A MULTI-OBJECTIVE STUDY FOR TRANSONIC AIRFOIL DESIGN

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Abstract.
The optimization process of engineering design usually involves problems in which the target has different objectives. Specifically in the airfoil design, the performance parameters used as targets, influence each other, competing among them. The optimization problems which deals with some objectives simultaneously are called multi-objective optimization problems. These techniques relates the domination of the objective targets, finding the called Pareto's front. In airfoils optimization problems, where there are competition among the target objectives, the multi-objective technique is extremely appropriate to find a family of good candidates to global solutions. In this work a multi-objective technique is applied to a set of airfoil solutions. These solutions were obtained by numerical simulations of the Euler equations. The Pareto's front was defined, generating excellent candidates for the problem approached. The target objectives aimed the maximization of the aerodynamic efficiency, the minimization of the pitching moment intensity and the restriction of the lift coefficient. The obtained optimized airfoil set has presented good characteristics, confirming that the multi-objective technique is a good tool in airfoil design.

Keywords: *Optimization, Euler equations, Finite volume method, multi-objective function.*

1. INTRODUCTION

For the development of modern aviation, researches are required to improve the aircraft performance. The most important motivation for this is related with the fuel consumption reduction. According to Mair W. A. and Birdsall D. L. (1998), in transonic airplanes, the specific range of an airplane is defined by Eq. 1:

$$r_a = \frac{V}{cW} \cdot \frac{L}{D} \tag{1}$$

where r_a is the specific range, W is the airplane weight and c is the specific consumption. Considering the air temperature constant, V is proportional to Ma, where Ma is the Mach number. So to improve r_a in a fixed Ma, is necessary to maximize $\frac{L}{D}$. Thinking at a constant weight and speed, the lift coefficient (Cl) of an airplane is defined at cruise speed. So to make the $\frac{L}{D}$ better is necessary the reduction of drag coefficient (Cd).

To perform the stabilization of the the airplane, its is necessary to use a horizontal tail. This device of aircraft generate a down force that reduce the total lift and generate a trim drag. To obtain the reduction of this drag and of the down force, it is necessary to reduce the intensity of the pitching moment (Cm).

The main aerodynamic structure of an aircraft, and the one which determine its performance, is the wing. The wing is responsible to generate the lift and for a great part of the drag. The most important geometric feature, which defines the aerodynamic, is the wing profile. Many works have been done in the development of new methods to optimize airfoil shapes and wing planforms with the objective of maximization of the aerodynamic efficiency (Azevedo J. L. F. Antunes A. P. and Santos L. C. C., 2003; Azevedo J. L. F. Antunes A. P. and Santos L. C. C., 2004; Holst T. L.,2004; Obayashi S., 1995; Oyama A. et al., 2000; Oyama A. et al., 2000; Oyama A., Obayashi S., and Nakamura T., 2001; Ray T. and Tsai H. M., 2004; Song W. et al., 2003;

In the commercial aviation, the high-speed aircraft with supercritical wings are predominant and the challenge is to minimize shock wave effects which increase the drag during cruiser flight. Studies have been addressed to determine good methods to optimize airfoils during design. Two different approaches are often employed in the aerodynamic design (Song W. and Keane A. J., 2004): the inverse design and the direct numerical optimization (DNO).

The first method, named inverse design, tries to find out a geometry which produces a prescribed pressure distribution. The second, named the direct numerical optimization (DNO) method, considers a set of geometries and an aerodynamic analysis code, in an iterative process, to obtain an optimum design (Yamamoto K. and Inoue., 1995).

Other works consider the unconstrained single-objective airfoil design and constrained design (Azevedo J. L. F. Antunes A. P. and Santos L. C. C., 2003; Souza L. F., Cuenca R. G. and Mello R. F., 2006). Some of them, analyze optimization problems using evolutionary algorithms (EA), genetic algorithms (GA) with real number encoding, and hybrids comprised of GA and gradient-based methods. Constrained single-objective airfoil design problems have also considered solutions based on GA such as non dominated sorting genetic algorithm (NSGA), multi-objective GA, and NSGA coupled with artificial neural networks (Ray T. and Tsai H. M., 2004; Azevedo J. L. F. Antunes A. P. and Santos L. C. C., 2004). Marler R. T. and Arora J. S. (2004) presents a good survey about multi-objective optimization methods.

