ON NONLINEAR DYNAMICS OF A NON-IDEAL RAYLEIGH-DUFFING TYPE VIBRATING SYSTEM

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Abstract. *In this paper we present analytical and numerical investigations of the dynamics interactions between cantilever beam, modeled with nonlinear damping and stiffness (Rayleigh´s and Duffing equation) and an electric motor with eccentricity and of limited power supply. We analyze the non-ideal system as two coupled nonlinear differential equations through of a perturbation method and tools as time history, phase portrait, Lyapunov exponents to determine some interesting nonlinear phenomena as Sommerfeld effect and nonlinear resonance including periodic, quasi-periodic and chaotic regime in non-stationary and steady process.*

Keywords: *non-ideal system, nonlinear system, averaging method, nonlinear dynamics*

1. INTRODUCTION

The excitation of the vibration systems analyzed here is always limited; on the one hand by the characteristics of a particular energy source and by other hand it is limited by the dependence of the motion of the oscillating system on the motion of the energy source. Note that this connection is expressed by coupling between the differential equations of motion of the vibrating system and the source.

When the excitation is not influenced by the response of a vibrating system, it is said to be an ideal system (traditional ones), or an ideal energy sources. Formally, the excitation may be expressed as a pure function of time. For an example, we consider a system of m mass, c damping, k stiffness and driven by a harmonic excitation $F = A\cos(\omega t)$ having frequency ω and amplitude A (in this case they are constants). Then, the dynamics of the ideal system may be described by the following classical (linear) differential equation for the displacement $x(t)$.

$$m\ddot{x} + c\dot{x} + kx = A\cos(\omega t) \tag{1}$$

For non-ideal dynamical systems, one must add an equation that describes how the energy source supplies the energy to the equations that governs the corresponding ideal dynamical system. We add the nonlinear damping and stiffness terms in the equation of the oscillating system. In this case the non-ideal nonlinear system can be described by the following coupled differential equations for the displacement $x(t)$ and angular displacement $\varphi(t)$:

$$\begin{aligned} &m\ddot{x} + f_1(x,\dot{x}) + f_2(x) = F(\varphi,\dot{\varphi},\ddot{\varphi},q) \\ &I\ddot{\varphi} + H(\dot{\varphi}) = L(\dot{\varphi}) + R(\varphi,\dot{\varphi},\ddot{x},q) \end{aligned} \tag{2}$$

where m is structure mass, $F(\varphi,\dot{\varphi},\ddot{\varphi},q)$ express the action of the source of energy on the oscillating system (angular velocity of motor is not constant), q is unbalanced coefficient (electric motor with eccentricity), I is the moment of inertia of mass, the function $R(\varphi,\dot{\varphi},\ddot{x},q)$ express the action of the oscillating system on the source of energy, the function $H(\dot{\varphi})$ is the resistive torque applied to the motor, the function $L(\dot{\varphi})$ is the driving torque of the source of energy (motor), the function $f_1 = (c_1 + c_2\dot{x}^2)\dot{x}$ is called Rayleigh's function and it describes a nonlinear damping of the system and the function $f_2 = k_1 x + k_2 x^3$ is called Duffing function and it describes a nonlinear stiffness.

Note that, usually, the inductance is much smaller than the mechanical time constant of the system and in stationary regime we can take $L(\dot{\varphi})$ as (linear) $L(\dot{\varphi}) = a - b\dot{\varphi}$, where are a related to voltage applied across the armature of the DC motor, that is, a possible control parameter of the problem and b is a constant for each model of DC motor considered.

For non-ideal systems if we consider the region before resonance on a typical frequency- response curve, we note that as the power supplied to the source increases, the RPM of the motor increases accordingly. However, this behavior does not continue indefinitely. That is, closer the motor speed moves toward the resonant frequency the more power require increasing the motor speed. More formally, a large change in the power supplied to the motor results in a small change in the frequency, but a large increase in the amplitude of the resulting vibrations. Thus, near resonance it appears that additional power supplied to the motor only increases the amplitude of the response while having little effect on the RPM of the motor.

We remarked that Jump phenomena and the increase in power required by a source operating near resonance are manifestations of a non-ideal energy source and are often referred as Sommerfeld effect (Sommerfeld, 1904) (of getting stuck in resonance: the motor may not have enough power to reach higher regimes with low energy consumption as most of its energy is applied to move the structure and not to accelerate the shaft.), that is, the structural response provide a certain energy sink (one of the problems often faced by designers is how to drive a vibrating system through resonance and avoid the energy sink described by Sommerfeld.).

The study of non-ideal vibrating systems, that is, when the excitation is influenced by the response of the system, has been considered a major challenge in theoretical and practical engineering research.

We mention that for more details on non-ideal systems theory, see (Kononenko, 1969), (Balthazar et al., 2003; Balthazar et al., 2004; Dantas and Balthazar 2006; Dantas and Balthazar, 2007), without undeserved others.

