VIBRO-ACOUSTIC MODAL ANALYSIS OF A PVC CAVITY CLOSED BY A FLEXIBLE PLATE

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Abstract. *The coupling between acoustical response in the cavity and structural excitation, whereas the structural response is also related to acoustical excitation source has been found in many systems in day-life. Car interiors, cabs of trucks and aircraft fuselages are just a few pratical examples of this sort of system. Coupling implies that the acoustical and vibrational system behavior are not independent from each other, and therefore they must be considered as global system behavior. The aim of this paper is to study the vibro-acoustical problem and to understand the interference and contribution between acoustical and structural modal analysis. The model is an irregular polyvinylchloride (PVC) cavity with some resemblance to a car body. The cavity is closed by a flexible plate to obtain a vibro-acoustical coupling. The modeling of fluid-structure interaction is based on the finite element theory and compact matrix technique. The FRF(s) of the modal model are defined by the relationship between pressure response in the cavity per structural force and the velocity response on the boundary condiction per excitation force applied on the flexible plate. The comparison of the numerical and experimental models shows the correlation of the results.*

Keywords*: vibro-acoustic, modal analysis, compact matrix, finite element.*

1. INTRODUCTION

In the vibro-acoustical system, the structural and acoustic behavior of the model are not independent. When the system is excited by a force or by a volume acceleration, acoustical and structural response are coupled. Acoustical peaks emerge because resonant frequencies in the structure, whereas the structural response is affected by acoustical modes. The examples of vibro-acoustical system can be found in cabin trucks, aircraft fuselage, building acoustic, etc.

The modeling of uncoupled acoustical case and uncoupled structural case is based on symetrical matrices and can be solved by standard method (Maia, 1997). The modeling of coupled system is more elaborate and the means is to calculate contribuitions of the subsystems to each other. Many researches have been worked in his subject and developed different methods to solve the vibro-acoustical system, Lyon (1963) analyzed the problem of sound transmission through a panel into a rectangular cavity, Pretlove (1965) investigate the free and force vibration of a rectangular panel backed by a regular cavity, Wyckaert *et al.* (1996) has consolidated the vibro-acoustical modal analysis and Luo *at al.* (2004) analyzed the problem using the green function to solve a non-regular cavity with one flexible wall. Almost all the previous researches were based on a regular model (normally rectangular enclosure with one flexible boundary). If the model is regular the system response is reasonably well-known and the analytical descriptions of natural frequencies and the related mode shapes exist. However, these solutions will face some difficulties if the model is irregular. The purpose of this paper is to solve the vibro-acoustical problem of a non-regular cavity by using the finite element method, FEM, and compact matrix technique to obtain frequency response function of the system. An experimental modal analysis is performed to validate the approach.

2. VIBRO-ACOUSTICAL MODEL

2.1 Finite element model approach

The vibro-acoustical behavior of enclosure problems with a flexible wall (Fig. 01) can be discussed in terms of equilibrium and coupling equations. The finite element equation that describes the movement of a plate under external structural loading in an acoustical cavity is given by (zienkiewicz, 1981):

$$\left[-\omega^2 M^s - i\omega C^s + K^s\right]\{u\} = \{f_{S_b}\} + \left[K^c\right]\{P\} \tag{01}$$

$$\left[K^c\right]\{P\} = \int_{S_b} \left[N^s\right]^T \{n\}\left[N^f\right] dS \tag{02}$$

The script s refers to structural system and the script f refers to acoustical system. Equation (02) represents the coupling term on the flexible surface S_b of the cavity, due to acoustic pressure. The vector f_{S_b} represents the external load applied to flexible boundary. The matrices M^s, C^s and K^s are the well know mass, damping and stiffness structural matrices, respectively.