A relevant question in the aircraft design is how to consider the multi-objective optimization to improve performance measures such as the lift, drag and others. Many researchers adopted GA's and EA's to commit these needs. These algorithms have been also successfully applied to aerodynamic shape optimization problems such as airfoil shape design (Quagliarella D. and Cioppa A. D., 1994; Yamamoto K. and Inoue., 1995), Multi-element airfoil shape design (Cao H. V. and Blom G. A., 1996), subsonic wing shape design (Obayashi S. and Oyama A., 1996) and supersonic wing shape design (Oyama A. et al., 1999). Besides, these algorithms also aims at solving non-linear problems.

Motivated by the solutions provided by genetic algorithm, this paper proposes the utilization of dominated multi-objective rank to optimize the aerodynamic performance of transonic airfoils. A Computational Fluid Dynamics (CFD) code was used to solve the governing equations. The multi-objective optimization is applied over airfoil generations until it reaches a profile which satisfies the performance needs. In the present work, the Parsec parametric airfoils (Sobieczky H., 1998) were adopted.

This paper is divided as follows: section 2.1 shows the aerodynamic formulation used to evaluate the airfoil performance; section 2.2 that explain the multi-objective problem and the dominance of solution; section 3. explains the numerical methods used of this work, the Jameson at 3.1and the Genetic Algorithm at 3.2; section 4.describes the geometry family that defines the airfoil shape; section 5. shows the code validation and the results obtained for three different fitness functions. The last section presents the main conclusions of the present work.

2. Formulation

2.1 Governing Equations

The Navier-Stokes (NS) equations represent the mathematical model for any kind of flow. For transonic flow simulations over airfoils, one of the most important phenomena is related with the compressibility, ie. shock wave. When using the supercritical airfoil, the interaction between boundary layer and shock wave is important to find the exact shock location and to analyze moderns wings profiles. These profiles are called *Natural Laminar Flow* (NLF), were the reduction of drag is caused by an increment of laminar flow percentage ($\geq 30\%$ of chord) on the foil surface (Selig W. S., Maughmer M. D., and Somers D. M., 1995). However, the viscous effects were neglected in the current work, simplifying the NS equations. This simplification leads do the called Euler equations. These equations in conservative form and in Cartesian coordinates are:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0, \tag{2}$$

where

$$Q = [\rho, \rho u, \rho v, e]^t, \tag{3}$$

$$E = \left[\rho u, \rho u^2 + p, \rho uv, (e+p)u\right]^t, \tag{4}$$

$$F = \left[\rho u, \rho uv, \rho v^2 + p, (e+p)v\right]^t, \tag{5}$$

$$e = \rho(e_i + \frac{1}{2}(u^2 + v^2)), \tag{6}$$

$$p = \rho RT, \tag{7}$$

$$e_i = \frac{p}{(\gamma - 1)\rho}. \tag{8}$$

These equations were simulated in a numerical code. The details of the numerical code is presented in the next section.

2.2 Multi-Objective Problem

The Multi-Objective Problem (MOP) is and problem that involves more than one objective function ($f_1(x)$). The MOP's should be solved by Multi-Objective Optimization (MOO). According to Marler R. T. and Arora J. S. (2004), there are many ways to solve a MOO. In the present work, two different MOO methods are compared: the Pareto rank (PR) and the weighted product methods (WP).

The PR method uses the concept of dominance of multi-objective solutions. A solution is called not-dominated by other one when these conditions, shown in Eq.9, are satisfied. These condition mean that when comparing the solution with all group of solution, its need to be better or equivalent at all objectives and better at least in one. In these equations f_i is the i-th objective, and x_1 and x_2 are the design variables vectors. For a group of solutions $F(x_j)$, where $F(x_j) = \{f_1(x_j)..f_N(x_j)\}$, with N the number of objectives, the solution is called Pareto solution $F(x^*)$ if it is non-dominated for all solutions group.

$$
\begin{aligned}
condition1 &\;:\; f_i(x_1) \leq f_i(x_2) \text{ for all } i \\
condition2 &\;:\; f_i(x_1) < f_i(x_2) \text{ for any } i
\end{aligned} \tag{9}
$$

In PW one have to choose a product weight global fitness to represent the solution. It is illustrated by Eq.10. At the equation, p_i is the weight of objective i. An group of optimizations with different weight generate a Pareto front. This method is often used as a single-objective optimization and is appropriate if it is known the adequate p_i's.

$$
F(x) = \prod_{i=1}^{N} f_i^{p_i}(x) \tag{10}
$$

At this paper, the dominance of solutions evaluated by GA (described below) performed with PR and WP are compared.

