COMPARISON BETWEEN THE FIRST ORDER UPWIND UNSTRUCTURED ALGORITHMS OF STEGER AND WARMING AND OF VAN LEER IN THE SOLUTION OF THE EULER EQUATIONS IN TWO-DIMENSIONS

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Abstract. *The present work performs comparisons between the Steger and Warming and the Van Leer algorithms applied to the solution of aeronautical and of aerospace problems, in two-dimensions. The Euler equations in conservative form, employing a finite volume formulation and an unstructured spatial discretization, are solved. The Steger and Warming and the Van Leer schemes are flux vector splitting ones and good robustness properties are expected. The time integration is performed by a Runge-Kutta method of five stages. Both schemes are first order accurate in space and second order accurate in time. The steady state physical problems of the transonic flow along a convergent-divergent nozzle and of the supersonic flows along a ramp and around a blunt body are studied. In all problems, the value of the entrance or attack angle is considered equal to zero. A spatially variable time step procedure is implemented aiming to accelerate the convergence of the schemes to the steady state condition. The results have demonstrated that the Van Leer scheme presents more severe pressure fields in the ramp and in the blunt body problems, as well more accurate values in the determination of the shock angle, ramp problem, and of the stagnation pressure, blunt body problem, than the Steger and Warming scheme, recommending the former to project calculations.*

Keywords: *Steger and Warming algorithm, Van Leer algorithm, Flux vector splitting algorithms, Euler equations, Two-dimensions.*

1. INTRODUCTION

Conventional non-upwind algorithms have been used extensively to solve a wide variety of problems (Kutler, 1975, and Steger, 1978). Conventional algorithms are somewhat unreliable in the sense that for every different problem (and sometimes, every different case in the same class of problems) artificial dissipation terms must be specially tuned and judicially chosen for convergence. Also, complex problems with shocks and steep compression and expansion gradients may defy solution altogether.

Upwind schemes are in general more robust but are also more involved in their derivation and application. Some upwind schemes that have been applied to the Euler equations are: Steger and Warming (1981) and Van Leer (1982). Some comments about these methods are reported below:

Steger and Warming (1981) developed a method that used the remarkable property that the nonlinear flux vectors of the inviscid gasdynamic equations in conservation law form were homogeneous functions of degree one of the vector of conserved variables. This property readily permitted the splitting of the flux vectors into subvectors by similarity transformations so that each subvector had associated with it a specified eigenvalue spectrum. As a consequence of flux vector splitting, new explicit and implicit dissipative finite-difference schemes were developed for first-order hyperbolic systems of equations.

Van Leer (1982) suggested an upwind scheme based on the flux vector splitting concept. This scheme considered the fact that the convective flux vector components could be written as flow Mach number polynomial functions, as main characteristic. Such polynomials presented the particularity of having the minor possible degree and the scheme had to satisfy seven basic properties to form such polynomials. This scheme was presented to the Euler equations in Cartesian coordinates and three-dimensions.

On an unstructured algorithm context, Maciel (2005a) and Maciel (2005b) have presented works involving the numerical implementation of two typical algorithms of the Computational Fluid Dynamics community. The Jameson and Mavriplis (1986) and the Frink, Parikh and Pirzadeh (1991) algorithms were implemented on an unstructured spatial discretization context. The Jameson and Mavriplis (1986) scheme was symmetrical and the Mavriplis (1990) artificial dissipation operator was implemented aiming to guarantee the scheme stability. The Frink, Parikh and Pirzadeh (1991) scheme was upwind and of flux difference splitting type based on Roe (1981) method. The Jameson and Mavriplis (1986) scheme was second order accurate in space and time, while the Frink, Parikh and Pirzadeh (1991) scheme was first order accurate in space and second order accurate in time. The Euler equations in conservative form were solved. The physical problems of the transonic flow around a NACA 0012 airfoil and the supersonic flow around a simplified version of the VLS (Brazilian Satellite Launcher Vehicle) were studied and good results were obtained, highlighting better solution quality and convergence acceleration features to the Jameson and Mavriplis (1986) scheme.

In the present work, the Steger and Warming (1981) and the Van Leer (1982) schemes are implemented, on a finite volume context and using an upwind and unstructured spatial discretization, to solve the Euler equations in the two-dimensional space applied to the steady state physical problems of the transonic flow along a convergent-divergent nozzle and of the supersonic flows along a ramp and around a blunt body. The Steger and Warming (1981) and the Van Leer (1982) schemes are flux vector splitting ones and good robustness properties are expected. The implemented schemes are first order accurate in space. The time integration uses a Runge-Kutta method of five stages and is second order accurate. Both algorithms are accelerated to the steady state solution using a spatially variable time step. This technique has proved excellent gains in terms of convergence ratio as reported in Maciel (2005c). The results have demonstrated that the Van Leer (1982) scheme presents more severe pressure fields in the ramp and in the blunt body problems, as well more accurate values in the determination of the shock angle, ramp problem, and of the stagnation pressure, blunt body problem, than the Steger and Warming (1981) scheme, recommending the former to project calculations.

An unstructured discretization of the calculation domain is usually recommended to complex configurations, due to the easily and efficiency that such domains can be discretized (Mavriplis, 1990, and Pirzadeh, 1991). However, the unstructured mesh generation question will not be studied in this work.

