MODE LOCALIZATION BY USING THE DYNAMICAL BASIS APPROACH

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Abstract. *Structures consisting of repeated unit cells are common in engineering applications such groups of buildings, bridges, etc. They can be classified under periodic or quasi periodic systems, which have unique structural dynamic characteristics. For the case of perfectly ordered systems, the vibration modes are extended to the whole set of substructures. When some of the building blocks are slightly imperfect and weak coupling is present between them, considerable changes in these modes are observed. Vibrating energy may be confined over part of the system while another part of it may remain motionless. This is called Mode Localization and may be used to control oscillations. Here the vibration modes of such systems are determined in a unified manner with the use of a fundamental free response. This later being referred to as the dynamical solution. Together with its first three derivatives it constitutes a dynamical basis of solutions for the fourth order differential equation that governs the shape of a vibrating mode.*

Keywords: *mode localization, Euler-Bernoulli beam, dynamical basis*

1. INTRODUCTION

Natural frequencies and mode shapes are the dynamic characteristics of structural systems which are functions of the geometric configuration and the material properties of the structures. The dynamic characteristics may be changed radically by structural changes in some structures when mode localization occurs. Mode localization and eigenvalue curve veering are the phenomena of rapid and even violent changes in vibrating dynamic modes. It is well known that weakly coupled periodic structures are sensitive to certain types of periodicity breaking disorder, resulting in mode localization with serious implication for the control problem. The occurrence of curve veering and mode localization suggests that the dynamic system is very sensitive to a parameter. Attention must be paid to the significance of the sensitivity, for it affects the dynamic modes greatly. A dynamic model can be far from the assumed prototype, caused by a small variation, such as a manufacturing error, a geometrical irregularity, or a mistuned parameter. For these reasons, the subject is worthy of both a theoretical study and a guide for engineering practice. Earlier studies showed that mode localization can be observed in linear periodic systems (As Anderson, 1958; Hodge, 1982; Bendiksen, 1987; Brasil and Hawwa, 1995). Rapid change of eigenvalues, known as curve veering or loci veering, has been noted in various structural systems, such as coupled oscillators (Perkins,1986), multi-span beams (Pierre, Tang and Dowell, 1987), among others. Here the vibration modes of such systems are determined in a unified manner with the use of a fundamental free response. This later being referred to as the dynamical solution (Claeyssen and Soder, 2003). Together with its first three derivatives it constitutes a dynamical basis of solutions for the fourth order differential equation that governs the shape of a vibrating mode.

An advantage of the proposed methodology is that different kind of evolution systems can be treated systematically in a compact form and in a convenient way for numeric simulation and data treatment. Besides, differently from most of the works found in the literature, this methodology is developed in its own physical space, that is, without passing to the denominated state space formulation. An appropriate formulation in the time domain has an immediate use in the frequency domain and in the applications (Claeyssen, Chiwiacowsky and Suazo 2002; Claeyssen, Ferreira and Copetti, 2003). The matrix formulation is done in such a way that the modes are obtained from the product of a boundary matrix and a basis matrix that involve values of the dynamical basis at points located on the boundary and points where the spans are located. The study of frequency equations usually involves the analysis of transcendental equations that arise by using the trigonometric hyperbolic representation of the elements of the dynamical basis. Other approaches could be considered by writing a frequency equation in terms of a dynamical basis. In this work we propose a simple and effective method to study the problem of the free vibrations and mode localization of a N-span Euler-Bernoulli beam, which is quite different from all previous studies.

2. FREE VIBRATION ANALYSIS

The example structures considered are continuous Euler-Bernoulli beam structures resting on N simple supports, which are constrained by torsional springs at intermediate supports. The transverse displacements of each substructure are represented by $v_j(t,x)$ in the j-th span $[x_{j-1}, x_j],\ j = 1, 2, .., N$ with $0 = x_0 \leq x_1 \leq \cdots \leq x_{N-1} \leq x_N = L$.

In each segment of the beam, we have the governing equations

$$m_j \frac{\partial^2 v_j(t,x)}{\partial t^2} + K_j v_j(t,x) = 0,\ x_{j-1} < x < x_j, \tag{1}$$

where

$$m_j = \rho_j A_j, \quad j = 1, 2, .., N, \tag{2}$$

$$K_j = \frac{\partial^2}{\partial x^2}\left[k_j(x)\frac{\partial^2}{\partial x^2}\right], \quad j = 1, 2, .., N. \tag{3}$$

In what follows the coefficients in the operators K_j are assumed constants. Thus, $K_j = EI_j \frac{\partial^4}{\partial x^4}$. Here EI_j are flexural rigidity and m_j are masses per unit length of each substructure.

