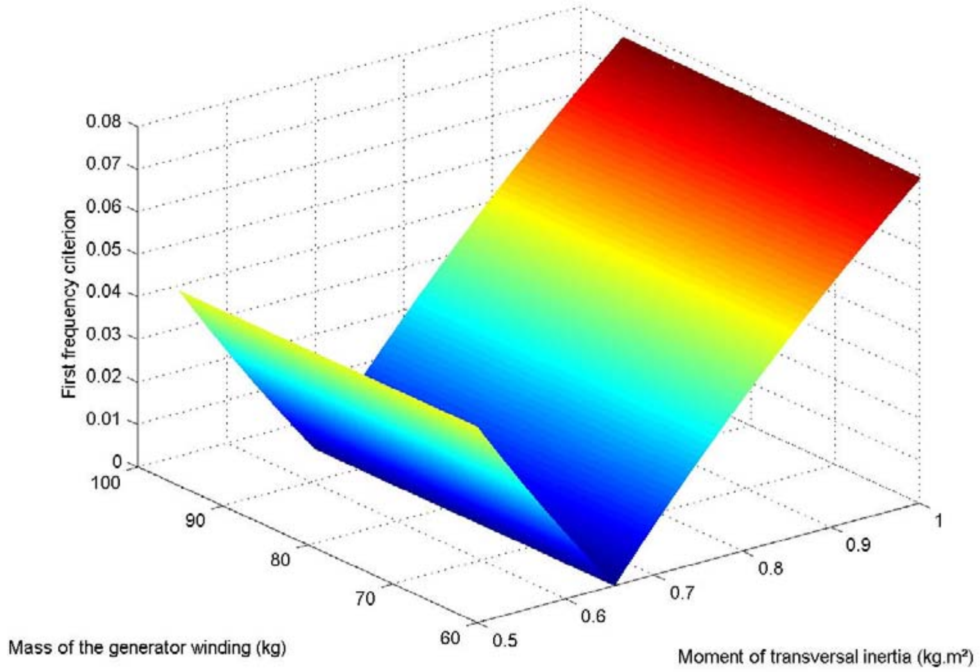

Figure 8: Optimization function for the first mode frequency (frequency criterion).

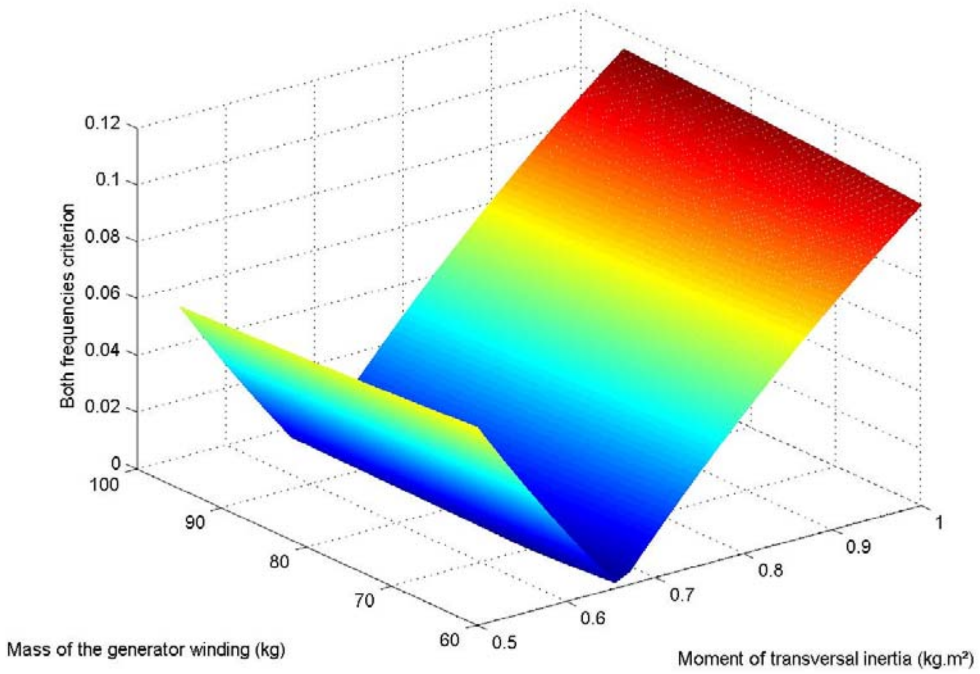

Figure 9: optimization function for both mode frequencies (frequency criterion).