2. EULER EQUATIONS

The fluid movement is described by the Euler equations, which express the conservation of mass, of linear momentum and of energy to an inviscid mean, heat non-conductor and compressible, in the absence of external forces. In integral and conservative forms, these equations can be represented by:

$$\partial/\partial t \int_V Q dV + \int_S \left[(E_e)n_x + (F_e)n_y\right] dS = 0\,, \tag{1}$$

with Q written to a Cartesian system, V is the cell volume, n_x and n_y are the components of the normal versor to the flux face, S is the flux area, and E_e and F_e are the convective flux vector components. The Q, E_e and F_e vectors are represented by:

$$Q = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{Bmatrix},\quad E_e = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e+p)u \end{Bmatrix} \quad and \quad F_e = \begin{Bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e+p)v \end{Bmatrix}, \tag{2}$$

being ρ the fluid density; u and v the Cartesian components of the velocity vector in the x and y directions, respectively; e the total energy per fluid volume unity; and p the static pressure of the fluid mean.

In the nozzle problem, the Euler equations were nondimensionalized in relation to the stagnation density and in relation to the critical speed of sound. In the others problems, the nondimensionalization is performed considering freestream density and freestream speed of sound. The matrix system of the Euler equations is closed with the state equation $p = (\gamma - 1)\left[e - 0.5\rho(u^2 + v^2)\right]$, assuming the ideal gas hypothesis. The total enthalpy is determined by $H = (e + p)/\rho$.

3. STEGER AND WARMING (1981) ALGORITHM

3.1. Theory for the one-dimensional case

If the homogeneous Euler equations are put in characteristic form

$$\partial W/\partial t + \Lambda \partial W/\partial x = 0\,, \tag{3}$$

where W is the vector of characteristic variables (defined in Hirsch, 1990) and Λ is the diagonal matrix of eigenvalues, the upwind scheme:

$$u_i^{n+1} - u_i^n = -\Delta t/\Delta x \left[\hat{a}^+\left(u_i^n - u_{i-1}^n\right) + \hat{a}^-\left(u_{i+1}^n - u_i^n\right)\right], \tag{4}$$

where u is a scalar property, $\hat{a}^+ = 0.5\left(\hat{a} + |\hat{a}|\right)$ and $\hat{a}^- = 0.5\left(\hat{a} - |\hat{a}|\right)$, can be applied to each of the three characteristic variables separately, with the definitions

$$\lambda_m^+ = 0.5\left(\lambda_m + \left|\lambda_m\right|\right) \quad \text{and} \quad \lambda_m^- = 0.5\left(\lambda_m - \left|\lambda_m\right|\right), \tag{5}$$

"m" assuming values from 1 to 4 (two-dimensional space), for each of the eigenvalues of Λ

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} = \begin{bmatrix} u & & \\ & u+a & \\ & & u-a \end{bmatrix}. \tag{6}$$

This defines two diagonal matrices $\Lambda^{\pm}$:

$$\Lambda^{\pm} = \begin{bmatrix} \lambda_1^{\pm} & & \\ & \lambda_2^{\pm} & \\ & & \lambda_3^{\pm} \end{bmatrix} = \begin{bmatrix} 0.5\left(u \pm |u|\right) & & \\ & 0.5\left(u + a \pm |u+a|\right) & \\ & & 0.5\left(u - a \pm |u-a|\right) \end{bmatrix}, \tag{7}$$

where Λ^+ has only positive eigenvalues, Λ^- only negative eigenvalues, and such that

$$\Lambda = \Lambda^+ + \Lambda^- \quad \text{and} \quad |\Lambda| = \Lambda^+ - \Lambda^- \quad \text{or} \quad \lambda_m = \lambda_m^+ + \lambda_m^- \quad \text{and} \quad \left|\lambda_m\right| = \lambda_m^+ - \lambda_m^- . \tag{8}$$

The quasi-linear coupled equations are obtained from the characteristic form by the transformation matrix P (defined in Hirsch, 1990), with the Jacobian A satisfying

$$A = P\Lambda P^{-1}\text{, resulting in } \partial Q/\partial t + A\partial Q/\partial x = 0 . \tag{9}$$

Hence an upwind formulation can be obtained with the Jacobians

$$A^+ = P\Lambda^+ P^{-1} \quad \text{and} \quad A^- = P\Lambda^- P^{-1}\text{, with: } A = A^+ + A^- \text{ and } |A| = A^+ - A^- . \tag{10}$$

The fluxes associated with these split Jacobians are obtained from the remarkable property of homogeneity of the flux vector $f(Q)$. $f(Q)$ is a homogeneous function of degree one of Q. Hence, $f = AQ$ and the following flux splitting can be defined:

$$f^+ = A^+ Q \quad \text{and} \quad f^- = A^- Q\text{, with: } f = f^+ + f^- . \tag{11}$$

This flux vector splitting, based on Eq. (5), has been introduced by Steger and Warming (1981). The split fluxes f^+ and f^- are also homogeneous functions of degree one in Q.

3.2. Arbitrary meshes

In practical computations one deal mostly with arbitrary meshes, considering either in a finite volume approach or in a curvilinear coordinate system.

In both cases, the upwind characterization is based on the signs of the eigenvalues of the matrix

$$K^{(n)} = \vec{A} \bullet \vec{n} = An_x + Bn_y . \tag{12}$$

The fluxes will be decomposed by their components

$$\widetilde{F}^{(n)} = \vec{\vec{F}} \bullet \vec{n} = En_x + Fn_y \tag{13}$$

and separated into positive and negative parts according to the sign of the eigenvalues of $K^{(n)}$ as described above, considering the normal direction as a local coordinate direction.