We organize this paper as follows. In Section 2, we present the mathematical model used and the derivation of the governing equations. In section 3, we obtain an analytical solution to the analyzed problem, by using an average procedure to the non-ideal dynamics of the adopted oscillating problem. In section 4 we did some acknowledgements. Finally we list the main bibliographic used references.

2. MODEL OF THE NON-IDEAL NONLINEAR SYSTEM

We consider the non-ideal problem consisting of a nonlinear cantilever beam supporting an unbalanced motor with limited power at its free end as is shown in Fig. 1. The motor is composed of an unbalanced mass m_0, an eccentricity e, of an angular displacement of the rotor ϕ, a resistance R, an inductor L, an electric current i and a voltage U. Its representative mathematical formulation is considered as a nonlinear differential equation of Rayleigh-Duffing type forced by a dynamic equation of electric motor.

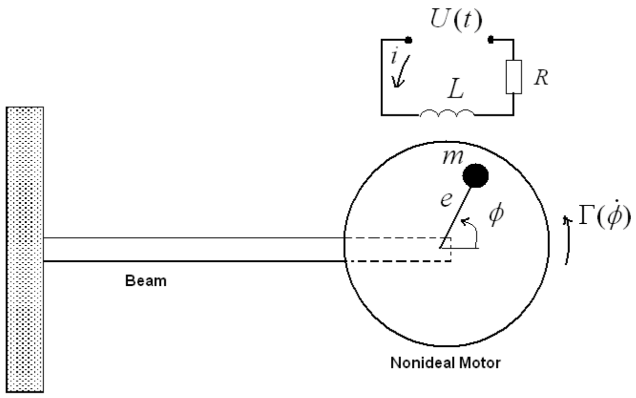


Figure 1. Non-ideal cantilever beam.

In this paper, we will extend the work of (Bolla et al., 2006) introducing the cubic nonlinear damping and the reduced model exhibits on (Warminski, et al., 2001) and (Warminski and Balthazar, 2003) with parametric coefficient $\mu = 0$. We obtain a nonlinear differential equation of Duffing-Rayleigh type driven by a non-ideal energy source in a dimensionless form:

$$\ddot{X} + (-\alpha + \beta \dot{X}^2)\dot{X} + (1 + \gamma X^2)X = q_1(\ddot{\phi}\cos\phi - \dot{\phi}^2 \sin\phi)$$
$$\ddot{\phi} = \Gamma(\dot{\phi}) + q_2 \ddot{X}\cos\phi \qquad (3)$$

where we assume that motor's torque is a linear function of its angular velocity $\Gamma(\dot{\phi}) = V_m - C_m\dot{\phi}$, V_m is a control parameter and it can be changed according to the voltage of the DC motor, C_m is a constant for each model of DC motor considered, α is coefficient of linear damping, β is nonlinear damping coefficient, γ is nonlinear stiffness coefficient, q_1 and q_2 unbalanced coefficients, the natural frequency is one unity, X is the coordinate oscillatory motion of the considered cantilever beam and $\dot{\phi}$ is the angular velocity of DC motor.

3. AN APPROXIMATE ANALYTICAL SOLUTION

In this section, we use the method of averaging (Nayfeh, 1979; Palacios, 2002) to determine an approximate solution of Eq. (3):

$$\ddot{X} + X = \varepsilon\{\hat{\alpha}\dot{X} - \hat{\beta}\dot{X}^3 - \hat{\gamma}X^3 + \hat{q}_1(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi)\}$$

$$\ddot{\phi} = \varepsilon\{\hat{\Gamma}(\dot{\phi}) + \hat{q}_2\ddot{X}\cos\phi\} \tag{4}$$

where $\alpha = \varepsilon\hat{\alpha}$, $\beta = \varepsilon\hat{\beta}$, $\gamma = \varepsilon\hat{\gamma}$, $q_1 = \varepsilon\hat{q}_1$, $q_2 = \varepsilon\hat{q}_2$, $\Gamma(\dot{\phi}) = V_m - C_m\dot{\phi} = \varepsilon\hat{\Gamma}(\dot{\phi})$. Here ε, $0 < \varepsilon \ll 1$, is an arbitrary small parameter. To this end, we apply the method of variation of parameters and let

$$X = a\cos(\phi + \xi) \tag{5}$$

$$\dot{X} = -a\sin(\phi + \xi) \tag{6}$$

$$\dot{\phi} = \Omega \tag{7}$$

In the regime near resonant the difference between the excitation frequency is close to the natural frequency:

$$\Omega = 1 + \varepsilon\sigma \tag{8}$$

where σ is a detuning parameter.