Free vibrations whose spatial distribution amplitude in each segment is $X_k(x)$, that is $v_j = e^{\lambda t} X_j(x), \quad x \in [x_{j-1}, x_j], \quad j = 1, 2, .., N$, can be found by substituting them into the above system. It turns out the spatial modal differential equation

$$X_j^{(iv)}(x) - \varepsilon_j^4 X_j(x) = 0,\ x \in [x_{j-1}, x_j],\ j = 1, 2, .., N, \tag{4}$$

for each segment of the beam. Here

$$\varepsilon_j^4 = \frac{m_j}{EI_j}\lambda^2 \tag{5}$$

The solution for each segment $[x_{j-1}, x_j]$, $j = 1, 2, .., N$, in the equation (4) can be conveniently written as

$$X_j(x) = d_{1j}\phi_{1j}(x) + d_{2j}\phi_{2j}(x) + d_{3j}\phi_{3j}(x) + d_{4j}\phi_{4j}(x) \tag{6}$$

Generic boundary conditions of classical or non-classical nature can be written as

$$\begin{array}{l} A_{11}X_1(0) + B_{11}X_1'(0) + C_{11}X_1''(0) + D_{11}X_1'''(0) = 0, \\ A_{12}X_1(0) + B_{12}X_1'(0) + C_{12}X_1''(0) + D_{12}X_1'''(0) = 0, \\ A_{21}X_N(L) + B_{21}X_N'(L) + C_{21}X_N''(L) + D_{21}X_N'''(L) = 0, \\ A_{22}X_N(L) + B_{22}X_N'(L) + C_{22}X_N''(L) + D_{22}X_N'''(L) = 0. \end{array} \tag{7}$$

General conditions of continuity or compatibility for the displacement, the slope, the bending moment or for the jump in the internal shear force when there is an applied force or a physical device at an intermediate location x_j, $j = 1, 2, .., N-1$ can be written as

$$\begin{array}{l} E_{11}^{(j)}X_j(x_j) + F_{11}^{(j)}X_j'(x_j) + G_{11}^{(j)}X_j''(x_j) + H_{11}^{(j)}X_j'''(x_j) = \\ E_{12}^{(j)}X_{j+1}(x_j) + F_{12}^{(j)}X_{j+1}'(x_j) + G_{12}^{(j)}X_{j+1}''(x_j) + H_{12}^{(j)}X_{j+1}'''(x_j), \\ \\ E_{21}^{(j)}X_j(x_j) + F_{21}^{(j)}X_j'(x_j) + G_{21}^{(j)}X_j''(x_j) + H_{21}^{(j)}X_j'''(x_j) = \\ E_{22}^{(j)}X_{j+1}(x_j) + F_{22}^{(j)}X_{j+1}'(x_j) + G_{22}^{(j)}X_{j+1}''(x_j) + H(j)_{22}X_{j+1}'''(x_j), \\ \\ E_{31}^{(j)}X_j(x_j) + F_{31}^{(j)}X_j'(x_j) + G_{31}^{(j)}X_1''(x_j) + H_{31}^{(j)}X_1'''(x_j) = \\ E_{32}^{(j)}X_{j+1}(x_j) + F_{32}^{(j)}X_{j+1}'(x_j) + G_{32}^{(j)}X_{j+1}''(x_j) + H_{32}^{(j)}X_{j+1}'''(x_j), \\ E_{41}^{(j)}X_j(x_j) + F_{41}^{(j)}X_j'(x_j) + G_{41}^{(j)}X_j''(x_j) + H_{41}^{(j)}X_j'''(x_j) = \\ E_{42}^{(j)}X_{j+1}(x_j) + F_{42}^{(j)}X_{j+1}'(x_j) + G_{42}^{(j)}X_{j+1}''(x_j) + H_{42}^{(j)}X_{j+1}'''(x_j) \\ +F_j, \end{array} \tag{8}$$

where F_j denotes the force exerted by the intermediate device. These equations include the case of an intermediate support at x_j. We simple consider the first two equations for zero displacement at x_j^+ and x_j^-. Then the continuity of the rotation or inertia moment and for the bending moment at a x_j^+ and x_j^-. For the case of a torsional spring or a rotational mass, the bending moment has a jump at an intermediate support (Claeyssen and Soder, 2003).