For a general eigenvalue splitting, as Eq. (5), the normal flux projection, Eq. (13), is decomposed by a Steger and Warming (1981) flux splitting as

$$\tilde{F}_{\pm}^{(n)} = \frac{\rho}{2\gamma}\left\{\begin{array}{c} \alpha \\ \alpha u + a\left(\lambda_2^{\pm} - \lambda_3^{\pm}\right)n_x \\ \alpha v + a\left(\lambda_2^{\pm} - \lambda_3^{\pm}\right)n_y \\ \alpha\frac{u^2 + v^2}{2} + a v_n\left(\lambda_2^{\pm} - \lambda_3^{\pm}\right) + a^2\frac{\lambda_2^{\pm} + \lambda_3^{\pm}}{\gamma - 1} \end{array}\right\}, \tag{14}$$

where the eigenvalues of the matrix K are defined as

$$\lambda_1 = \vec{v} \bullet \vec{n} \equiv v_n,\ \lambda_2 = \vec{v} \bullet \vec{n} + a \quad \text{and} \quad \lambda_3 = \vec{v} \bullet \vec{n} - a, \tag{15}$$

with $\vec{v}$ is the flow velocity vector, and $\pm$ sign indicates the positive or negative parts respectively. The parameter α is defined as

$$\alpha = 2(\gamma - 1)\lambda_1^{\pm} + \lambda_2^{\pm} + \lambda_3^{\pm}. \tag{16}$$

3.3. Numerical scheme

The numerical scheme of Steger and Warming (1981) implemented in this work is based on an unstructured finite volume formulation, where the convective numerical fluxes at interface are calculated as

$$\tilde{F}_l^{(m)} = \left[\left(\tilde{F}_R^{-}\right)^{(m)} + \left(\tilde{F}_L^{+}\right)^{(m)}\right]S^l, \tag{17}$$

where "R" and "L" represent right and left states, respectively, S is the cell face area and "l" indicates the flux interface. The subscript "L" is associated to properties of a given "i" cell and the subscript "R" is associated to properties of the "ne" neighbor cell of "i". The cell volume and the respective cell face area components are defined in Maciel (2007a), as well an illustration of a typical unstructured cell.

The time integration is performed by an explicit method, second order accurate, Runge-Kutta type of five stages and can be represented of generalized form by:

$$\begin{aligned} Q_i^{(0)} &= Q_i^{(n)} \\ Q_i^{(k)} &= Q_i^{(0)} - \alpha_k\, \Delta t_i / V_i \times C\left(Q_i^{(k-1)}\right), \\ Q_i^{(n+1)} &= Q_i^{(k)} \end{aligned} \tag{18}$$

with k = 1,...,5; α_1 = 1/4, α_2 = 1/6, α_3 = 3/8, α_4 = 1/2 and α_5 = 1. The contribution of the convective numerical flux vectors is determined by the C_i vector:

$$C_i^{(m)} = \tilde{F}_1^{(m)} + \tilde{F}_2^{(m)} + \tilde{F}_3^{(m)}. \tag{19}$$

This version of the flux vector splitting algorithm of Steger and Warming (1981) is first order accurate in space.

4. VAN LEER (1982) ALGORITHM

The approximation of the integral equation (1) to a triangular finite volume yields a system of ordinary differential equations with respect to time:

$$V_i\, dQ_i / dt = -C_i, \tag{20}$$

with C_i representing the net flux (residue) of conservation of mass, of linear momentum and of energy in the V_i volume. The residue is calculated as:

$$C_i = F_1 + F_2 + F_3, \tag{21}$$

with $F_1 = F_1^c$, where "c" is related to the flow convective contribution at l = 1 interface.

As shown in Liou and Steffen (1993), the discrete convective flux calculated by the AUSM scheme ("Advection Upstream Splitting Method") can be interpreted as a sum involving the arithmetical average between the right (R) and the left (L) states of the "l" cell face, related to cell i and its neighbor, respectively, multiplied by the interface Mach number, and a scalar dissipative term. Hence, to the "l" interface:

$$F_l = |S|_l \left(\frac{1}{2} M_l \left(\begin{bmatrix} \rho a \\ \rho a u \\ \rho a v \\ \rho a H \end{bmatrix}_L + \begin{bmatrix} \rho a \\ \rho a u \\ \rho a v \\ \rho a H \end{bmatrix}_R \right) - \frac{1}{2} \phi_l \left(\begin{bmatrix} \rho a \\ \rho a u \\ \rho a v \\ \rho a H \end{bmatrix}_R - \begin{bmatrix} \rho a \\ \rho a u \\ \rho a v \\ \rho a H \end{bmatrix}_L \right) \right) + \begin{bmatrix} 0 \\ S_x p \\ S_y p \\ 0 \end{bmatrix}_l , \tag{22}$$

where $S_l = \begin{bmatrix} S_x & S_y \end{bmatrix}_l^T$ defines the normal area vector to the "l" surface. The "a" quantity represents the speed of sound, defined as $a = \sqrt{\gamma p / \rho}$. M_l defines the advection Mach number at the "l" face of the "i" cell, which is calculated according to Liou and Steffen (1993) as:

$$M_l = M_L^+ + M_R^- , \tag{23}$$

where the separated Mach numbers $M^{+/-}$ are defined by Van Leer (1982):

$$M^+ = \begin{bmatrix} M, & \text{if } M \geq 1; \\ 0.25(M+1)^2, & \text{if } |M| < 1; \\ 0, & \text{if } M \leq -1; \end{bmatrix} \quad \text{and} \quad M^- = \begin{bmatrix} 0, & \text{if } M \geq 1; \\ -0.25(M-1)^2, & \text{if } |M| < 1; \\ M, & \text{if } M \leq -1. \end{bmatrix} \tag{24}$$