3. Numerical Methods

To accomplish the present work, it was necessary the implementation of some numerical methods to simulate and optimize the wing section leaving to computer the job to calculate and evaluate the performance of airfoil and decide which are the best results. This section is divided in two parts, the first one describes the numerical method used to solve the Euler equations and the second describes the Genetic Algorithm adopted.

3.1 Euler solver

To evaluate the performance of a wing in a inviscid compressible flow, it is necessary to simulate the flow around the airfoil, using any method of solution for the Euler equations. There are a lot of these methods, that differ about the mesh, the discretization of equation and the accuracy.

In the current work, it was adopted a finite volume (FV) method proposed by Jameson (Jameson A., Schimidt W., and Turkel E., 1981; Jameson A.,1982 and Jameson A. and Mavriplis D., 1986) on a O structured mesh. This mesh allows an easy definition of the geometry around the airfoil and a fast convergence and calculation. The mesh generation were performed by an elliptic partial differential equation. To improve the convergence speed and the stability of method, it was implemented the residual smooth leading the increase of CFL, and a free stream correction for reduce the computational domain with less lost of precision. A detailed description of Jameson can be found in Hirsch C. (1981).

3.2 Genetic Algorithms

Genetic Algorithms (GA) are applied as search and optimization techniques in several domains. These algorithms are based on nature select mechanisms focusing at survival of the most capable individuals. GA does not always give the best possible solution, however provides good local solutions and is robust for non-linear problems.

The problem solution using genetic algorithms involves two different aspects: solution encoding into the form of chromosomes, where each chromosome represents a possible solution, and a fitness function or rank applied to evaluate the solution.

Different encoding techniques can be used for different kind of problems, such as binary strings, bitmaps, real numbers, and so on. The fitness function is responsible for the evaluation of possible solutions. This function receives a chromosome as parameter and returns a real number, informing the quality of the obtained solution, e.g., how adequate is the solution for the currently studied problem.

The most adequate chromosomes are identified and stored during the evolution process. The weakest ones, on the other side, are eliminated. Different techniques can be applied for the identification of the best chromosomes, such as the proportional selection, ranking selection and tournament-based selection (Back T., Fogel D. B., and Michalewicz Z.,1999a,Back T., Fogel D. B., and Michalewicz Z.,1999b).

In the proportional selection, individuals are transferred to the next generation according to their fitness or rank value probability proportion. One of the possible implementations of this technique consists in the usage of a roulette, divided into N parts, N being the number of individuals (chromosomes) in the current population. The size of each part is proportional to the fitness (or rank) value of each individual. The roulette is rotated N times afterward, and at each turn the appointed individual is selected and inserted into the new population.

Ranking-based selection can be subdivided into two steps. During the first one, the solutions are ordered according to their fitness function values. Once the list is ordered, each individual receives a new fitness function value equivalent to its position in the ranking. After that, a procedure that selects the individuals, according to their ranking position, is applied. Thus, the individuals with better ranking position have more chances to be selected.

In the multi-objective context, the individual rank is defined by the dominance among solutions (Van Veldhuizen D. A., Coello C. A. and Lamont G. B., 2002). In a population of solution, the domination test is carried through and the non-dominated solutions are identified, afterwards, they are removed from the group and the test is performed again in order to find the solution group which presents one level of dominance. This process is repeated until there is no solution in the population. The rank is defined as follow: the non-dominated solutions receive the greater rank r, the solutions with one level of dominance receive rank $r-1$, and so on up to solutions ranked at 1.

Once selected the individuals for reproduction, it is necessary to modify their genetic characteristics using techniques known as genetic operators. The most common operators are crossover and mutation.

The crossover operator allows to exchange genetic material among two individuals, known as parents, combining their information in a way that provides a significant chance of creating new improved individuals (Hinterding R., 2000).

The single-point crossover operator is the most used. In order to apply it, two individuals (parents) are selected and two new individuals are created from them (children). A single random splitting point is selected in parent chromosomes, and the new chromosomes are created from the combination of the parents, as shown in Tab. 1. In this table, label (a) shows the parent individuals and the splitting point marked by | symbol. The new individuals created are shown in the same table with label (b), illustrating the crossover operator.