The first derivative of Eq. (5)

$$\dot{X} = \dot{a}\cos(\phi + \xi) - a(\Omega + \dot{\xi})\sin(\phi + \xi) \tag{9}$$

it follows from Eq. (6) and Eq. (9) that

$$\dot{a}\cos(\phi + \xi) - a\dot{\xi}\sin(\phi + \xi) = a(\Omega - 1)\sin(\phi + \xi) \tag{10}$$

Differentiating Eq. (6) gives

$$\ddot{X} = -\dot{a}\sin(\phi + \xi) - a(\Omega + \dot{\xi})\cos(\phi + \xi) \tag{11}$$

Substituting Eqs. (8), (9) and (11) into Eq. (4) yields

$$-\dot{a}\sin(\phi + \xi) - a\dot{\xi}\cos(\phi + \xi) = a(\Omega - 1)\cos(\phi + \xi) + \varepsilon f_1 \tag{12}$$

where $f_1 = -\hat{\alpha}a\sin(\phi + \xi) + \hat{\beta}a^3\sin^3(\phi + \xi) - \hat{\gamma}a^3\cos^3(\phi + \xi) + q_1\Omega^2\sin\phi$.

Resolving the Eqs. (10) and (12) as a system of equations with $\dot{a}$ and $\dot{\xi}$ and using the trigonometric identities: $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$, $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$, $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$, $\sin^3\theta\cos\theta = \frac{1}{8}(2\sin 2\theta - \sin 4\theta)$, $\cos^3\theta\sin\theta = \frac{1}{8}(2\sin 2\theta + \sin 4\theta)$ and considering the second equation of Eq. (4), gives the following variational equations:

$$\dot{a} = -\varepsilon f_1\sin(\phi + \xi) = \varepsilon[\frac{1}{2}\hat{\alpha}a - \frac{3}{8}\hat{\beta}a^3 + \frac{1}{2}\hat{q}_1\Omega^2\cos\xi + A_1(a, \xi, \phi, \Omega)] \tag{13a}$$

$$a\dot{\xi} = -\varepsilon a\sigma - \varepsilon f_1\cos(\phi + \xi) = \varepsilon[-a\sigma + \frac{3}{8}\hat{\gamma}a^3 - \frac{1}{2}\hat{q}_1\Omega^2\sin\xi + A_2(a, \xi, \phi, \Omega)] \tag{13b}$$

$$\dot{\Omega} = \varepsilon[\hat{\Gamma}(\Omega) - \hat{q}_2 a\Omega\cos(\phi + \xi)\cos\xi + A_3(a,\xi,\phi,\Omega)] \tag{13c}$$

where $A_1(a,\xi,\phi,\Omega)$, $A_1(a,\xi,\phi,\Omega)$ and $A_1(a,\xi,\phi,\Omega)$ are small periodic functions defined by

$$\begin{aligned} A_1(a,\xi,\phi,\Omega) &= \frac{1}{2}\hat{\alpha}a\cos(2\phi+2\xi) + \frac{1}{8}\hat{\beta}a^3[\cos(4\phi+4\xi) - 4\cos(2\phi+2\xi)] \\ &\quad -\frac{1}{8}\hat{\gamma}a^3[2\sin(4\phi+4\xi) + \sin(4\phi+4\xi)] - \frac{1}{2}\hat{q}_1\Omega^2\cos(2\phi+\xi) \end{aligned}$$

$$\begin{aligned} A_2(a,\xi,\phi,\Omega) &= -\frac{1}{2}\hat{\alpha}a\sin(2\phi+2\xi) + \frac{1}{8}\hat{\beta}a^3[2\sin(2\phi+2\xi) - \sin(4\phi+4\xi)] \\ &\quad -\frac{1}{8}\hat{\gamma}a^3[\cos(4\phi+4\xi) + 4\cos(2\phi+2\xi)] + \frac{1}{2}\hat{q}_1\Omega^2\sin(2\phi+\xi) \end{aligned}$$

$$A_3(a,\xi,\phi,\Omega) = -\frac{1}{2}\hat{q}_2 a\Omega\cos(2\phi+\xi)$$

We determine a first approximation from the average equations of Eq. (13) (considering a, ξ and Ω to be constants over one cycle and integrate (average) the equations over one cycle), the result is

$$\begin{aligned} \dot{a} &= \frac{1}{2}\alpha a - \frac{3}{8}\beta a^3 + \frac{1}{2}q_1\Omega^2\cos\xi \\ \dot{\xi} &= -(\Omega - 1) + \frac{3}{8}\gamma a^2 - \frac{1}{2}q_1\frac{\Omega^2}{a}\sin\xi \\ \dot{\Omega} &= \Gamma(\Omega) - \frac{1}{2}aq_2\Omega\cos\xi \end{aligned} \tag{14}$$

Here the first equation describes the variation of the amplitude of the oscillation (the behavior of the envelope of the oscillatory motion of the coordinate X), the second equation describes the variation of the initial phase of the motion ξ), the third equation describes the variation of the frequency Ω (the average value of the angular velocity of the motor).

We first determine the amplitude, phase and average angular velocity of the response of Eq. (14) when $q_1 = q_2 = 0$ (there is not interaction between structure support and motor).