M_L and M_R represent the Mach number associated with the left and right states, respectively. The advection Mach number is defined by:

$$M = \left(S_x u + S_y v \right) / \left(|S| a \right). \tag{25}$$

The pressure at the "l" face of the "i" cell is calculated by a similar way:

$$p_l = p_L^+ + p_R^- , \tag{26}$$

with $p^{+/-}$ denoting the pressure separation defined according to Van Leer (1982):

$$p^+ = \begin{bmatrix} p, & \text{if } M \geq 1; \\ 0.25p(M+1)^2(2-M), & \text{if } |M| < 1 \\ 0, & \text{if } M \leq -1; \end{bmatrix} \quad \text{and} \quad p^- = \begin{bmatrix} 0, & \text{if } M \geq 1; \\ 0.25p(M-1)^2(2+M), & \text{if } |M| < 1; \\ p, & \text{if } M \leq -1. \end{bmatrix} \tag{27}$$

The definition of the dissipative term ϕ determines the particular formulation of the convective fluxes. According to Radespiel and Kroll (1995), the choice below corresponds to the Van Leer (1982) scheme:

$$\phi_l = \phi_l^{VL} = \begin{cases} |M_l|, & \text{if } |M_l| \geq 1; \\ |M_l| + 0.5(M_R - 1)^2, & \text{if } 0 \leq M_l < 1; \\ |M_l| + 0.5(M_L + 1)^2, & \text{if } -1 < M_l \leq 0. \end{cases} \tag{28}$$

The equations above clearly show that to a supersonic Mach number at the cell face, the Van Leer (1982) scheme represents a purely upwind discretization, using either the left state or the right state to the convective and pressure terms, depending of the Mach number signal. The time integration follows the method described in the Steger and Warming (1981) scheme (Eq. 18). The Van Leer (1982) scheme presented in this work is first order accurate in space.

5. SPATIALLY VARIABLE TIME STEP

The basic idea of this procedure consists in keeping constant the CFL number in all calculation domain, allowing, hence, the use of appropriated time steps to each specific mesh region during the convergence process. Details of the present implementation are found in Maciel (2007a).

6. INITIAL AND BOUNDARY CONDITIONS

Stagnation values are used as initial condition to the nozzle problem (Maciel, 2002). Only at the exit boundary is imposed a reduction of 1/3 to the density and to the pressure to start the flow along the nozzle. To the others problems, values of freestream flow are adopted for all properties as initial condition, in the whole calculation domain (Jameson and Mavriplis, 1986, Maciel, 2005a, Maciel, 2005b, and Maciel, 2002). To a detailed description of these initial conditions, see Maciel (2007b).

The boundary conditions are basically of three types: solid wall, entrance and exit. These conditions are implemented in special cells named ghost cells. Details of the present implementation are described in Maciel (2007b).

7. RESULTS

Tests were performed in a CELERON - 1.2 GHz and 640 Mbytes of RAM memory microcomputer. Converged results occurred to four (4) orders of reduction in the value of the maximum residue. The value used for γ was 1.4. To all problems, the entrance or attack angle adopted a value 0.0°.

The meshes used in the simulations were structured generated, using rectangular cells, and posteriorly were transformed in meshes of triangles through specific subroutines implemented in the calculation algorithms, where the connectivity, neighboring, node coordinate and ghost cell tables were generated to the simulations. On this context, only the advantages of unstructured meshes were not appreciated; however, the unstructured algorithms could be tested on a context of unstructured spatial discretization. Information about the meshes is found in Maciel (2007a).

7.1. Nozzle physical problem

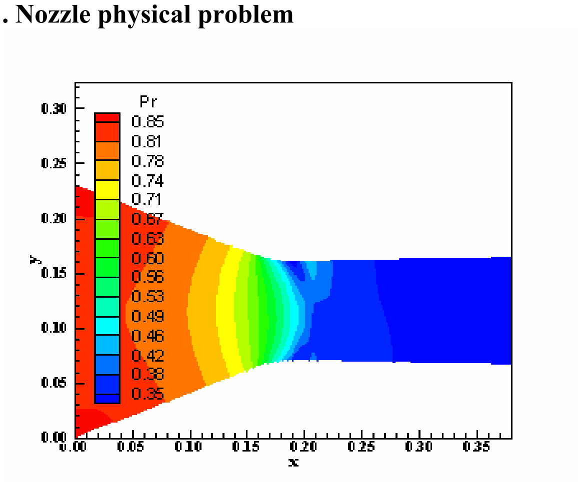

Figure 1. Pressure contours (SW).

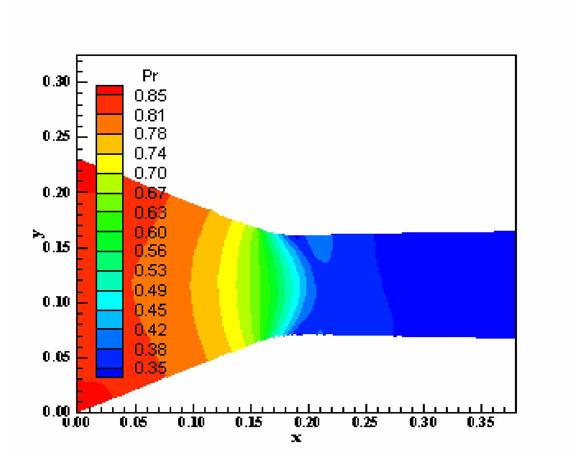

Figure 2. Pressure contours (VL).

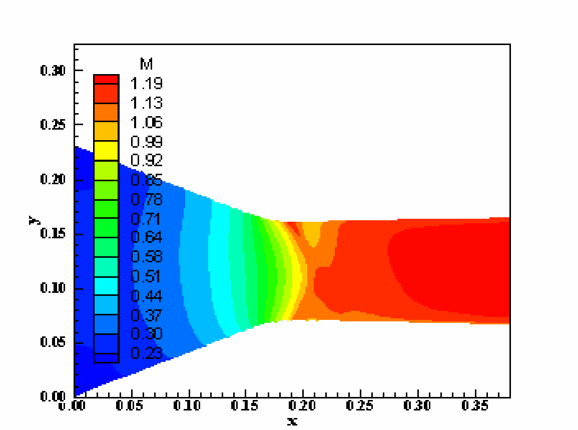

Figure 3. Mach number contours (SW).