Table 1. Crossover operator

$X_1X_2\|X_3X_4X_5X_6$	$X_1X_2\|Y_3Y_4Y_5Y_6$
$Y_1Y_2\|Y_3Y_4Y_5Y_6$	$Y_1Y_2\|X_3X_4X_5X_6$
(a) Before the crossover	**(b) After the crossover**

The mutation operator is used for changing a single gene value for a new random one. When an individual is represented by a bitmap, this operation consists of a random choice of a chromosome gene and the swapping of its value from 1 to 0 (or from 0 to 1, correspondingly). The goal of the mutation operator is to maintain the population diversity, always allowing a chromosome to cover a significantly large result space (Hinterding R., 2000). It is usually applied at a low rate, as at high ones the results tend to be random.

4. Parametric airfoil shape

Aiming the generation of a big number of airfoils shapes for the use with GA, the parametric airfoils families (Parsec) were adopted. These families are very appropriated because allow the generation of different shapes using a limited number of parameters. These parameters are used for the GA as the chromosomes of the individuals. According to Song W. and Keane A. J. (2004) and Ray T. and Tsai H. M. (2004) there are many functions proposed to evaluate the shape, like: analytical functions (PARSEC, NACA, etc); splines, B-splines and Bezier curves via interpolation methods; and others.

The Parsec family of wing sections defined by 11 geometric parameters was adopted. According to Sobieczky H. (1998), a blend of two or more airfoils schemes can be used to improve the number of shapes representations and consequently the possibilities of geometries. To make a simple analysis, only the Parsec shapes were analyzed. This family is detailed bellow.

4.1 The Parsec Family

The PARSEC representation is particularly attractive as it uses a small number of design variables, all of which are related to some properties of the shape (Ray T. and Tsai H. M., 2004). It parameterizes the upper and the lower airfoil surfaces using polynomials in coordinates X and Z as:

$$Z = \sum_{n=1}^{6} a_n X^{n-\frac{1}{2}} \tag{11}$$

where a_n are real coefficients. The parameters of a PARSEC representation include the leading-edge radius r_{le}, upper and lower crest heights Z_{UP}, Z_{LO} and location X_{UP}, X_{LO}, curvatures at the upper and lower crests Z_{XXUP}, Z_{XXLO}, trailing-edge thickness ΔZ_{TE} and ordinate Z_{TE}, and direction and wedge angle α_{TE}, β_{TE}. The parameters are schematically shown in Fig.1.

For this paper, the design variables' limits used are presented at Tab.2, according to Ray T. and Tsai H. M. (2004), except by the lower r_{le} limit which the original is 0.0085.

The angle of attack of airfoil is fixed at zero. The reason is to reduce the number of variables projects and because the objective of this studie is to compare methods of MOO.

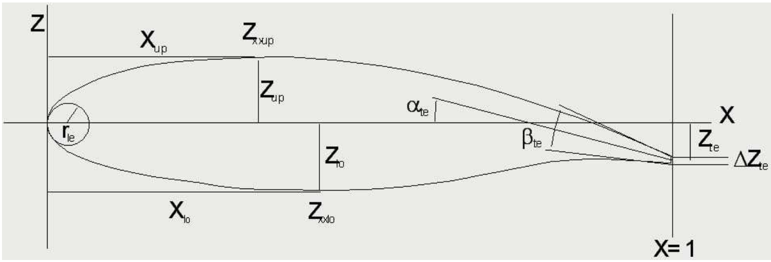

Figure 1. Variables in the Parsec representation scheme

Table 2. Design variables limits adopted

Param.	r_{le}	X_{up}	Z_{up}	Zxx_{up}	X_{lo}	Z_{lo}	Zxx_{lo}	α_{te}	β_{te}	Z_{te}	dZ_{te}
Lower	0.0055	0.3	0.05	-0.5	0.3	-0.05	0.55	8^o	10^o	0	0
top	0.01	0.5	0.12	0.1	0.45	0.1	1.35	17^o	11.5^o	0.05	0

5. Results:

In this work, the MOO was performed by the Pareto rank method and the weighted product method. Tree weighted product were used and compared with the Pareto rank method. The multi-objective problem was defined by:

Minimize: $f_1 = |cl - 0.2|$
$f_2 = cd$
$f_3 = |cm|$

Subject to:
$camber \leq 30\%$
$thickness \leq 20\%$

The cases analyzed was named: MOGA; SOGA 1; SOGA 2; SOGA 3. The MOGA is the Pareto rank GA. The others are the weighted product GA with tree different functions. The descriptions of any case follows below.