In this case Eq. (14) can be written as

$$\begin{aligned} \dot{a} &= \frac{1}{2}\alpha a - \frac{3}{8}\beta a^3 \\ \dot{\xi} &= -(\Omega - 1) + \frac{3}{8}\gamma a^2 \\ \dot{\Omega} &= \Gamma(\Omega) \end{aligned} \tag{15}$$

Solving the third equation of Eq. (15), yields

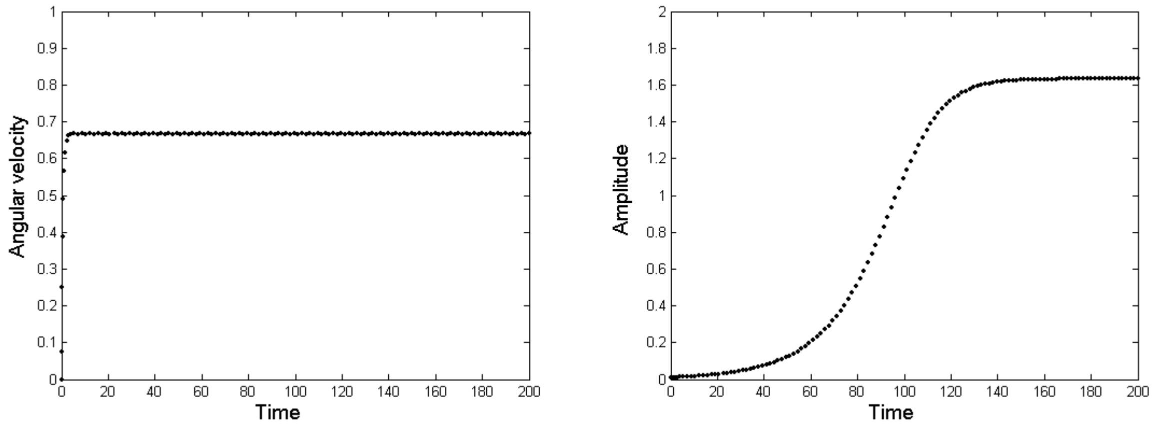
$$\Omega = \frac{V_m}{C_m} + c_1 e^{-C_m t}, \quad c_1 = \Omega_0 - \frac{V_m}{C_m} \tag{16}$$

Solving the first equation of Eq. (15), yields

$$a = \frac{\sqrt{\frac{\alpha}{2}}e^{\alpha(t+c_2)}}{(1+\frac{3}{8}\beta e^{\alpha(t+c_2)})^{1/2}}, \; c_2 = \frac{1}{\alpha}\ln\left(\frac{a_0}{\frac{\alpha}{2}-\frac{3}{8}\beta a_0^2}\right) \tag{17}$$

where a_0 and Ω_0 are the initial conditions.

Solving numerically the Eq. (15) with initial conditions $a = 0.01$, $\xi = 0$, $\Omega = 0$ and $\alpha = 0.1$, $\beta = 0.05$, $\gamma = 0.1$, $V_m = 1$ and $C_m = 1.5$. The results are shown in the Fig. 2 that verifies the Eqs. (16) and (17).

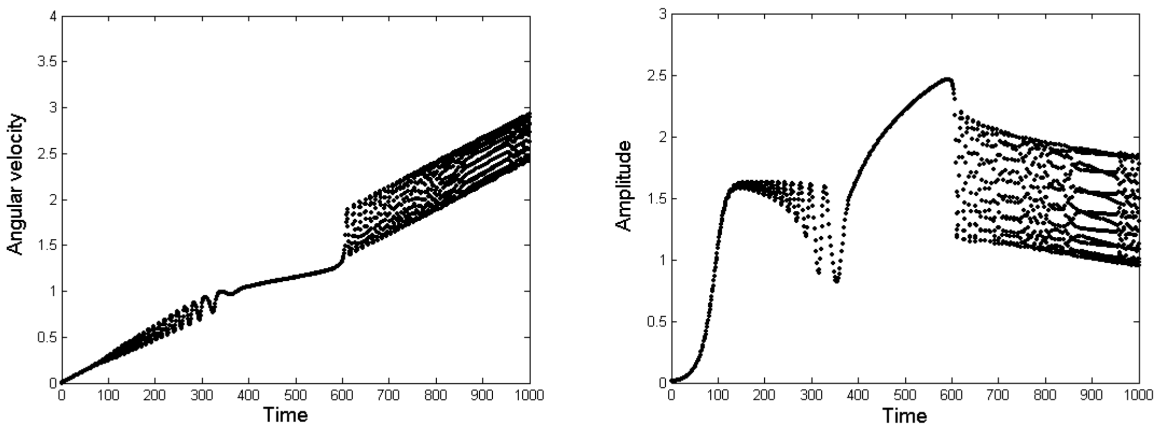


a) Average angular velocity Ω b) Amplitude a

Figure 2. Response of the Eq. (15).

Considering $q_1 \neq 0$ and $q_2 \neq 0$ (q_1=0.2 and q_2=0.3, case of interaction between cantilever beam and electric motor). In this case is response of Eq. (14).