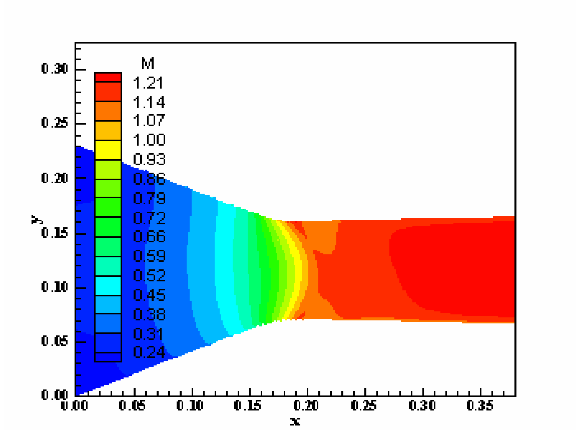

Figure 4. Mach number contours (VL).

The geometry of the convergent-divergent nozzle studied in this work is described in Maciel (2007a). Figures 1 and 2 show the pressure contours obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes, respectively. Loss of symmetry is observed in both solutions at the throat region. The Van Leer (1982) solution presents better symmetry characteristics at the throat region than the Steger and Warming (1981) solution. The pressure field generated by the Steger and Warming (1981) scheme is more severe than that generated by the Van Leer (1982) scheme. Figures 3 and 4 exhibit the Mach number contours obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes, respectively. Loss of symmetry is again observed in both solutions, with the Van Leer (1982) scheme presenting better behavior. The Mach number field generated by the Van Leer (1982) scheme is more intense than that generated by the Steger and Warming (1981) scheme.

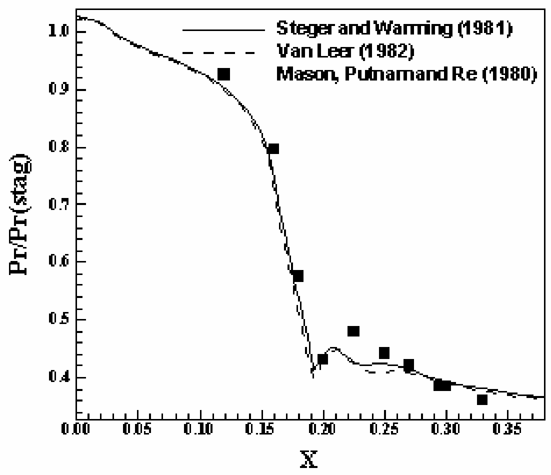

Figure 5. Wall pressure distributions.

Figure 5 exhibits the nozzle lower wall pressure distributions obtained by both schemes. These pressure distributions are compared with the experimental results of Mason, Putnam and Re (1980). The Steger and Warming (1981) solution is closer to the experimental results than the Van Leer (1982) scheme, although the later is better at the nozzle end.

7.2. Ramp physical problem

The ramp configuration is described in detail in Maciel (2007a). The initial condition adopted for this problem was a freestream Mach number of 2.0, characterizing a supersonic flow.

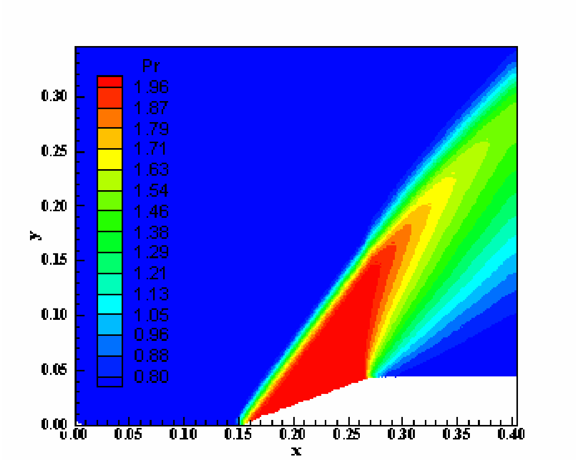

Figure 6. Pressure contours (SW).

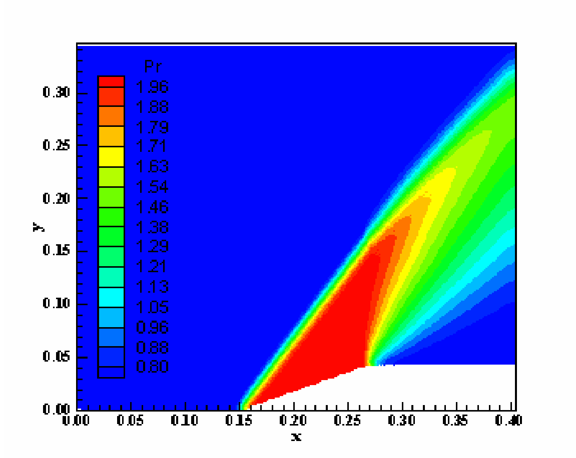

Figure 7. Pressure contours (VL).

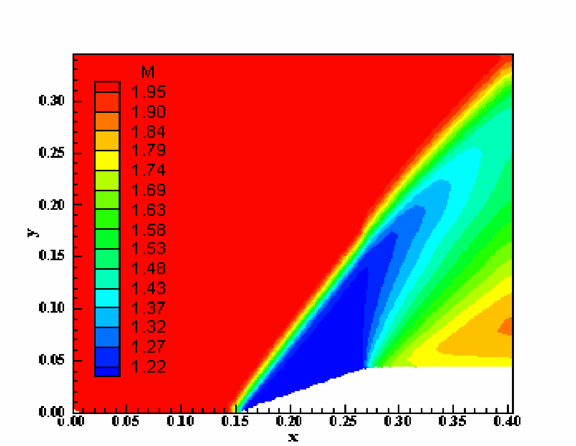

Figure 8. Mach number contours (SW).