The substitution of (6) into (7) and (8), the boundary and continuity conditions leads to a linear algebraic system

$$\mathcal{U}(\lambda)\mathbf{d} = \mathbf{0}, \tag{9}$$

for the vector $\mathbf{d}$ of order $4N \times 1$

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_N \end{bmatrix}, \quad \mathbf{d_j} = \begin{bmatrix} d_{1j} \\ d_{2j} \\ d_{3j} \\ d_{4j} \end{bmatrix}, \;\; j = 1, 2, .., N.$$

Here, the matrix $\mathcal{U}$ is of order $4N \times 4N$ and it has the form $\mathcal{U} = \mathcal{B}\Phi$ where $\mathcal{B}$ is a matrix of order $4N \times 8N$ formed with the coefficients associated to the boundary and continuity conditions and Φ is a matrix of order $8N \times 4N$ whose components are values of the solution basis at the ends and the conditions at the discontinuity. Non-zero solutions of (9) are obtained for frequency values that satisfy the characteristic equation

$$det(\mathcal{U}) = 0. \tag{10}$$

2.1 Block Matrix Formulation

The description of the matrix $\mathcal{U}$ will be given in a block formulation. The matrix coefficients corresponding to the boundary values can be written as

$$\mathcal{B}_0 = \begin{bmatrix} A_{11} & B_{11} & C_{11} & D_{11} \\ A_{12} & B_{12} & C_{12} & D_{12} \end{bmatrix}, \qquad \mathcal{B}_L = \begin{bmatrix} A_{21} & B_{21} & C_{21} & D_{21} \\ A_{22} & B_{22} & C_{22} & D_{22} \end{bmatrix} \tag{11}$$

The matrix coefficients corresponding to the continuity conditions at $x = x_j, \; j = 1, 2, .., N$, can be described in terms of the matrices

$$\mathcal{B}_{1j} = \begin{bmatrix} E_{11}^{(j)} & F_{11}^{(j)} & G_{11}^{(j)} & H_{11}^{(j)} \\ E_{21}^{(j)} & F_{21}^{(j)} & G_{21}^{(j)} & H_{21}^{(j)} \\ E_{31}^{(j)} & F_{31}^{(j)} & G_{31}^{(j)} & H_{31}^{(j)} \\ E_{41}^{(j)} & F_{41} & G_{41}^{(j)} & H_{41}^{(j)} \end{bmatrix}, \qquad \mathcal{B}_{2j} = \begin{bmatrix} E_{12}^{(j)} & F_{12}^{(j)} & G_{12}^{(j)} & H_{12}^{(j)} \\ E_{22}^{(j)} & F_{22}^{(j)} & G_{22}^{(j)} & H_{22}^{(j)} \\ E_{32}^{(j)} & F_{32}^{(j)} & G_{32}^{(j)} & H_{32}^{(j)} \\ E_{42}^{(j)} & F_{42}^{(j)} & G_{42}^{(j)} & H_{42}^{(j)} \end{bmatrix} \tag{12}$$

while the values of the basis solutions at x_j can be written in terms of the matrices

$$\Phi_j = \Phi_j(x) \;\; = \;\; \begin{bmatrix} \phi_{1j}(x) & \phi_{2j}(x) & \phi_{3j}(x) & \phi_{4j}(x) \\ \phi'_{1j}(x) & \phi'_{2j}(x) & \phi_{3j}(x) & \phi'_{4j}(x) \\ \phi''_{1j}(x) & \phi''_{2j}(x) & \phi_{3j}(x) & \phi''_{4j}(x) \\ \phi'''_{1j}(x) & \phi'''_{2j}(x) & \phi'''_{3j}(x) & \phi'''_{4j}(x) \end{bmatrix}, \quad x = x_0, .., x_N, \;\; j = 1, 2, .., N. \tag{13}$$

It is clear that for a multi-span beam with N segments, we shall have the block matrices