The results of the updating process using the shape modes are organized in Table 5. This table shows the two natural frequencies when the updated parameters (generator winding mass and moment of inertia) obtained by the optimization with the MAC criterion are used in the FE model. Three cases were considered: optimizations of the MAC coefficient for the first mode, for the second mode and for both modes simultaneously. Compared with the measured frequencies, all the calculated frequencies show good approximation. The updated masses obtained in the second and third optimizations make the generator winding too heavy.

Table 5. Updating (comparing the mode shape) using MAC.

	Experimental Frequencies	Shape criterion optimizing first mode	Shape criterion optimizing second mode	Shape criterion optimizing both modes
1st frequency Hz]	139.5300	142.1215	138.3071	140.6617
2nd frequency Hz]	487.2193	502.9616	496.3565	500.2808
M updated [kg]	- - - - - - -	61.21	87.87	70.90
I updated [kg.m^2]	- - - - -	0.56	0.65	0.59

Figures 10 and 11 show the optimization functions for each mode separately.

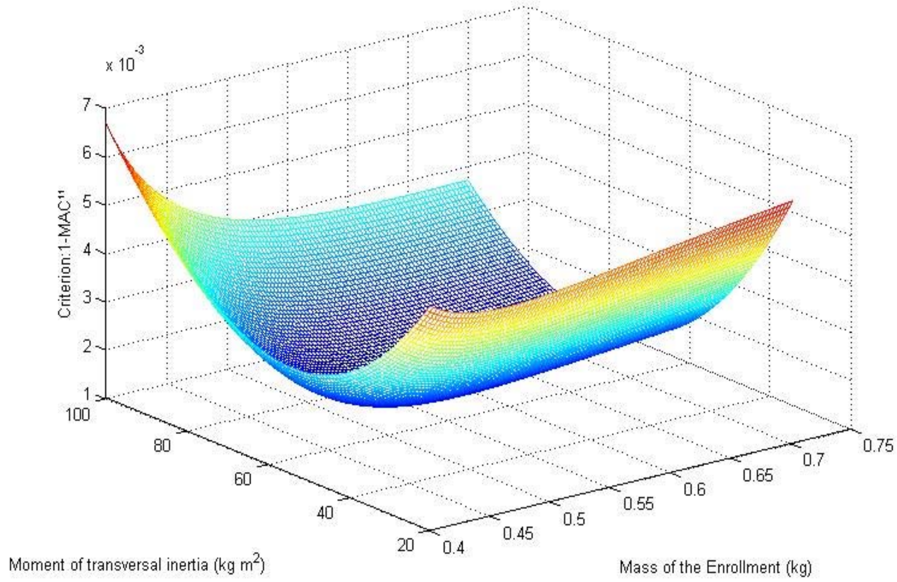

Figure 10: Optimization function for the first mode (MAC criterion).

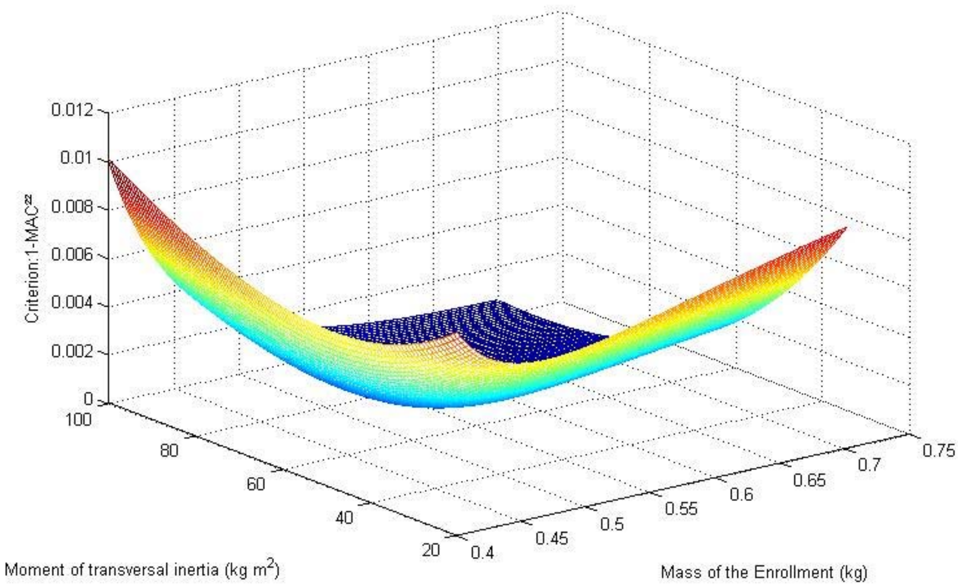

Figure 11: Optimization function for the second mode (MAC criterion).