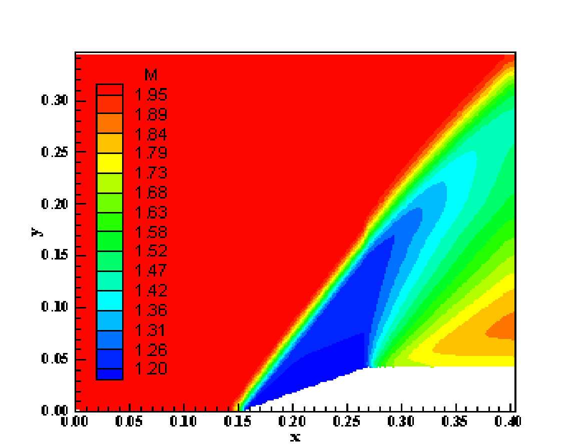

Figure 9. Mach number contours (VL).

Figures 6 and 7 show the pressure contours obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes, respectively. The pressure field generated by the Van Leer (1982) scheme is more severe than that generated by the Steger and Warming (1981) scheme. In both solutions the shock is well captured. Figures 8 and 9 exhibit the Mach number contours obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes, respectively. The Steger and Warming (1981) generates a more intense Mach number field.

Figure 10 shows the ramp pressure distributions obtained by the schemes of Steger and Warming (1981) and of Van Leer (1982). They are compared with the oblique shock wave and the Prandtl-Meyer expansion theories. As can be observed, both schemes slightly overpredict the pressure plateau generated by the shock at the ramp in comparison with the theory results. The same behavior is noted in the prediction of the pressure after the expansion fan. Both schemes slightly overpredict this pressure. Even so, the results are of good quality and the Steger and Warming (1981) and the Van Leer (1982) schemes capture appropriately the shock and the expansion fan.

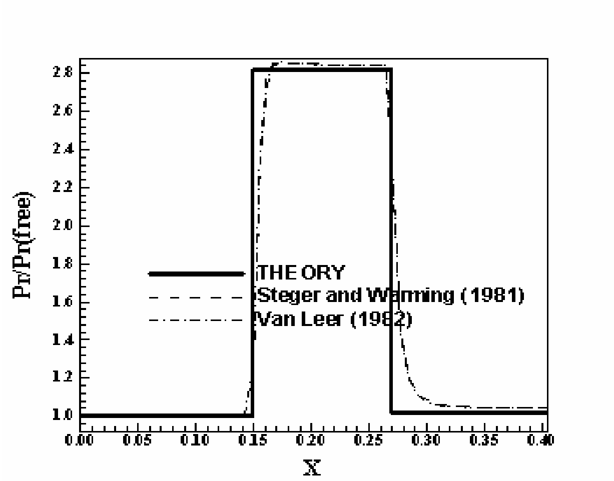

Figure 10. Wall pressure distributions.

One way to quantitatively verify if the solutions generated by each scheme are satisfactory consists in determining the shock angle of the oblique shock wave, β, measured in relation to the initial direction of the flow field. Anderson Jr. (1984) presents a diagram with values of the shock angle, β, to oblique shock waves. The value of this angle is determined as function of the freestream Mach number and of the deflection angle of the flow after the shock wave, ϕ. To ϕ = 20º (ramp inclination angle) and to a freestream Mach number equals to 2.0, it is possible to obtain from this diagram a value to β equals to 53.0º. Using a transfer in Figures 6 and 7, it is possible to obtain the values of β to each scheme, as well the respective errors, shown in Tab. 1. The Van Leer (1982) scheme yields the best result.

Table 1. Shock angle and percentage errors to the ramp problem.

Algorithm:	β (°):	Error (%):
Steger and Warming (1981)	55.0	3.8
Van Leer (1982)	54.3	2.5

7.3. Blunt body physical problem

The blunt body configuration is described in detail in Maciel (2007a). The freestream flow Mach number adopted for this simulation was 3.0, characterizing a supersonic flow regime.

Figures 11 and 12 exhibit the pressure contours obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes, respectively. Good symmetry properties are observed in both solutions in opposite behavior as observed in the nozzle problem. The Van Leer (1982) scheme presents a more severe pressure than that obtained with the Steger and Warming (1981) scheme. Figures 13 and 14 show the Mach number contours obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes, respectively. Both solutions present good symmetry properties and capture the shock appropriately. Figure 15 exhibits the –Cp distributions around the blunt body wall obtained by the Steger and Warming (1981) and by the Van Leer (1982) schemes. Both schemes present similar –Cp distributions. The small divergence involving the solutions occurs at x = 1.0m, but the Cp peak at the shock is equal to both schemes and

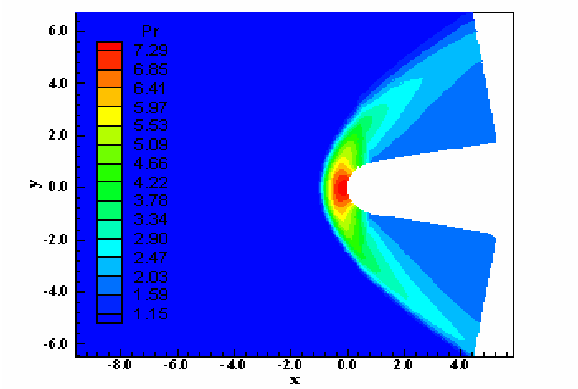

Figure 11. Pressure contours (SW).

