# ON THE DETERMINATION OF COLLAPSE LOADS OF TRANSMISSION LINE TOWERS THROUGH NONLINEAR MODEL

**First Author's Name:** João Kaminski Junior
Institution and address: Federal University of Santa Maria – UFSM, Department of Civil Engineering
e-mail: jkj@smail.ufsm.br

**Second Author's Name:** Letícia Fleck Fadel Miguel
Institution and address: Federal University of Rio Grande do Sul – UFRGS, Department of Mechanical Engineering
e-mail: letffm@ufrgs.br

**Third Author's Name:** Leandro Fleck Fadel Miguel
Institution and address: Federal University of Rio Grande do Sul – UFRGS, Department of Civil Engineering
e-mail: leandroffm@gmail.com

**Fourth Author's Name:** Gerson Moacyr Sisniegas Alva
Institution and address: Federal University of Santa Maria – UFSM, Department of Civil Engineering
e-mail: alva_gerson@yahoo.com.br

***Abstract.*** *The prediction of the structural behavior until the rupture is a complex task, even for latticed structures, because it involves second order effects, physical nonlinearities, besides other phenomena. This paper reports a methodology developed for analysis of transmission line (TL) latticed steel towers with the objective of evaluating their collapse loads. In this numerical approach, towers are subjected to a ramp-loading until the rupture, in a time sufficiently large to consider the loading as static, i.e., to avoid dynamic amplifications. The collapse loads can be compared with available experimental results. The direct numerical integration of the equations of motion using an explicit approach (central finite differences) was adopted in this study, because it offers several advantages in connection with nonlinear problems. Since it does not require assembling or updating of the global stiffness matrix, the integration is performed at elements level. After each integration step, all coordinates of the system are updated, therefore, taking into account geometrical nonlinearity. Physical nonlinearity is incorporated into the analysis with little additional effort. For these analyses it was developed a FORTRAN program, taking into account the nonlinear constitutive relations between normal load and axial displacement in tension and compression of the steel tower members. The results showed that the predicted collapse loads were very close to the experimental results, proving the efficiency of this approach.*

***Keywords****: latticed steel towers, collapse load, nonlinear analysis, explicit numerical integration, central finite differences.*

## 1. INTRODUCTION

The classic discrete formulations of structural systems, as the finite element method (Bathe, 1982), require assembling of the global stiffness matrix and solution of very large resulting system of equations. The direct numerical integration of the equations of motion, in the explicit form, presents some attractive features in the solution of structural problems, such as it does not require mounting global stiffness matrix, since the integration can be performed at elements level (Menezes *et al.*, 1998).

In this context, the explicit numerical integration of the equations of motion - derived for dynamic problems - appears as an attractive option to obtain the response of structures like latticed towers, subjected to static loads. Thus, the loads, considered time-dependent, are increased from zero to their final static value, using a ramp-loading in a time sufficiently large to consider the loading as static, so the results converge to the static response.

This approach is known as dynamic relaxation technique and it is not attractive in the solution of linear systems, due to the large number of required integration steps in the analysis. However, when nonlinear systems must be analyzed using the classical approach, stiffness matrix must be assembled at the beginning and updated every time that the load increases (load step). In addition, the solution of the equilibrium equations often requires iterations after each load step, rendering dynamic relaxation a competitive option.

It is important to observe that geometrical nonlinearities are always considered, since the nodal coordinates are updated after each integration step, and physical nonlinearities may be easily implemented, since a global stiffness matrix is not needed.

Herein, a *TL* latticed steel tower is analyzed, subjected to a ramp-loading until the rupture, using the dynamic relaxation technique with a nonlinear model to predict the behavior of the tower members. Experimental results are available in order to evaluate the efficiency of this approach.

## 2. SOLUTION METHOD

Direct explicit numerical integration of the equations of motion in the time domain was adopted, using the finite central differences scheme, because it does not require assembling or updating the system global stiffness matrix. Integration is carried out at elements level, which constitutes an advantage in nonlinear problems.