As in the previous case (frequency criterion), the results from the first optimization are more coherent. The results from the second and the third optimizations confirm the fact that the FE model does not take into account the mass distribution along the third region of the rotor shaft. Also, the fact that there are no points of measurement on this region in the experimental test does not allow for a good description of the second vibration mode.

For the studied rotor-generator unit and the actual FE model, the updating process using the experimental data of first vibration mode was conclusive. The comparison in the table 6 allows concluding that frequencies obtained from both optimizations (with the frequency criterion and the MAC criterion) approximate well the experimental frequencies. It is also clear that the frequencies obtained from the frequency criterion are closer to the experimental ones than those obtained from the MAC criterion. However, considering the first evaluation made for the generator winding mass (item 3.1) the optimization using MAC criterion is the best one. Indeed a mass of the generator of 65.19 leads to a mass of the rotor shaft of 23.41 against 27.39 when the MAC criterion was used.

Table 6. Comparison between experimental frequencies and the calculated ones after updating.

Experimental Frequencies	Frequency criterion optimizing first mode	Relative Difference[%]	MAC Criterion optimizing the 1st mode	Relative Difference[%]
139.5300	139.53	0	142.1215	0.01
487.2193	487.6354	0.0008	502.9616	0.03

5. CONCLUSION

A technique of FE model updating, using Experimental Modal Analysis data, was used to adjust the FE model of a rotor-generator test rig used to simulate the dynamical behavior of hydrogenerators in hydroelectric power plants. The updating process used optimizes the mass and moment of inertia of the generator rotor shaft. These parameters were chosen for the rotor-generator model updating, because they are physical parameters describing the generator and they can be easily included in global matrices of the rotor-generator unit. The optimization process considered firstly the frequencies and secondly the modal shapes of vibration.

The optimization results using the first mode (frequency and shape mode) are more coherent than those that used the second mode or both modes. This is justified by the fact that the second vibration mode was not well represented by numerical nor by experimental models, the numerical model did not consider the generator properties distributed along the appropriate region of the rotor shaft. In the same way the experimental model lacks measurement points in this region.

6. REFERENCES

Alvin, K. F., Robertson, A. N., Reich, G. W., Park, K. C., 2003, “Structural system identification: from reality to models”, computers and structures, Elsevier Science Ltd.

Ewins, D.J., 1985, “Modal testing theory and pratice”, R. S. P.

Friswell, M.I. and Mottershead, J.E.,1995, “Finite Element Model Updating in Structural Dynamics”, Kluwer Academic publishers, Dordrecht, The Netherleands.

Imregun, M., Visser, W., J. and Ewins, D., J.,1995, “Finite Element Model Updating using Frequency Response Function data”, Mechanical Systems and Signal Processing, 9(2), 187-202.

Lin, R.M., Zhu, J., 2006, “Model updating of damped structures using FRF data”, Mechanical Systems and Signal Processing, 20(2006), 2200-2218.

Modak, S.V., Kundra, T.K. and Nakra B.C., 2002, “Comparative study of model updating methods using simulated experimental data”, Computers and Structures, 8(2002), 437-447.

Modak, S.V., Kundra, T.K. and Nakra B.C., 2002, “Predictions dynamics characteristics using updated finite element model”, Journal of Sound and Vibration, 254(3), 447-467.

Mottershead, J.E. and Friswell, M.I., 1993, “Model updating in Structural Dynamics”, Journal of Sound and Vibration, 167(2), 347-375.

Randall J. Allemang, February 2002, “Modal Parameter Estimation overview review”, Structural Dynamics Research Laboratory, University of Cincinnati.

Steenackers, G., Guillaume, P., 2006, “Finite Element Model Updating taking account the uncertainty on the modal parameters estimates”, Journal of Sound and Vibration, 296(2006), 919-934.

Thomas Gmür, 1997, Dynamique des structures: Analyse modale numérique, Presse polytechniques et universitaires Roamndes.

Tshilidzi Marwala, 2002, ” Finite Element Model Updating using Wavelet data and Genetic Algorithm”, Journal of Aircarft, Vol.39, No 4, July-August 2002.

7. RESPONSIBILITY NOTICE

Only the authors are responsible for the printed material included in this paper.