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_0 & 0 & 0 & 0 & \cdot & 0 & 0 & 0 \\ 0 & \mathcal{B}_{11} & -\mathcal{B}_{21} & 0 & \cdot & 0 & 0 & 0 \\ 0 & 0 & \mathcal{B}_{12} & -\mathcal{B}_{22} & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \mathcal{B}_{1N-1} & -\mathcal{B}_{2N-1} & 0 \\ 0 & 0 & 0 & 0 & \cdot & 0 & 0 & \mathcal{B}_L \end{bmatrix}. \tag{14}$$

and

$$\Phi = \begin{bmatrix} \Phi_1(0) & 0 & 0 & \cdot & 0 \\ \Phi_1(x_1) & 0 & 0 & \cdot & 0 \\ 0 & \Phi_2(x_1) & 0 & \cdot & 0 \\ 0 & \Phi_2(x_2) & 0 & \cdot & 0 \\ 0 & 0 & \Phi_3(x_2) & \cdot & 0 \\ 0 & 0 & \Phi_3(x_3) & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \Phi_N(x_{N-1}) \\ 0 & 0 & 0 & \cdot & \Phi_N(L) \end{bmatrix}. \tag{15}$$

In the above, 0 denote null matrices with appropriate dimensions, that is, 2×4 or 4×4. Thus

$$\mathcal{U} = \mathcal{B}\Phi \tag{16}$$

Since the boundary conditions depending on each given problem, it is clear that a simplification or sparsity of the matrix $\mathcal{U}$ could be achieved by a proper choose of the matrix basis.

3. THE DYNAMICAL BASIS

According to Claeyssen and Soder, 2003 and Claeyssen and Costa, 2006, to solve the modal equation (9), it is necessary to introduce a basis suitable for determining the matrix (13) or (15). From the many mathematical bases available for the fourth-order equation

$$X^{(iv)}(x) - \varepsilon^4 X(x) = 0, \tag{17}$$

it is convenient to choose one that makes (15) as sparse as possible. This is accomplished by choosing the *dynamical* or *fundamental* basis which is generated by the solution $h(x)$ of the initial value problem (Claeyssen and Soder, 2003)

$$\begin{array}{l} h^{(iv)}(x) - \varepsilon^4 h(x) = 0 \\ h(0) = 0,\ h'(0) = 0,\ h''(0) = 0,\ h'''(0) = 1, \end{array} \tag{18}$$

and it first three derivatives $h'(x)$, $h''(x)$, $h'''(x)$. In terms of the traditional spectral basis, constructed using the roots of the associated characteristic polynomial $P(s) = s^4 - \varepsilon^4$, that is,

$$\Psi = [sin(\varepsilon x),\ \ cos(\varepsilon x),\ \ sinh(\varepsilon x),\ \ cosh(\varepsilon x)],$$

we have that the fundamental solution $h(x)$ has the following representation with respect to the spectral Euler basis,

$$h(x) = \frac{sinh(\varepsilon x) - sin(\varepsilon x)}{2\varepsilon^3}. \tag{19}$$

From (18), higher derivatives of h can be written in terms of the dynamical solution and it first three derivatives. Thus

$$\Phi_j(x_{j-1}) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad j = 1, 2, .., N. \tag{20}$$

It follows that (15) becomes more sparse.

The fundamental response $h(x, \varepsilon)$, has the same shape for each segment, but depends on different values for the involved physical parameters such as flexural rigidity and masses per unit length of each substructure. Now we will apply it in an example.

4. MODE LOCALIZATION ANALYSIS

The engineering application structures we consider are continuous beam structures resting on the three simple supports, which are constrained by torsional spring at mid support as shown schematically in Figure 1. The torsional spring plays the role of a decoupler. As $k_r \to \infty$ the spans are fully decoupled from each other because no moment can be transmitted from one substructure to the other. For $k_r \to 0$ the substructures are strongly coupled since no restoring moment is exerted. The system can be divided into two substructures and for the convenience of simple analysis the coordinates of the substructures are determined as in Figure 1. The eigenvalue problems for free bending vibrations of each substructure can be written as (4), that is,

$$X_j^{(iv)}(x) - \varepsilon_j^4 X_j(x) = 0, j = 1, 2 \tag{21}$$

for each segment of the beam, with ε_j^4 as (5). The general solutions can be written as (6), that is,

$$X_j(x) = d_{1j}h_{1j}(x) + d_{2j}h_{2j}(x) + d_{3j}h_{3j}(x) + d_{4j}h_{4j}(x) \tag{22}$$

where $h(x)$ was defined in (19).