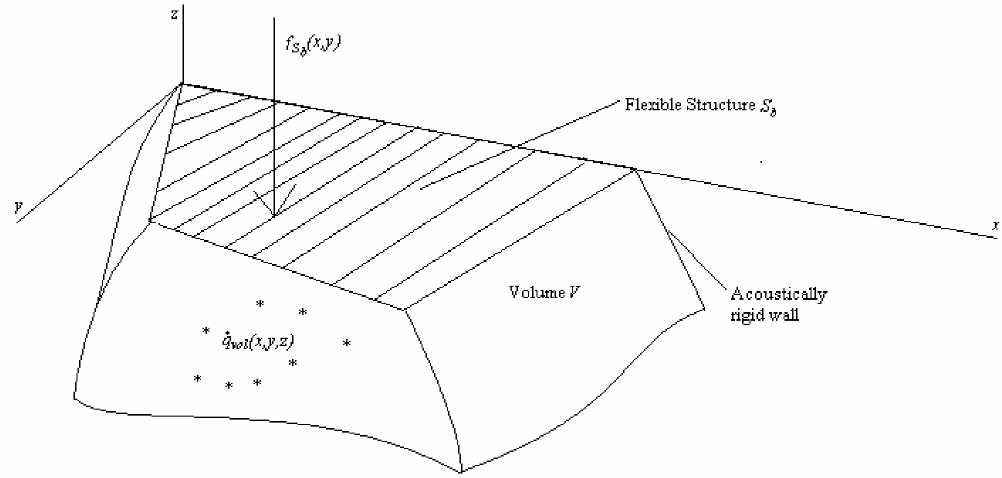


Figure 1. A vibro-acoustic system with structural excitation and acoustical source

The vibro-acoustical system can also be described in terms of the acoustical pressure variable (Göransson, 1994). In this case one can defined a pair of equations similar to Eq. (01) and (02).

$$\left[-\omega^2 M^f - i\omega C^f + K^f\right]\{P\} = \rho_0\{\dot{q}_{vol}\} - \omega^2\left[M^C\right]\{u\} \tag{03}$$

$$\left[M^C\right] = \rho_0 \int_{S_b} \left[N^f\right]^T \left[N^s\right]\{n\} dS \tag{04}$$

Equation (04) represents the loading due to the normal vibration on the flexible boundary surface S_b of the cavity. The term $\dot{q}_{vol}$ represents the acoustical load applied into the cavity. The terms M^f , C^f and K^f are, indirectly, the mass, damping and stiffness fluid matrices. These matrices (Göransson, 1994) can be calculating by the following equations:

$$M^f = \frac{1}{c^2}\int_V N^f {N^f}^T dV \tag{05}$$

$$C^f = \frac{\beta}{c}\int_S N^f {N^f}^T dV \tag{06}$$

$$K^f = \int_V \nabla {N^f}^T \nabla N^f dV \tag{07}$$

Where $\beta = \dfrac{r}{\rho_0 c}$ is the boundary admittance on the wall, r is the impedance of the wall surface and c is the sound velocity in the fluid.

By combining Eq. (01), (02), (03) and (04) one can write a set of equation for the coupled vibro-acoustic problem, Eq. (08).

$$-\omega^2\begin{bmatrix} M^s & 0 \\ M^c & M^f \end{bmatrix}\begin{Bmatrix} u \\ P \end{Bmatrix} - i\omega\begin{bmatrix} C^s & 0 \\ 0 & C^f \end{bmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix} + \begin{bmatrix} K^s & -K^c \\ 0 & K^f \end{bmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f_{S_b} \\ \rho\dot{q}_{vol} \end{Bmatrix} \tag{08}$$

Equation (08) is an unsymmetrical equation and its solution is not straightforward. Wyckaert *et al.* (1996) discusse this problem of unsymmetry and show that for low frequencies applications, the problem can be solved by a modal analysis approach, defining a scale factor to correct the difference between the left and right eigenvectors, in order to take into account the non-symmetry of the equation. By using FEM approach (Zienkiewicz, 1981), the solution of the unsymmetrical problem can be obtained numerically, but it has been shown very heavy time computationally. It can also be discussed by using a compact matrix formulation (Kim and Brennan, 1998).

The compact matrix formulation discusses the vibro-acoustical problem using a compact matrix, where the coupled response can be represented in terms of the uncoupled structural acoustical modes, as well as, the respective resonant frequencies. The technique presents some advantages such as a low computational time and the possibility of use in active control of noise (Luo *et all.* 2004). This paper will be focus in this approach.