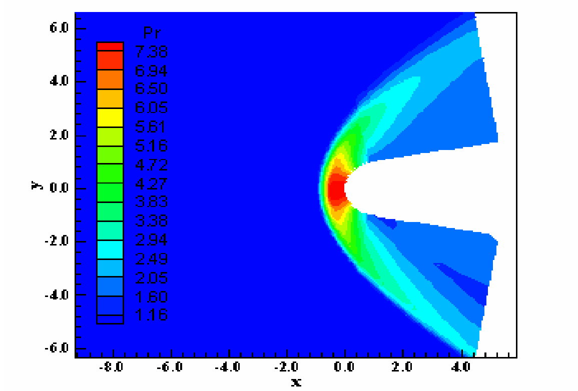

Figure 12. Pressure contours (VL).

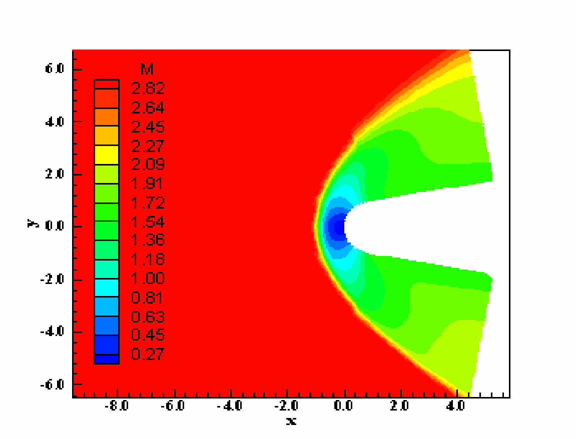

Figure 13. Mach number contours (SW).

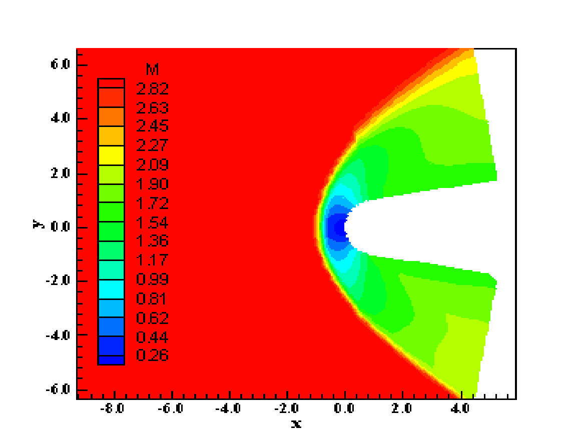

Figure 14. Mach number contours (VL).

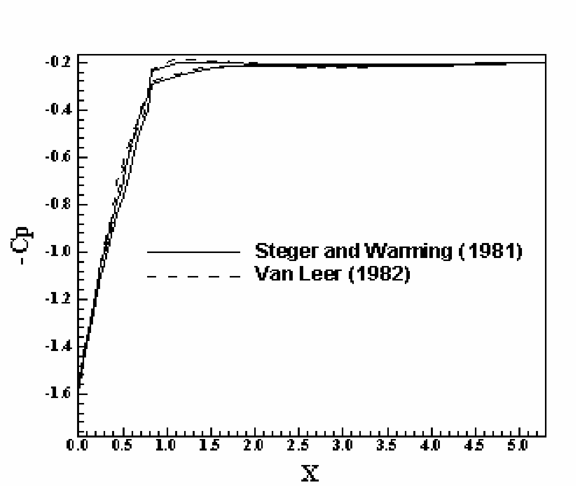

Figure 15. Wall -Cp distributions.

the constant pressure plateau after x = 1.0m, the rectilinear wall blunt body region, is well captured after 1.5m.

Another possibility to quantitative comparison of all schemes is the determination of the stagnation pressure ahead of the configuration. Anderson Jr. (1984) presents a table of normal shock wave properties in its B Appendix. This table permits the determination of some shock wave properties as function of the freestream Mach number. In front of the blunt body configuration, the shock wave presents a normal shock behavior, which permits the determination of the stagnation pressure, behind the shock wave, from the tables encountered in Anderson Jr. (1984). It is possible to determine the ratio pr_0/pr_∞ from Anderson Jr. (1984), where pr_0 is the stagnation pressure in front of the configuration and pr_∞ is the freestream pressure (equals to $1/\gamma$ with the present nondimensionalization).

Hence, to this problem, $M_\infty = 3.0$ corresponds to $pr_0/pr_\infty = 12.06$ and remembering that $pr_\infty = 0.714$, it is possible to conclude that $pr_0 = 8.61$. Values of the stagnation pressure, with respective percentage errors, to each scheme are shown in Tab. 2. The Van Leer (1982) scheme presents the best result.

Table 2. Values of the stagnation pressure and percentage errors.

Algorithm:	pr_0:	Error (%):
Steger and Warming (1981)	7.29	15.3
Van Leer (1982)	7.38	14.3

7.4. Numerical data of the simulations

Table 3. CFL numbers, iterations to convergence and computational costs of the schemes.

	Nozzle		Ramp		Blunt body		
Scheme	CFL	Iterations	CFL	Iterations	CFL	Iterations	Cost(1)
Steger and Warming (1981)	0.4	6,946	0.8	1,264	0.4	2,289	0.0000346
Van Leer (1982)	0.4	7,447	0.8	1,275	0.4	2,454	0.0000369

(1): measured in seconds/per cell/per iteration

Table 3 shows the numerical data of the simulations performed with the Steger and Warming (1981) and the Van Leer (1982) schemes. The later is approximately 6.6% more expensive than the former.