The variation of the amplitude a and excitation frequency Ω (average angular velocity) with time in the non-stationary regime is shown in Fig. 3 during the passage through resonance. In the time [200, 350] and [600, 1500] exist a synchronization of motion and oscillations between a and Ω in this case the response of X is quasi-periodic motion. In the time <350, 600> the amplitude and angular velocity are constant then the response of $X = a\cos(\Omega t + \xi)$ is periodic motion. In resonance region we observe the phenomenon of Sommerfeld in the time [400, 650].



a) Average angular velocity Ω b) Amplitude a

Figure 3. Response of the Eq. (14).

Following, when the parameter $V_m \in$ [0.5, 1.4] the response of X is of quasi-periodic motion, for example, for V_m = 1.0 is justified by a close curve shown on phase portrait (Fig. 4a). For $V_m \in$ [1.4, 2.4] the response of X is of periodic motion in this case chosen V_m =1.8 that justify by one point shown on phase portrait (Fig. 4b). For $V_m \in$ [2.24, 6.0] the response de X has quasi-periodic motion in this case we chosen V_m = 3.0 that justify by a close curve shown on phase portrait (Fig. 4c).

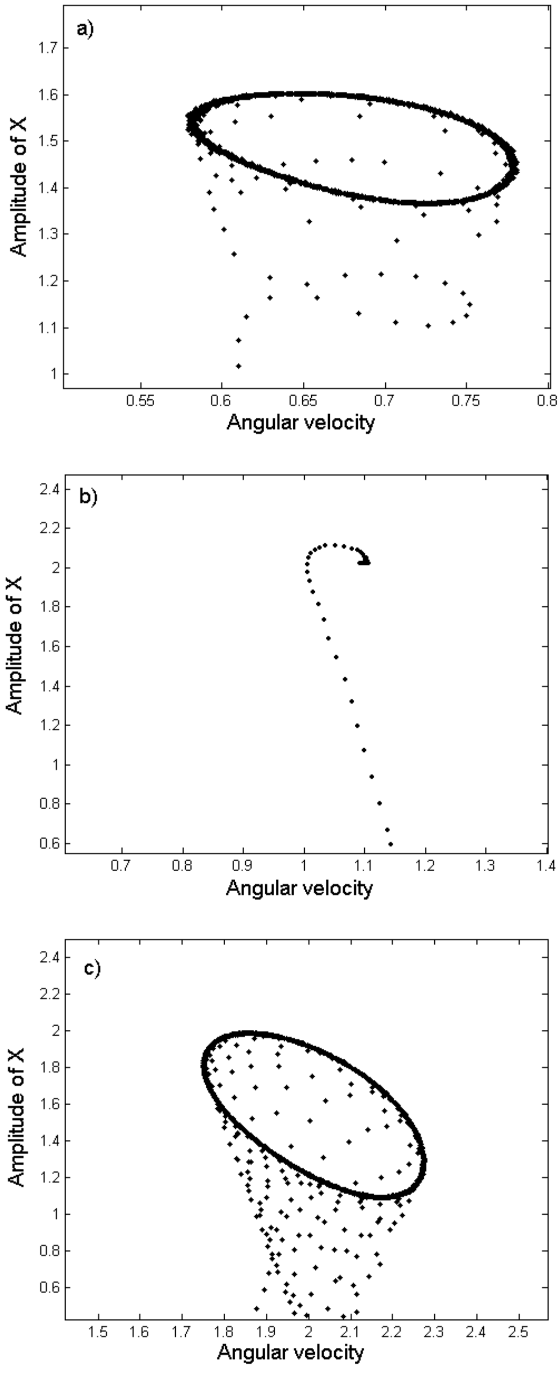


Figure 4. Phase portrait of amplitude versus angular velocity: a) V_m =1.0, b) V_m =1.8, c) V_m =3.0

The values of a , Ω , ξ for the stationary conditions of motion are determined as the roots of the system of equations

$$\begin{aligned}
&\frac{1}{2}\alpha a-\frac{3}{8}\beta a^{3}+\frac{1}{2}q_{1}\Omega^{2}\cos\xi=0\\
&-(\Omega-1)+\frac{3}{8}\gamma a^{2}-\frac{1}{2}q_{1}\frac{\Omega^{2}}{a}\sin\xi=0\\
&\Gamma(\Omega)-\frac{1}{2}aq_{2}\Omega\cos\xi=0
\end{aligned}\tag{18}$$

The criterion of the stationary solutions of according (Nayfeh, 1979): to determine the steady-state motion of an ideal system have that integrated numerically the Eq. (14) for long time then a , Ω , and ξ tend to constant values or we use the fact that a , Ω , and ξ are constant, set $\dot{a}=0$, $\dot{\Omega}=0$ and $\dot{\xi}=0$ from Eq. (18). But in this case, for a non-ideal system is different, for example,