2.2 A compact matrix formulation

The compact matrix formulation approach is defined in terms of the uncoupled structural and uncoupled acoustic equations. From the Eq. (08), taking $\left(K^c = M^c = 0\right)$ one can see that both acoustical and vibrational uncoupled problems are described by symmetrical sets of second order equations. So, the solution of the uncoupled problems can be obtained by the traditional numerical modal analysis tools. Therefore, the solution of the uncoupled structural and acoustical problems permits to obtain the uncoupled structural modes vectors $\varphi's$ and the uncoupled acoustical modes vectors $\psi's$.

Defined the modes vectors, the acoustic pressure can be described by a summation of the N uncoupled acoustic modes multiplied by a complex coefficient *a* (Dowell *at al.*, 1977) expression (09).

$$P(x,y,z,\omega) = \sum_{n=1}^{N} \psi_n(x,y,z)a_n(\omega) \tag{09}$$

In the same way, normal velocity can be expressed by:

$$v(x,y,\omega) = \sum_{m=1}^{M} \varphi_m(x,y)b_m(\omega) \tag{10}$$

Where N is the number of the acoustical modes and M is the number of the structural modes in the frequency band of interest. The coefficients *a* and *b* are complex amplitude of the acoustic pressure modes and normal vibration velocity modes respectively. According to Luo *et al.* (2004), they can be expressed by:

$$a = A_n\left(q - Cb\right) \tag{11}$$

$$b = B_n\left(C^T a - f\right) \tag{12}$$

The terms q and f represent the acoustical generalized modal source and the structural modal force respectively and they are given by the volumetric acceleration (acoustic source), structural external load and mode shapes, Eq. (13) and (14).

$$q = \dot{q}_{vol}\psi \tag{13}$$

$$f = f_{S_b}\varphi \tag{14}$$

The term C is the coupling coefficient matrix. In the Eq. (11) it represent the loading effect of the structure in the cavity, however, in the Eq. (12) the transpose of C represents the effect of the cavity on the vibration of the flexible structure. The matrix is computed by the Eq. (15).

$$C = \int_{S_b} \psi(x,y)\varphi(x,y)dS \tag{15}$$

The coefficients A_n is a (N x N) diagonal matrix that represents the acoustical impedance of the acoustical boundary. It was defined (Kim and Brennan, 1998) by expression (16) and (17).

$$A_1 = \frac{\rho_0 c_0^2}{V_n(1/5 + i\omega)} \tag{16}$$

$$A_n = \frac{i\omega\rho_0 c_0^2}{V_n\left(\omega_n^2 - \omega^2 + 2i\xi_n\omega_n\omega\right)} \quad \left(n > 1\right) \tag{17}$$

Where $V_n = \int_V \psi_n^2(x,y,z)dV$. The terms ρ_0 and c_0 are the mass density and sound velocity of the air, respectively. ω_n and ξ_n are the natural frequency and damping ratio of the *n*th acoustical mode, respectively. As show in Kim and Brennan (1998), the mobility of the boundary surface is represented by B_m-terms as a (M x M) diagonal matrix shows below:

$$B_m = \frac{i\omega}{\rho_s h_s M_m\left(\omega_m^2 - \omega^2 + 2i\zeta_m\omega_n\omega\right)} \tag{18}$$

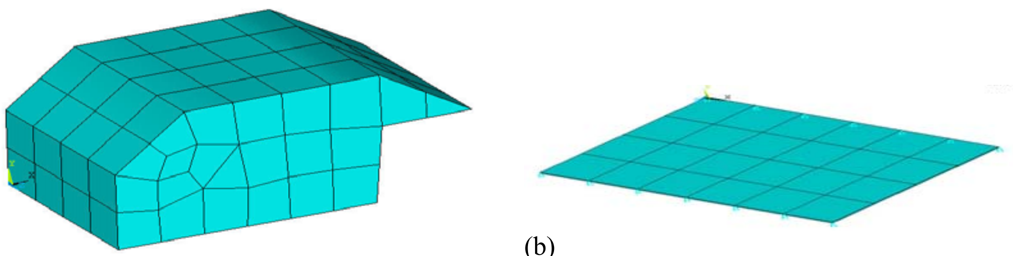
Where $M_m = \int_{S_b} \psi_m^2(x,y)dS$. The terms ρ_s , h_s , ω_m and ξ_m are the mass density, thickness, natural frequency and damping ratio of the flexible structure, respectively