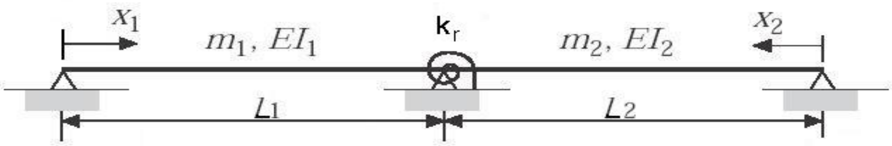

Figure 1. Simply supported two-span beam with rotational stiffness at mid support.

The matrix (14) corresponding to the boundary conditions for this particular case can be written considering that the deflections and the moments at $x_1 = 0$ and $x_2 = 0$ are zeros, deflections at the mid support are zeros and the two continuity conditions such as

$$X'(L_1) = X'(L_2), \qquad EI_1 X''(L_1) + EI_1 X''(L_2) = k_r X'(L_2). \tag{23}$$

By using the block matrix formulation, we have the blocks with the boundary conditions coefficients

$$\mathcal{B}_{x_1=0} = B_{x_2=0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{24}$$

and the blocks for the continuity conditions

$$\mathcal{B}_{x=L_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & EI_1 & 0 \end{bmatrix}, \quad \mathcal{B}_{x=L_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -k_r & -EI_2 & 0 \end{bmatrix}. \tag{25}$$

The matrix formed with solution basis for each span of the beam, by using the initial values of $h(x, \epsilon)$, (15) becomes more sparse

$$\Phi = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h(L_1) & h'(L_1) & h''(L_1) & h'''(L_1) & 0 & 0 & 0 & 0 \\ h'(L_1) & h''(L_1) & h'''(L_1) & h^{(iv)}(L_1) & 0 & 0 & 0 & 0 \\ h''(L_1) & h'''(L_1) & h^{(iv)}(L_1) & h^{(v)}(L_1) & 0 & 0 & 0 & 0 \\ h'''(L_1) & h^{(iv)}(L_1) & h^{(v)}(L_1) & h^{(vi)}(L_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h(L_2) & h'(L_2) & h''(L_2) & h'''(L_2) \\ 0 & 0 & 0 & 0 & h'(L_2) & h''(L_2) & h'''(L_2) & h^{(iv)}(L_2) \\ 0 & 0 & 0 & 0 & h''(L_2) & h'''(L_2) & h^{(iv)}(L_2) & h^{(v)}(L_2) \\ 0 & 0 & 0 & 0 & h'''(L_2) & h^{(iv)}(L_2) & h^{(v)}(L_2) & h^{(vi)}(L_2) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

Solving $\mathcal{B}\Phi$ and after simplifications and algebraic manipulations, we obtain the reduced system

$$\mathcal{U} = \begin{bmatrix} h'(L_1) & -h'(L_2) \\ EI_1 h''(L_1) & -k_r h'(L_2) - EI_2 h''(L_2) \end{bmatrix}. \tag{27}$$

This gives an equation for the determination of natural frequencies which is called the frequency equation

$$det(\mathcal{U}) = EI_1 h'(L_2) h''(L_1) - h'(L_1) \left[k_r h'(L_2) + EI_2 h''(L_2) \right], \tag{28}$$

or using the representation of the fundamental solution $h(x)$ (19)

$$det(\mathcal{U}) = k_r \varepsilon_2^4 \theta_1 \theta_2 - \varepsilon_2^2 \gamma_2 \theta_1 + \varepsilon_1^2 \gamma_1 \theta_2 \tag{29}$$

where

$$\theta_i = \frac{cosh(\varepsilon_i L_i) sen(\varepsilon_i L_i) - cos(\varepsilon_i L_i) senh(\varepsilon_i L_i)}{senh(\varepsilon_i L_i) + sen(\varepsilon_i L_i)}, \quad \gamma_i = -2EI_i \frac{senh(\varepsilon_i L_i) sen(\varepsilon_i L_i)}{\varepsilon_i \left(senh(\varepsilon_i L_i) + sen(\varepsilon_i L_i) \right)} \tag{30}$$

with $i = 1, 2$.