For V_m =1.8 the values of a , Ω are constant for long time (see Fig. 5)

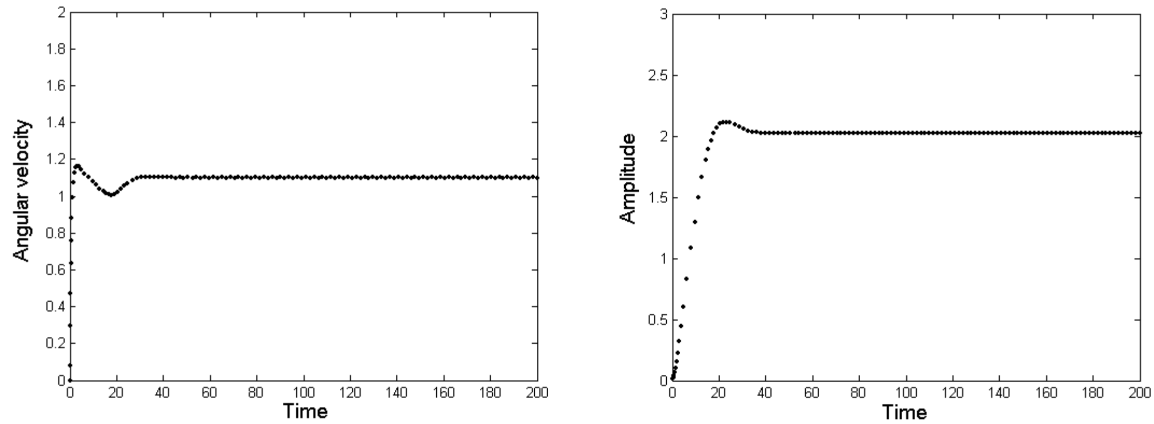


Figure 5. Response of the Eq. (14) for V_m =1.8

For V_m =3.0 the values of a , Ω are oscillating for long time (see Fig. 6)

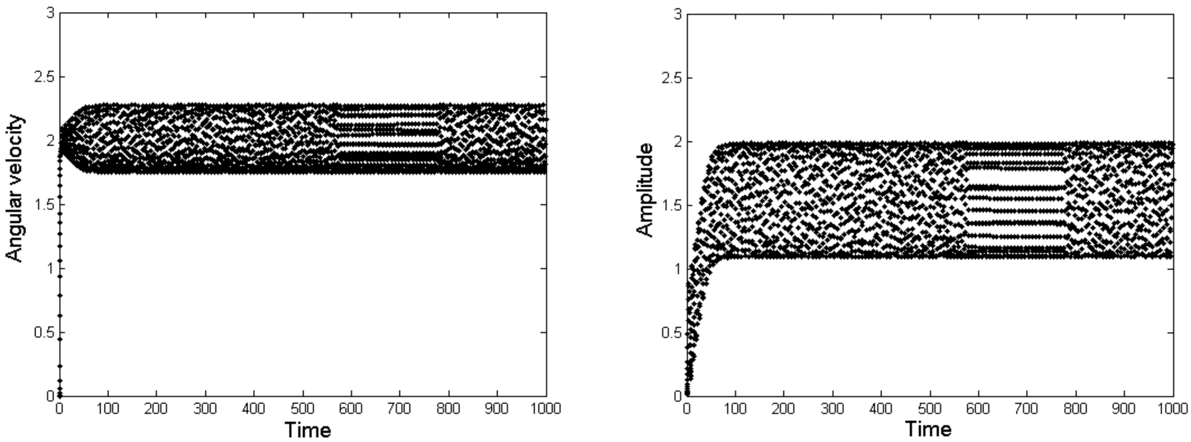
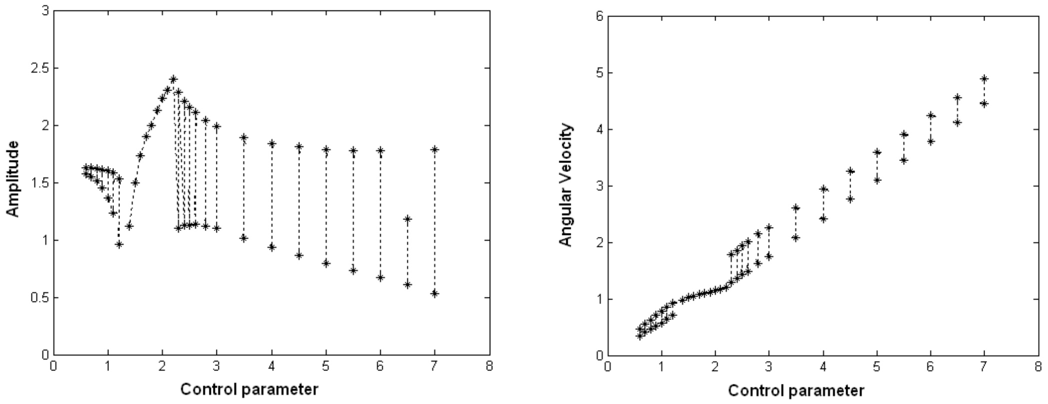