3. NUMERICAL SIMULATION AND EXPERIMENTAL MODEL

The analyzed system is an enclosure model defined by a non-regular rigid cavity with some resemblance to a car cavity of ¼ scale compared with a popular Brazilian auto. The numerical simulation of vibro-acoustic behavior of the model is discussed in terms of the finite element modeling of the uncoupled system and by using compact matrix technique. The finite element model was obtained by commercial finite element code and the compact matrix formulation by a proper matlab code. The experimental model was conducted, in order, to compare and validate the numerical simulation.

3.1 Numerical simulation

The studied physical system was separated in two uncoupled subsystem: a rigid walled cavity and a plate supported in two sides. The rigid walled enclosure cavity presents maximum dimensions of 560mm x 325mm x 250mm and the plate has dimensions L1 x L2, where L1 = 400mm and L2 = 325mm, and thickness is 3mm. Table 1 shows the materials properties used in the two subsystems. The modal analysis of the uncoupled models is performed by using commercial finite element software. The enclosure cavity was modeled by fluid element, which has pressure as DOF, totalizing 88 elements (Fig. 2.a). The plate was modelated by shell element with six degrees of freedom (DOF's) per node in the Cartesian coordinate system (Fig. 2.b). It was discretized in 24 elements with 4 nodes per element, a boundary condition (supported) was applied at the two sides of higher length of the plate.



(a) (b)

Figure 2. Finite element modeling of the two subsystems: (a) rigid walled cavity and (b) steel plate

Table 1. Material properties

Material	air	steel
Poisson's Ratio		0.33
Young's modulus (N/m^2)		2.1 x 10^{11}
Sound Speed (m/s)	340	
Density (kg/m^3)	1.21	7860

In the analyzed frequency range (0 to 500 Hz) it was found a total of seven uncoupled structural modes and two uncoupled acoustic modes. The related natural frequencies are listed in Table 2.

Table 2. Natural frequencies of the both subsystems

Mode	1	2	3	4	5	6	7
Flexible plate	71.25	105.5	207.27	290.26	329.38	388.8	444.49
Cavity	0	398					

The response of the model was obtained by aplying a point force at position (L1/2, L2/3) of the plate. No acoustic source was considered in this case, $\dot{q}_{vol} = 0$. Eq. (11) and (12) were combined to obtain the complex amplitude coeficients *a* and *b* leading to equation (19) and (20).

$$a = \left(I + A_n C B_m C^T\right)^{-1} A_n f \tag{19}$$

$$b = \left(I + B_m C^T A_n C\right)^{-1} B_m f \tag{20}$$

The coefficients in the above equations were calculated from the acoustical and structural modal parameters. The values of pressure were computed by using Eq. (09) and (11) and the values of normal velocity by Eq. (10) and (12).

3.2 Experimental model

The experimental modal analysis was performed. The test was conducted in a two-side simply supported plate condition. The enclosure cavity was made in PVC thickness of 10mm to obtain rigid condition on the wall. The bottom of the cavity is backed by a steel plate of 3mm to obtain the vibro-acoustical effect. The plate placed in the bottom of the enclosure cavity was excited by a shaker with random force in the frequency range of interest (0 to 500Hz). A force sensor placed on the excitation point and an accelerometer roving on the measuring point of the plate was used to obtain the FRF(s). An integrator was used to obtain velocity response. Table 3 shows the instrumentation and material used in the experiment.