5.1 VALIDATION

The Euler solver was implemented using Fortran 77. The validation of the code was made comparing the coefficient of pressure over the foil RAE2822. The result is presented at Fig.2. As expected, the shock obtained numerically is stronger than the experimental result because the numerical code is based in Euler equations, and hence there is no boundary layer shock wave interaction. The experimental data Slater J. W. (2005) is at $\alpha = 2.31$ and $Mach = 0.725$.

5.2 MOGA

The MOO performed by MOGA has the configurations parameters: 100 individuals at population; Crossover probability of 0.6; and Mutation Probability of 0.3. No elitism was used at this work.

5.3 SOGA's

The configuration values and functions used for each SOGA is at Tab.3.

Table 3. Genetic optimization configuration for weighted product method.

	Population size	Cross over Prob.	Mutation Prob.	Function
SOGA 1	150	0.7	0.05	$\frac{L}{D} \cdot \frac{1}{(\lvert cl-0.2\rvert+1.0)\cdot(\lvert cm\rvert+1.0)}$
SOGA 2	100	0.7	0.05	$\frac{L}{D}$
SOGA 3	100	0.7	0.05	$\frac{1}{cd\cdot(\lvert cl-0.2\rvert+1.0)\cdot(\lvert cm\rvert+1.0)}$

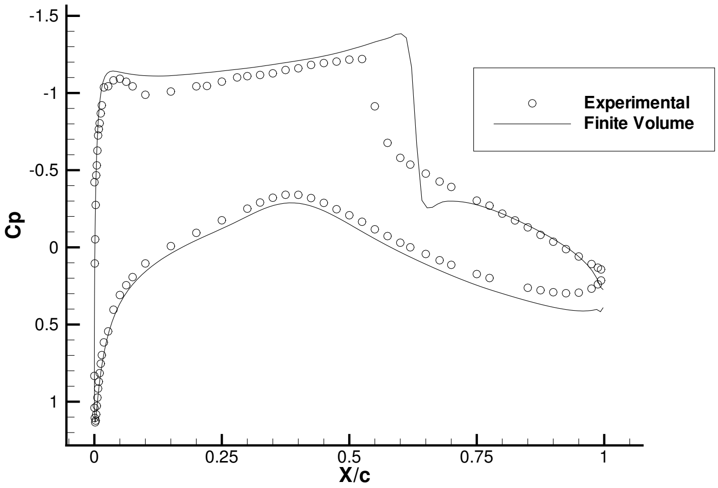

Figure 2. The Cp distribution over Airfoil RAE2822.

5.4 Paretos front

The Pareto front obtained for each case are presented at Fig.s from 3 to 5.

The results show that the SOGA optimizations were more apropriated to find solution at Pareto than MOGA. Comparing the dominance among the different results of optimizations, the SOGA 1 found the greater number of solutions at non-dominance. The MOGA gived the worse result. This can be infered by analising the figures, but other fact that is visible is that the search at domain of solution is performed differently by each method. Its show that the characteristic of all methods are needed, therefore a method that search through all domain. According to Obayashi S. (1998) the MOGA with fitness sharing method and *best-N* elitism have these characteristics defining the fitness though a union of dominance and objectives.

Table 4. Number of Solution at Pareto (a) and the number of solution at Pareto that dominate the Pareto from of others cases (b).

	Number of solution at Pareto (a)	Number of Pareto solution dominant
MOGA	18	3
SOGA 1	53	44
SOGA 2	34	12
SOGA 3	39	10

6. CONCLUSION

To solve optimization problems at aerodynamic design, many methods are proposed, and the performance of the method differ for each problem. The genetic algorithm are one of the most used methods and it can lead to the multi-objective optimization by many ways. Two of these method, weight product of objectives and the Pareto Rank, were compared to evaluate which one is best for the case studied. The genetic algorithm was used with Euler equations solver to perform the objectives functions. The results obtained here show that the weight product method performed better the search of Pareto front. The results show too that different method perform the search though the solution domain by different ways. According to Obayashi S. (1998) the use of MOGA with fitness sharing and $best-N$ elitism make better way to perform the optimization for aerodynamic airfoil.