Now we define a mode localization factor according to Kim, 1998. It can be used as a measure of the degree of mode localization of each mode. As aforementioned, the mode localization is the vibration energy confinement, so the degree of mode localization can be represented by logarithmic value of the ratio of the mean squared vibrational magnitude of the second substructure to that of the first substructure as $\Gamma = log\frac{\eta_2}{\eta_1}$, where Γ is the mode localization factor, and η_1 and η_2 are the mean squared free vibrational magnitude of the first and second substructures respectively. It is well know in the literature that Γ can be approximate as $\Gamma = \frac{d_{21}^2}{d_{11}^2}$.

The mode localization phenomenon caused by the small changes in system parameters in ordered and disordered systems are now discussed. The effects of the coupling - (a) strongly coupled and (b) weakly coupled - of the substructures and the mode localization are studied by considering the cases: *case 1*: ordered system; *case 2*: disordered system, with disorder on L_1 and L_2 lengths; *case 3*: disordered system, with disorder only on L_2. The example structures have the same material properties and each of the cases is classified by the rigidity k_r of the torsional spring and the span length of the substructures.

	Case 2-a		Case 2-b	
ε	f	Γ	f	Γ
-5%	1313.89	0.3778	1802.32	0.3583
-4%	1343.67	0.3014	1872.30	0.2839
-3%	1366.04	0.2255	1947.07	0.2107
-2%	1388.65	0.1499	2024.69	0.1389
-1%	1400.56	0.0749	2106.36	0.0685
0%	1403.14	0.0000	2191.11	0.0000
1%	1401.11	-0.0749	2150.16	-0.0681
2%	1387.96	-0.1500	2108.14	-0.1371
3%	1368.26	-0.2254	2067.27	-0.2070
4%	1339.10	-0.3015	2027.53	-0.2778
5%	1314.05	-0.3778	1989.02	-0.3492

Table 1. Natural frequencies and degrees of mode localization for case 2: (a) strongly coupled system; (b) weakly coupled system.

The curve veering and mode localization phenomena in the ordered and disordered periodic two span beams are examined for the following coupling conditions: in the disordered cases, a geometric disorder is introduced by varying the lengths L_1 and/or L_2 of the substructures according to (A) $L_1 = L_1(1+\epsilon)$, (B) $L_2 = L_2(1-\epsilon)$, where $-0.05 \leq \epsilon \leq 0.05$ is a non dimensional control parameter measuring the amount of disorder introduced. The masses per unit length of each substructure are $m_1 = m_2 = 25$ kg/m, the flexural rigidities $EI_1 = EI_2 = 2.5 \times 10^7 Nm^2$. The span length $L_1 = L_2 = 0.5$ m in ordered case 1; L_1 and L_2 according to (A) and (B) defined above in disordered case 2 and $L_1 = 0.5$ m and L_2 according only to (A) in disordered case 3. The torsional spring constants are k_r= 0 Nm and $k_r = 2.5 \times 10^8$ Nm in strongly and weakly coupled systems, respectively. Observe that this example is performed on the assumption that some errors such as manufacturing errors or structural damages make an ordered system into disordered one. The frequencies for $-0.05 \leq \epsilon \leq 0.05$ and the localization factor are showed in Table 1 for case 2.

Figure 2 shows the curve veering phenomena. Subcases a and b imply a strongly coupled system and a weakly coupled system, respectively. In the graphs on the first line only L_2 is disturbed by $\epsilon \in [-0.05, 0.05]$. In the graphs on the second line both L_1 and L_2 are disturbed. The eigenvalues are much close, but not equal, to each other at $\epsilon \approx 0$ when the substructures are weakly coupled. A rapid change of eigenvalues, showing the occurrence of the eigenvalue curve veering, is observed in both cases. The more rapid one comes from closer eigenvalues. The change happens in a range, which is small when two eigenvalues are getting close. As two eigenvalues become close, their curves repel each other, suggesting a violent veering.

The curve veering and mode localization in case 2 *a* are proved to be more violent than case 2 *b* by comparison of the figures. Selected mode shapes of each strongly and weakly coupled cases are shown in Figs. 3 and 4 for a disturb $\epsilon = -5\%$ on L_2, respectively.