8. CONCLUSIONS

The present work performs comparisons between the Steger and Warming (1981) and the Van Leer (1982) schemes applied to the solution of aeronautical and of aerospace problems, in two-dimensions. The Euler equations in conservative form, employing a finite volume formulation and an unstructured spatial discretization, are solved. The Steger and Warming (1981) and the Van Leer (1982) schemes are flux vector splitting ones and good robustness properties are expected. The time integration is performed by a Runge-Kutta method of five stages. Both schemes are first order accurate in space and second order accurate in time. The steady state physical problems of the transonic flow along a convergent-divergent nozzle and of the supersonic flows along a ramp and around a blunt body are studied. In all problems, the value of the entrance or attack angle is considered equal to zero. A spatially variable time step procedure is employed aiming to accelerate the convergence of the numerical schemes to the steady state solution. This technique has proved excellent gains in terms of convergence ratio as reported in Maciel (2005c).

The results have demonstrated that the Van Leer (1982) scheme yielded more severe pressure fields in the ramp and in the blunt body problems. Moreover, the Van Leer (1982) scheme presented better estimations of the shock angle, in the ramp problem, and of the stagnation pressure, in the blunt body problem, than the Steger and Warming (1981) scheme, resulting in more accurate results. The Steger and Warming (1981) scheme is only 6.6% cheaper than the Van Leer (1982) scheme, does not damaging the later as a more efficient algorithm.

9. REFERENCES

Anderson Jr., J. D., 1984, "Fundamentals of Aerodynamics", McGraw-Hill, Inc., EUA, 563p.

Frink, N. T., Parikh, P. and Pirzadeh, S., 1991, "Aerodynamic Analysis of Complex Configurations Using Unstructured Grids", AIAA 91-3292-CP.

Hirsch, C., 1990, "Numerical Computation of Internal and External Flows – Computational Methods for Inviscid and Viscous Flows", John Wiley & Sons Ltd, 691p.

Jameson, A. and Mavriplis, D., 1986, "Finite Volume Solution of the Two-Dimensional Euler Equations on a Regular Triangular Mesh", AIAA Journal, Vol. 24, No. 4, pp. 611-618.

Kutler, P., 1975, "Computation of Three-Dimensional, Inviscid Supersonic Flows", Lecture Notes in Physics, Vol. 41, pp. 287-374.

Liou, M. and Steffen Jr., C. J., 1993, "A New Flux Splitting Scheme", Journal of Computational Physics, Vol. 107, pp. 23-39.

Maciel, E. S. G., 2002, "Simulação Numérica de Escoamentos Supersônicos e Hipersônicos Utilizando Técnicas de Dinâmica dos Fluidos Computacional", Doctoral Thesis, ITA, CTA, São José dos Campos, SP, Brazil, 258 p.

Maciel, E. S. G., 2005a, "Comparison Between a Centered and a Flux Difference Split Schemes Using Unstructured Strategy", JBSMSE – Journal of the Brazilian Society of Mechanical Sciences and Engineering, Brazil, Vol. XXVII, No. 3, pp. 223-235.

Maciel, E. S. G., 2005b, "Comparison Between a Centered and a High Resolution Upwind Schemes in the Solution of Aerospace Problems Using Unstructured Strategy", Proceedings of the XVIII International Congress of Mechanical Engineering (XVIII COBEM), Ouro Preto, MG, Brazil.

Maciel, E. S. G., 2005c, "Analysis of Convergence Acceleration Techniques Used in Unstructured Algorithms in the Solution of Aeronautical Problems – Part I", Proceedings of the XVIII International Congress of Mechanical Engineering (XVIII COBEM), Ouro Preto, MG, Brazil.

Maciel, E. S. G., 2007a, "Comparison Between the First Order Upwind Unstructured Algorithms of Roe and of Harten in the Solution of the Euler Equations in Two-Dimensions", Proceedings of the XIX Congress of Mechanical Engineering (XIX COBEM), Brasília, DF, Brazil.

Maciel, E. S. G., 2007b, "Comparison Among Predictor-Corrector, Symmetrical and TVD Upwind Schemes in the Solution of the Euler Equations in Two-Dimensions – Theory", Proceedings of the XIX Congress of Mechanical Engineering (XIX COBEM), Brasília, DF, Brazil.

Mason, M. L., Putnam, L. E. and Re, R. J., 1980, "The Effect of Throat Contouring on Two-Dimensional Converging-Diverging Nozzles at Sonic Conditions", NASA Technical Paper 1704.

Mavriplis, D. J., 1990, "Accurate Multigrid Solution of the Euler Equations on Unstructured and Adaptive Meshes", AIAA Journal, Vol. 28, No. 2, pp. 213-221.

Pirzadeh, S., 1991, "Structured Background Grids for Generation of Unstructured Grids by Advancing Front Method", AIAA Paper 91-3233-CP.

Radespiel, R. and Kroll, N., 1995, "Accurate Flux Vector Splitting for Shocks and Shear Layers", Journal of Computational Physics, Vol. 121, pp. 66-78.

Steger, J. L., 1978, "Implicit Finite-Difference Simulation of Flow About Arbitrary Two-Dimensional Geometries", AIAA Journal, Vol. 16, No. 7, pp. 679-686.

Steger, J. L. and Warming, R. F., 1981, "Flux Vector Splitting of the Inviscid Gasdynamic Equations with Application to Finite-Difference Methods", Journal of Computational Physics, Vol. 40, pp. 263-293.

Van Leer, B., 1982, "Flux-Vector Splitting for the Euler Equations", Proceedings of the 8th International Conference on Numerical Methods in Fluid Dynamics, E. Krause, Editor, Lecture Notes in Physics, Vol. 70, pp. 507-512, Springer Verlag, Berlin.