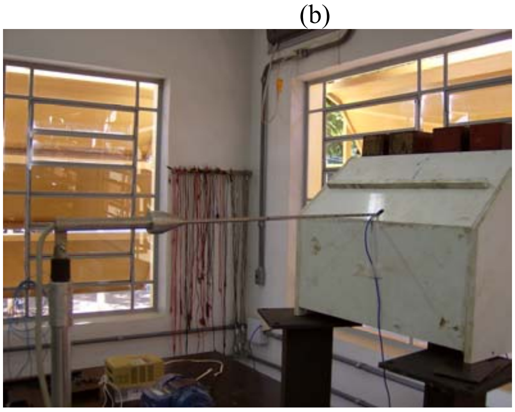
Table 3. Instruments and material used in the experiment

Item	Specifications
Acquisitions System	Data Physics Corporation Signal Calc Ace – 32Bits Maximum frequency range: 20 KHz 2 inputs-2 outputs
Axial accelerometer	PCB Piezoeletronics Type 325C68
Microphone	Robotron Type MK 201
Pre-amplifier (Mic.)	Robotron Type MV 201
Sound pressure meter	Robotron Type 00026
Signal conditioner	PCB Piezoeletronics Type 480E09
Integrator	PCB Piezoeletronics Type 480B10
Shaker	Operation frequency band: 15 Hz to 5 KHz Transition factor: 15 N/A Maximum peak: ± 3mm
Force Sensor	PCB Piezoeletronics Type 208C02
Computer	Pentium/128 MB-RAM

The measurements were realized by using the DATA PHYSICS System Acquisition. The accelerometer was calibrated in the laboratory by the handheld shaker calibrator, the sound pressure meter was calibrated by a pistonphone and the sensitivity of the force sensor was taken from the manufacturer specification sheet. The FRF(s) were measured in 21 point with the accelerometer and in 5 points with the microphone, the acquisition was made using 100 averages. Figure (3) shows the experimental setup of the modal test.

(a)

(b)



(c)

Figure 3. Experimental model. (a) general view (b) detailed of the connection plate – shaker (c) probe microphone

Two cases of measured FRF(s) (velocity and sound pressure) are present herein. In the first one, velocity was measured at drive point (L1/2, L2/3), in the second one, acoustic pressure was measured in an interior point of the cavity, both cases it used structural excitation force. Figure (4) shows the superimposed experimental and numerical calculate FRF(s) of the model for the drive point. For the simulation of the numerical FRF(s), it was considered a unity structural excitation force. The location of the excitation point is symmetric in one side that means some modes are do not excited. However, in the experiment text, some of these modes may be appears in the response since it difficult to exert the excitation in the exact defined exaction point. This does not occur in the numerical analysis, as can be seeing in the Fig. (04).

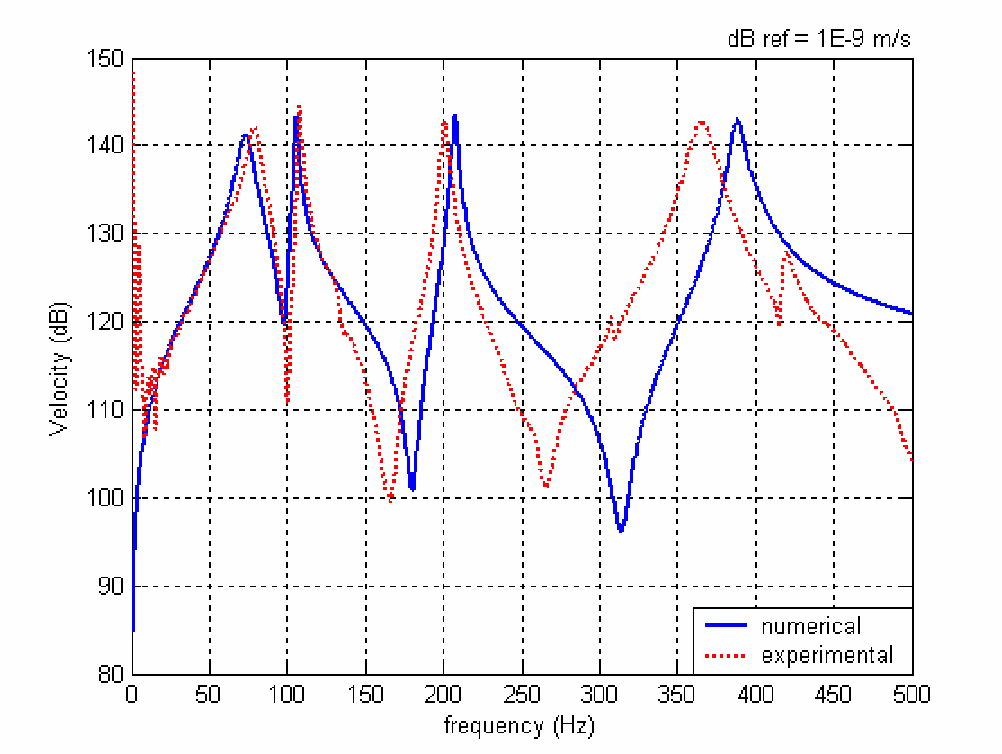


Figure 4. Numerical and experimental vibro-acoustic velocity response in the drive point.