### 2.1. Central Finite Differences

Starting from the equation of motion of structural systems with multiple degrees of freedom, the expression of the central finite differences, used to solve the equations of dynamic equilibrium in every discrete time and to obtain the nodal coordinates of the structure in the directions $x$, $y$ and $z$, is written as:

$$\left[\frac{1}{\Delta t^2}[M]+\frac{1}{2\,\Delta t}[C]\right]\left\{q_{(t+\Delta t)}\right\}=\{F(t)\}-\left[[K]-\frac{2}{\Delta t^2}[M]\right]\left\{q_{(t)}\right\}-\left[\frac{1}{\Delta t^2}[M]-\frac{1}{2\,\Delta t}[C]\right]\left\{q_{(t-\Delta t)}\right\} \qquad (2.1)$$

in which: $\Delta t$ is the integration time interval, $\{q_{(t)}\}$ is the nodal coordinate vector in the time $t$, $\{F_{(t)}\}$ is the nodal external forces vector in the time $t$, $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively.

When the system mass and damping matrices $[M]$ and $[C]$ are both diagonal, there is no need to use any solution process of system of equations to determine the vector $\{q_{(t+\Delta t)}\}$ in the expression (2.1). In this case, the method is considered explicit and the expression of the central finite differences can be written as:

$$q_{(t+\Delta t)}=\frac{1}{1+\dfrac{c_m\ \Delta t}{2}}\left[\frac{f_{(t)}\ \Delta t^2}{m}+2\,q_{(t)}\ -\left(1-\frac{c_m\ \Delta t}{2}\right)q_{(t-\Delta t)}\right] \qquad (2.2)$$

in which: $q$ denotes the nodal coordinate in either the $x$, $y$ or $z$ direction, $f$ the resultant nodal force component in the corresponding direction, $m$ the nodal mass, $c_m$ the damping coefficient, which is proportional to mass: $c_m = c\,/\,m$ and $c$ the viscous damping coefficient.

The resultant nodal force $f(t)$ consists of the gravitational forces $f_g(t)$ (dead weight and external nodal forces) and axial forces $f_a(t)$ in the truss elements, due to axial deformations.

In each integration step, *i.e.*, in the evaluation of the Eq. (2.2) for all nodes, in the $x$, $y$ and $z$ directions, the updated nodal coordinates lead to axial deformations of the truss elements, which react with axial forces $f_a(t)$ opposing the displacements. For instance, in a element of axial stiffness $E.A$ and instantaneous length $L(t)$, with linear behavior, the axial force at any time is given by:

$$f_a(t)\ =\ E\ A\left[\frac{L(t)-L(0)}{L(0)}\right] \qquad (2.3)$$

where: $L(0)$ is the initial length of the element (in $t = 0$)

To get the $f(t)$ components, $f_a(t)$ must be multiplied by director cosines of the element axis in the deformed state, and such components added with the respective gravitational force $f_g(t)$ in the $x$, $y$ and $z$ direction, acting on the node. It is important to quote that physical nonlinearities may be easily implemented, just replacing the linear relation given by Eq. (2.3), and the geometrical nonlinearity is always considered, since the nodal coordinates are updated after each integration step.

Convergence and accuracy of the solution depend basically on the integration time interval $\Delta t$. Since the method is only conditionally stable (Bathe, 1982), it is necessary that $\Delta t \le \Delta t_{crit}$. The difficulty in the determination of $\Delta t_{crit}$ consists on calculate the smallest vibration period, which corresponds to the largest eigenvalue of the structure. However, for latticed structures, the critical time interval can be estimated by (Groehs, 2001):

$$\Delta t \le \Delta t_{crit}=\frac{L_{min}(0)}{\sqrt{E/\rho}} \qquad (2.4)$$

in which: $L_{min}(0)$ is the initial length (in $t = 0$) of the smallest truss element, $E$ is the elastic modulus (Young's modulus) and $\rho$ is the material mass density. It should be pointed out that the expression (2.4) serves only to give an approximated value to $\Delta t$. To assure precision, without calculating the largest eigenvalue of the structure, it is necessary to compare the results for at least two different values of $\Delta t \le \Delta t_{crit}$.

More details about this numerical integration method, applied to analysis of *TL* towers and cables, can be found in Kaminski *et al.* (2005), Riera *et al.* (2005) and Miguel *et al.* (2005).