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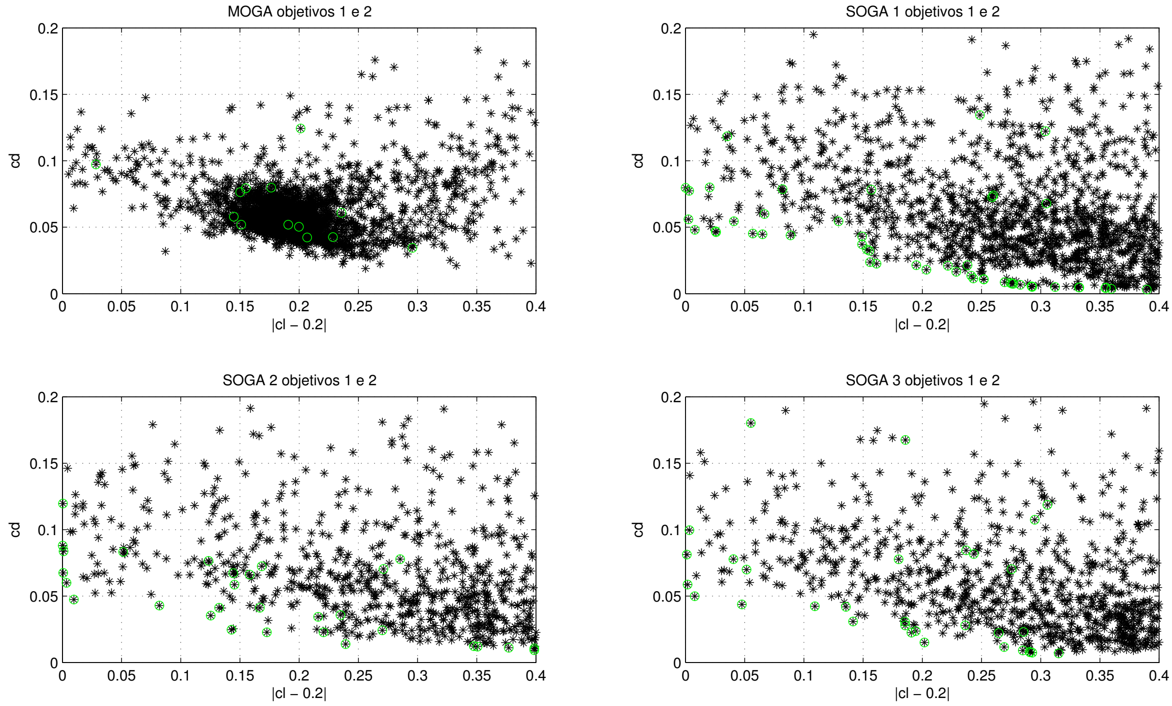

Figure 3. All Solutions (black cross) and Pareto front (green circle) projection for objectives f_1 and f_2