In the second case, due to the limitation of the laboratory and the lack of an anechoic chamber, the test was conducted in the early morning, when the external sound is more quite. In this case, the measured FRF present a higher noise level compare with the case before, but it is still possible to observe the representation of the numerical model compared with the experimental one. The superimposed experimental and numerical FRF(s) for the sound pressure level of the vibro-acoustical system is shown in Fig. (05).

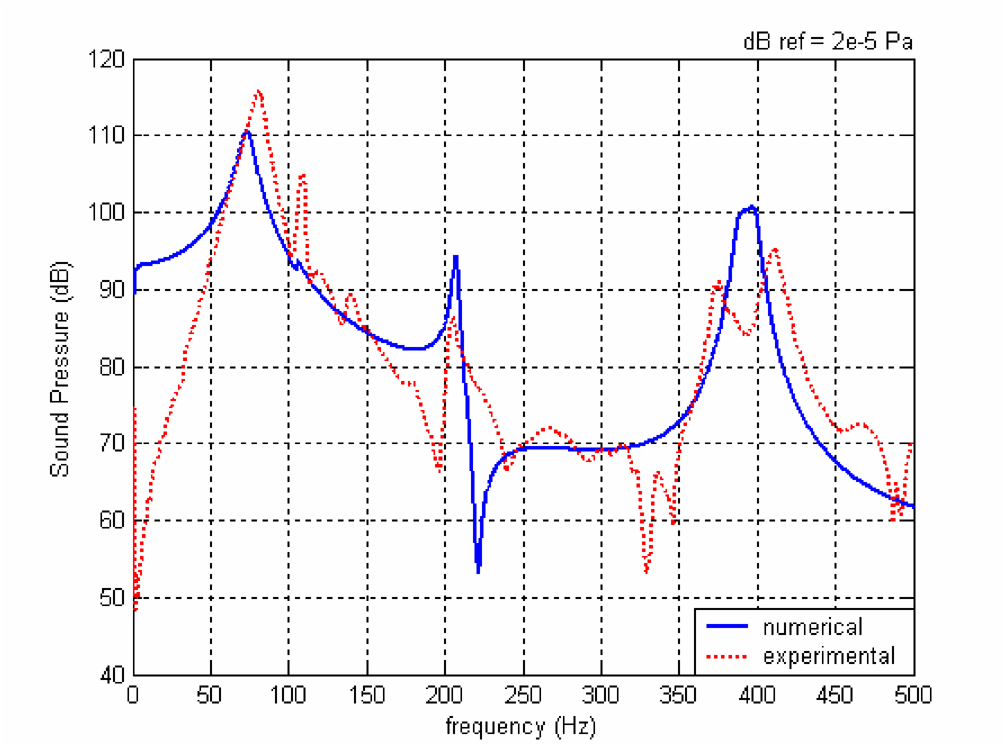


Figure 5. numerical and experimental vibro-acoustic sound pressure response in the interior of the cavity. The reference value for the pressure is $20\mu Pa$.

4. CONCLUSIONS

This paper discussed the solution of the vibro-acoustical problem using a compact matrix formulation based in a combination of the uncoupled modal parameters of the model. The uncoupled acoustic and structural modal parameters are obtained by finite element method. In the numerical simulation it used a commercial finite element code and a proper matlab code. Solely structural force excitation was adopted in this analysis. An experimental modal analysis was performed to validate the approach. Velocity and sound pressure was measured and compared with the numerical predictions. The agreement between the FRF(s) shows the potential of the compact matrix formulation to solve vibro-acoustical coupled systems.

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