Figure 6. Result of the Eq. (14) for V_m = 3.0

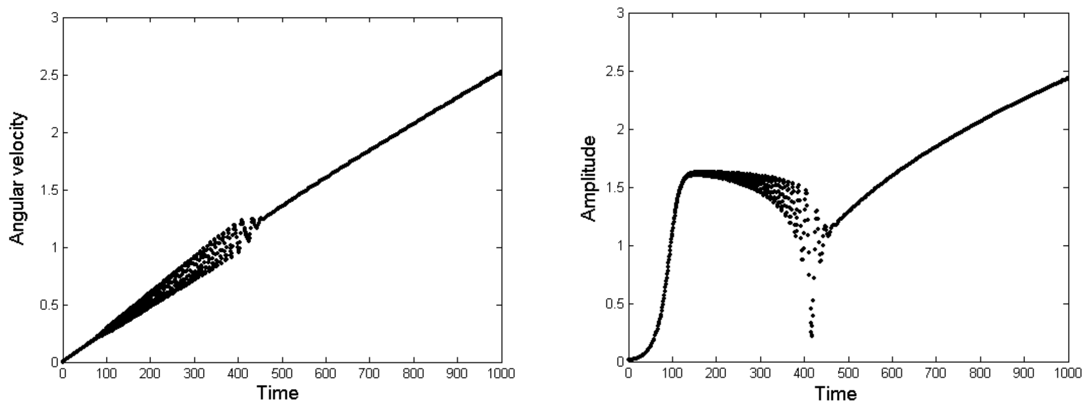
Figure 7, shows the results in steady-state motion with increasing control parameter during passage through resonance for the values of $V_m \in$ [0.6, 1.31] $\cup$ [2.24, 7] the response of the amplitude (Fig. 7a) and angular velocity (Fig. 7b) are oscillating then the non-ideal system is quasi-periodic motion. For $V_m \in$ [1.32, 2.23] the response of the amplitude and angular velocity are constant then the non-ideal system is periodic motion.



a) Amplitude of vibrating oscillator   b) Angular velocity of the motor

Figure 7. Response in steady-state process of amplitude and angular velocity versus control parameter

The following results show the influence of parameter of the nonlinear stiffness γ and the parameters other are fixed.



a) Angular velocity Ω b) Amplitude of X

Figure 8. Response in non-stationary process of the amplitude and angular velocity of Eq. (14) with γ =0.8.

Figure 8, shows the elimination of Sommerfeld effect and the synchronization outside of resonance region when nonlinear stiffness γ =0.8.

In order to complete, the dynamic analysis, we evaluate the Lyapunov exponents, using the classical method described in (Wolf, 1985), next. The main fórmula is

$$\lambda = \frac{1}{tN}\sum_{i=1}^{N} \ln\left(\frac{d_i(t)}{d_i(0)}\right) \tag{19}$$

where λ denotes the Lyapunov exponents, the index i, consecutive initial positions, N represents the total step number of evolution and d is the separation between two close trajectories, chosen. We obtain the Jacobian matrix of system of differential equations (14) in the following form:

$$J = \begin{bmatrix} \frac{\alpha}{2} - \frac{9}{8}\beta a^2 & -\frac{1}{2}q_1\Omega^2 \sin(\xi) & q_1\Omega\cos(\xi) \\ \frac{6}{8}\gamma a + \frac{q_1}{8a^2}\Omega^2 \sin(\xi) & -\frac{q_1}{2a}\Omega^2\cos(\xi) & -1 - \frac{q_1}{a}\Omega\sin(\xi) \\ -\frac{q_2}{2}\Omega\cos(\xi) & \frac{q_2 a\Omega}{2}\sin(\xi) & -b - \frac{q_2}{2}a\cos(\xi) \end{bmatrix} \tag{20}$$

We remarked that the Eqs. (14) and (20) are suitable adapted, in order to use the MATDS® program (Govorukhin, 2003), based with the routine of Matlab® of Matworks® , in order to evaluating Lyapunov's exponents, those are shown in Table 1. Note that if we taken into account V_m = 1.0 and V_m = 3.0 we obtain the confirmation that the considered non-ideal system vibrates in quasi-periodically motion and in the case where we taken V_m = 1.8, one confirms that the non-ideal system vibrates periodically.

Table 1. Lyapunov's Exponents.

V_m	Attractor's type	λ_1	λ_2	λ_3
1.0	Quasi-periodic	0.0	-0.079135	-1.459637
1.8	Periodic	-0.2192	-0.227961	-1401873
3.0	Quasi-periodic	0.0	-0.089585	-1.544268

In next case, to obtain chaotic regime, we need to assume that the natural frequency is negative of value -1. It means that the linear part of the spring stiffness is negative (Warminski, 2003). Here, we consider the parameter γ = 3.0. Solving, Numerically Eq. (3) we obtain Fig. 9, It shows that the phase portrait and Fourier Spectrum. From Fig. 9a and 9c, we can see that the non-ideal system is in periodic regime with V_m = 3.0 and V_m = 6.0. From Fig. 9b we can see that the non-ideal system is in chaotic regime with V_m = 4.0.