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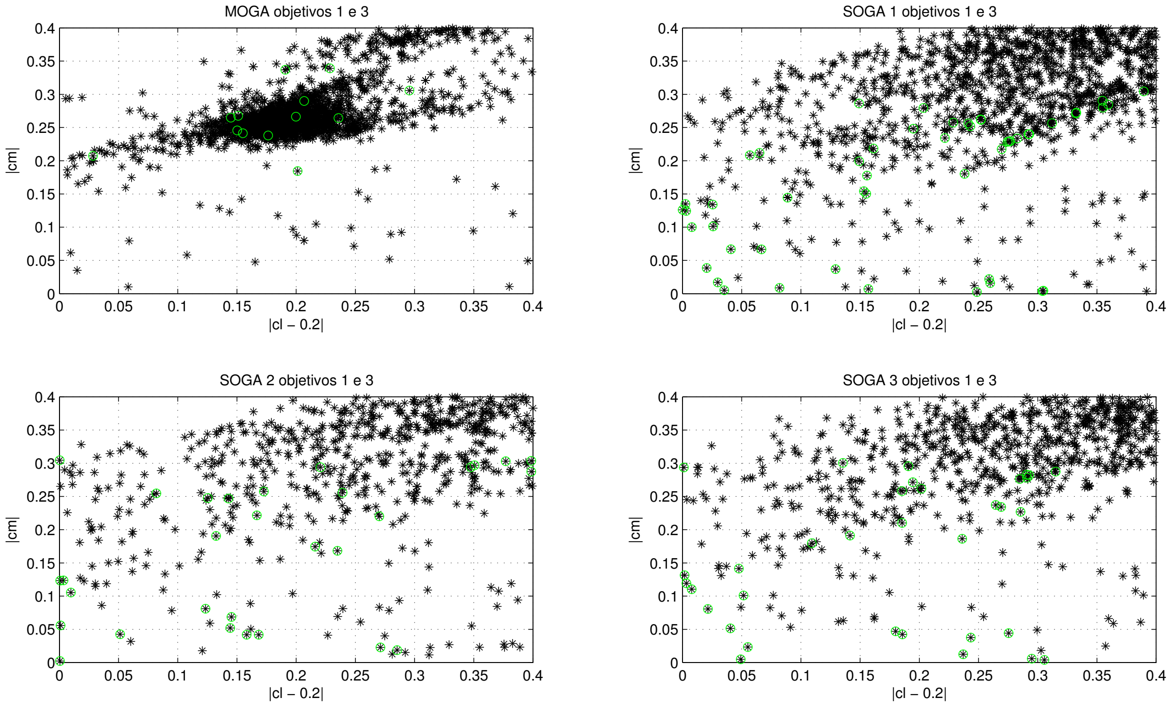

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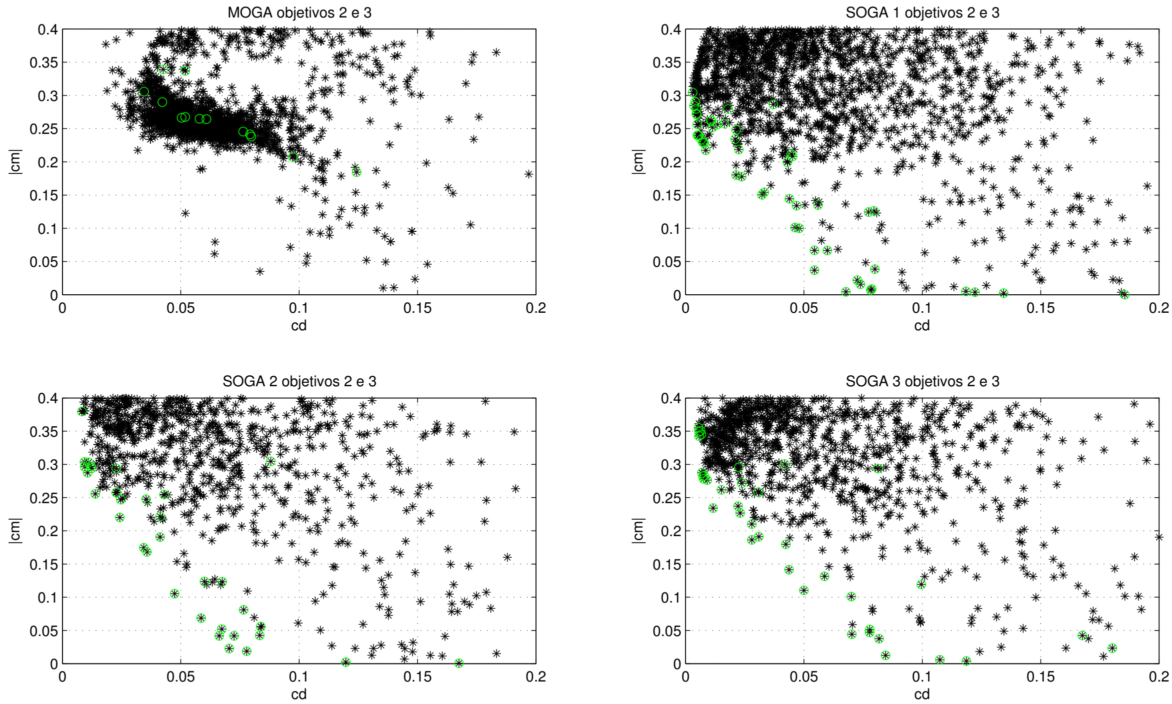

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