5. CONCLUSIONS

In this paper we use a different methodology for eigenanalysis of general beam problems, which permits that different evolution systems can be treated systematically in a compact form and in a convenient way for numeric simulation and data treatment. It permits we work in the own physical space of the problem. Additionally, this methodology can be applied to other kinds of beams that result from diverse approximations such as Rayleigh, shear or Timoshenko beams. Our method is simple but effective because *explicit formulas are obtained for the system of fundamental solutions*, which are very useful for other purposes such as stability analysis. As an application we study the mode localization and curve veering phenomena of two-span beams for ordered and disordered periodic structures for different coupling conditions. In the disordered cases, a geometric disorder is introduced by varying the lengths L_1 and/or L_2 of the substructures.

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(a) (b)

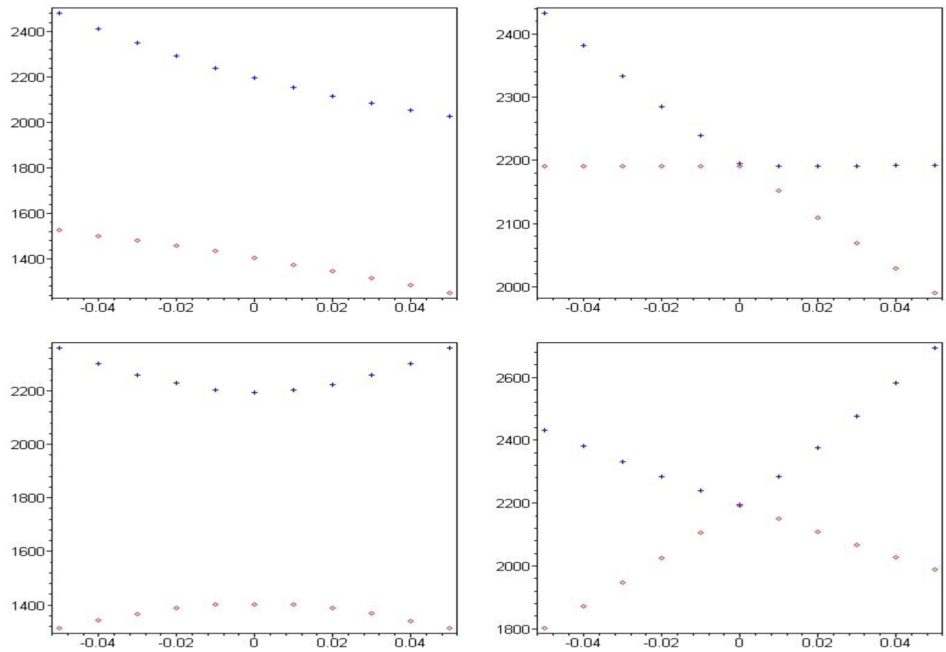

Figure 2. Frequencies versus perturbation ϵ for (a) strongly coupled system (b) weakly coupled system

(a) (b)