## 3. APLICATION TO *TL* LATTICED STEEL TOWERS

The difficulty in the use of an explicit numerical integration method in the analysis of *TL* latticed steel towers is the computational effort required for solving the displacement equations during a time compatible with the response period of the structure. The more high the tower, more flexible and longer such response period.

The integration time interval $\Delta t$ must be chosen according to Eq. (2.4), in which the denominator is the P-wave propagation velocity along the element. For instance, for a steel member with $L(0) = 0.5$ m, $E = 2.1 \times 10^{11}$ N/m$^2$ and $\rho = 7850$ kg/m$^3$, the $\Delta t$ should be smaller than $1 \times 10^{-4}$ s, or $1 \times 10^{-5}$ s for accuracy, which is a quite short time interval in comparison with usual *TL* tower vibration periods, in the order of one second.

In order to reduce any dynamic amplification of response, load should be gradually applied (ramp-loading) during a time covering a few natural vibration periods (5 to 10 periods). This means that, for a self-supporting tower with a fundamental vibration period of one second (around of 60 m height), load application may consume from half million to one million integration steps.

This difficulty, however, may be regarded as of minor importance nowadays, since even personal computers are able to undertake calculations of this type within a reasonable processing time.

## 4. CONSTITUTIVE LAW OF TOWER MEMBERS

Members of *TL* latticed steel towers are usually angle bars, often directly connected through galvanized bolted joints. To predict the loading capacity of an angle bar it is necessary to take residual stresses and initial imperfections into account, which are needed to determine the entire nonlinear load-deflection curve of the member. For members that fail in the inelastic range of column behavior, the effect of initial imperfections are not clear-cut (Galambos ,1998). Initial imperfections (out-of-straightness or initial curvatures) and residual stresses are always present in steel bars. In this paper, a constitutive law relating the compressive force $P$ in a steel angle member with the total apparent shortening $u$ was adopted, which accounts for initial imperfections, inelastic deformations and nonlinear effects.

The lateral deflection $y_{tc}$ induced by a compression load $P$ on a column with an initial imperfection $y_{oc}$ (Fig. 4.1), is closely approximated by Eq. (4.1), in which $P_E$ denotes the elastic bucking load of the column (Gere and Timoshenko, 1997):

$$P = P_E \left[ 1 - \frac{y_{oc}}{y_{tc}} \right] = P_E \left[ 1 - \frac{y_{oc}}{y_{oc} + y_c} \right] \tag{4.1}$$

where:

$$P_E = \frac{\pi^2 \; E \; I}{L^2} \tag{4.2}$$

in which: $E.I$ is the elastic flexural stiffness and $L$ is the length of the column. For other end conditions, the length $L$ should be replaced by $K.L$, *i.e.,* by the column effective length.

Since $L_S = \ell + u_i$ and assuming that the shortening of the column is due to its initial imperfection only and moreover that $y(x)$ is a half wave sine given by Eq. (4.3), Eq. (4.4) may be obtained:

$$y(x) = y_c \; sin\left[ \frac{\pi \, x}{\ell} \right] \tag{4.3}$$

$$y_c = \frac{2}{\pi} \sqrt{L_S \, u_i - u_i^2} \tag{4.4}$$

Substituting Eq. (4.4) in (4.1) the resulting compression $P$ may be fitted by the function:

$$P = -P_E \left[ 1 - e^{-\frac{E\,A}{L\,P_E} |u|} \right] \tag{4.5}$$

in which $A$ is the cross-section area, $u$ is the total shortening of the column $(u = u_i + u_{as})$, $u_i$ and $u_{as}$ the shortenings due to initial imperfection and axial stiffness of the column, respectively, as shown in Fig. 4.2. $P_E$ was calculated for all members of the tower using the effective-length factor $K$, prescribed by the Brazilian code NBR 8.850:1985.