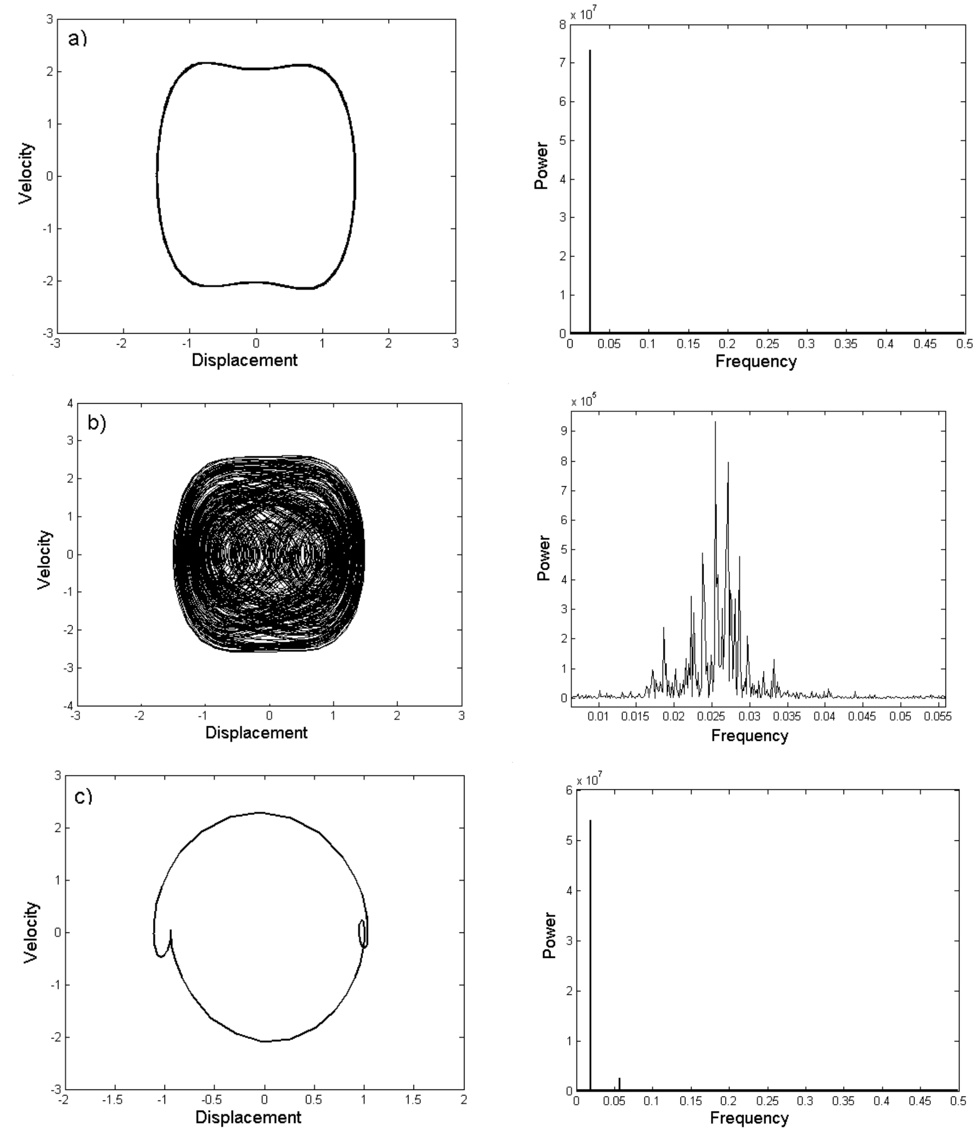


Figure 9. Periodic regime when a) V_m = 3.0 and c) V_m = 6.0; b) Chaotic regime when V_m = 4.0

4. CONCLUSIONS

In this paper, we have demonstrated conclusively that nonlinear stiffness and damping in cantilever beam oscillator, subject to a non-ideal excitation (unbalanced motor with power limited supply) may play an important role. This fact makes this problem more realistic, according to the experimental results. The method of averaging presented here, may allow a systematic study of non-ideal nonlinear system which may help revealing the underlying physical mechanisms. The Sommerfeld phenomenon, jump phenomenon, nonlinear stiffness and damping effect, unbalanced parameter effects, were obtained and analyzed during the passage through resonance region to the non-stationary and steady state process. In the resonance region of the considered oscillating it has periodic motion and outside the resonance region it has quasi-periodic motion. We also analyze, according to engineering point of view, the elimination of the jump phenomenon and control of the chaotic motion.

5. ACKNOWLEDGEMENTS

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7. RESPONSIBILITY NOTICE

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