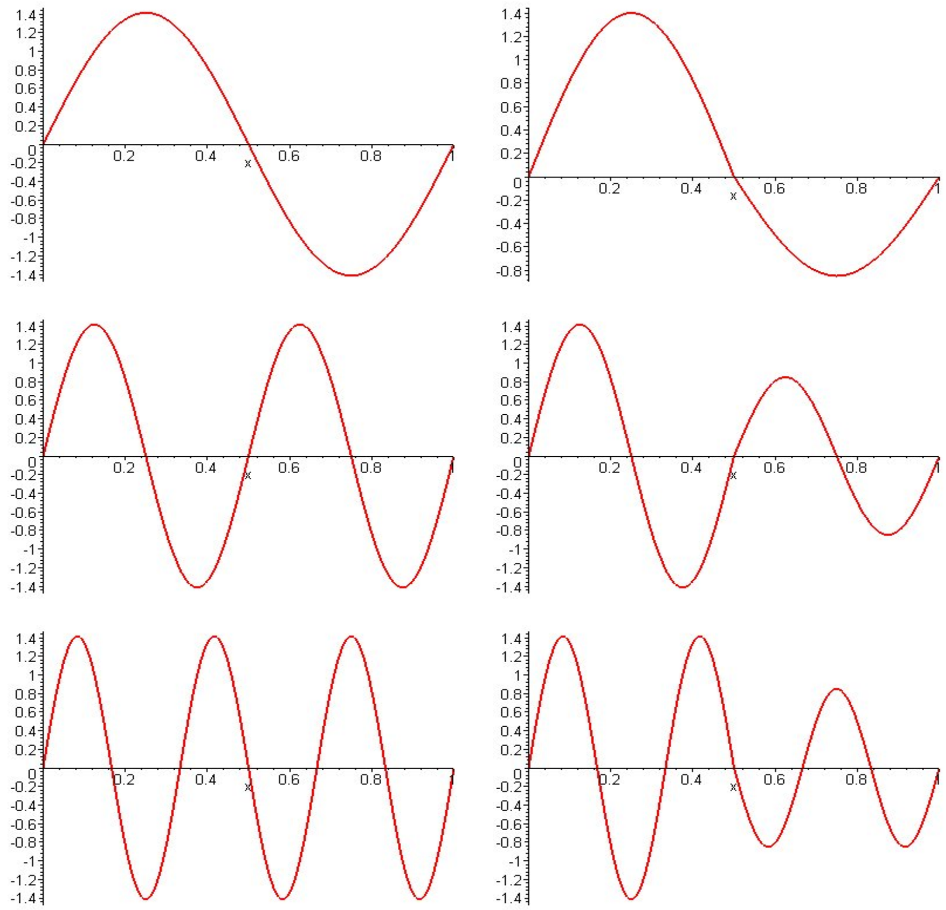

Figure 3. Mode shapes of strongly coupled system: (a) case 1; (b) case 3.

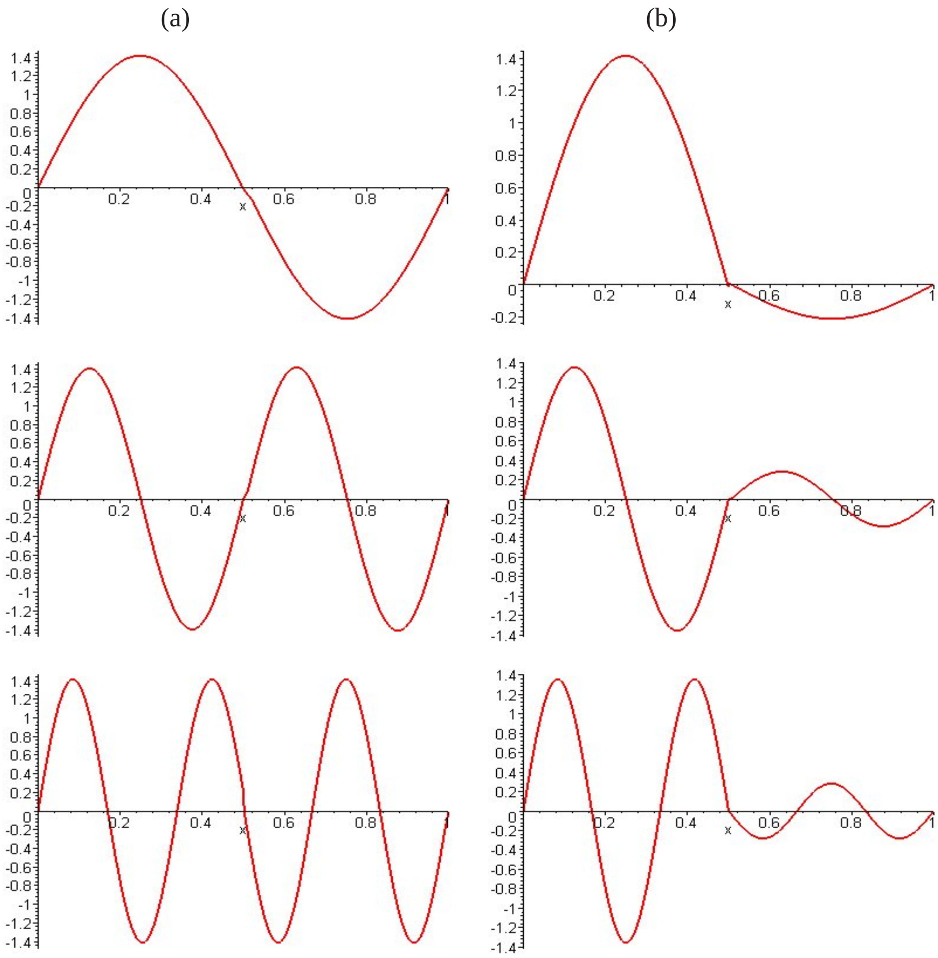

Figure 4. Mode shapes of weakly coupled system: (a) case 1; (b) case 3.

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