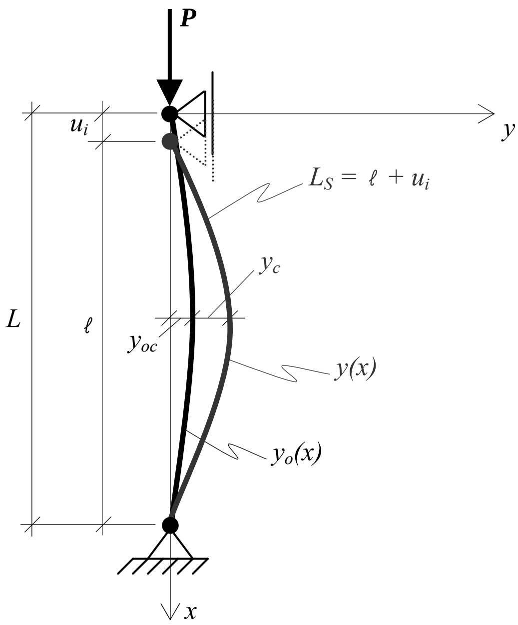



Figure 4.1. Compressed member with initial imperfection $y_{oc}$.

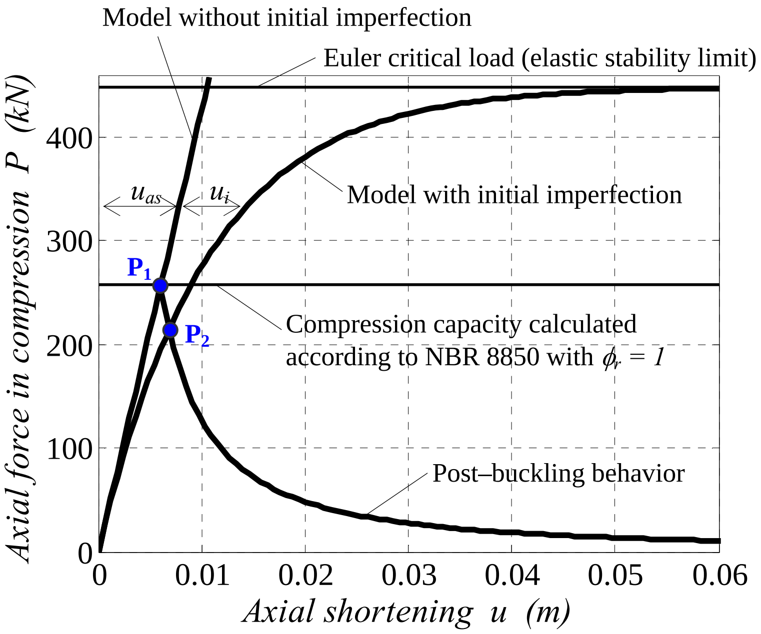



Figure 4.2. Predicted behavior in compression of a 6m long steel angle, with and without initial imperfections.

The magnitude of the initial imperfection $y_{oc}$ is limited by construction specifications, normally expressed as a fraction of the member length, *e.g.*, the Brazilian code NBR 6.109:1994 establishes that: $y_{oc} < L / 500$ for steel angle members with $b \geq 75\ mm$ and $y_{oc} < L / 250$ when $b < 75\ mm$, where $L$ is the initial member length and $b$ is the full width of the angle leg. Due to residual stresses, Euler's formula is restricted to stresses below the limit ($fy$ - $fr$), in which $fr$ is the maximum compression residual stress and $fy$ is the yield stress of the material.

A typical compression member performance curve is composed of three regions: elastic, inelastic and post-bucking, as shown in Fig. 4.2 for a steel angle with and without initial imperfection. The compression capacity of the tower members, which defines the beginning of the post-bucking branch, was determined according to Brazilian code NBR 8.850:1985, which is similar to ASCE Standard 10-97 (2000).

Post-bucking $P$ *vs.* $u$ curves were obtained for each member of the tower, starting from the pin-ended column shown in Fig. 4.3, with initial imperfection $y_{oc}$, which is supposed equal to half the maximum value established by the Brazilian code NBR 6.109:1994. When the column is loaded, it will be subjected to flexo-compression, since the initial imperfection introduces a moment $M = P\ (y_{oc} + y_c) = P\ .\ y_{tc}$, where $y_c$ is a function of $P$. The loading capacity of this imperfect column is reached when the entire central section is fully plastified. For simplicity, the elastic bending strains in the two half of the column are ignored in presence of the deformations due to the plastic hinge formed in the central section, as shown in Fig. 4.3. The stress diagram in the plastic hinge may be decomposed in a central zone, in equilibrium with the axial load *P*, and two lateral zones, that account for the bending moment *Mp*, as illustrated in Fig. 4.4.

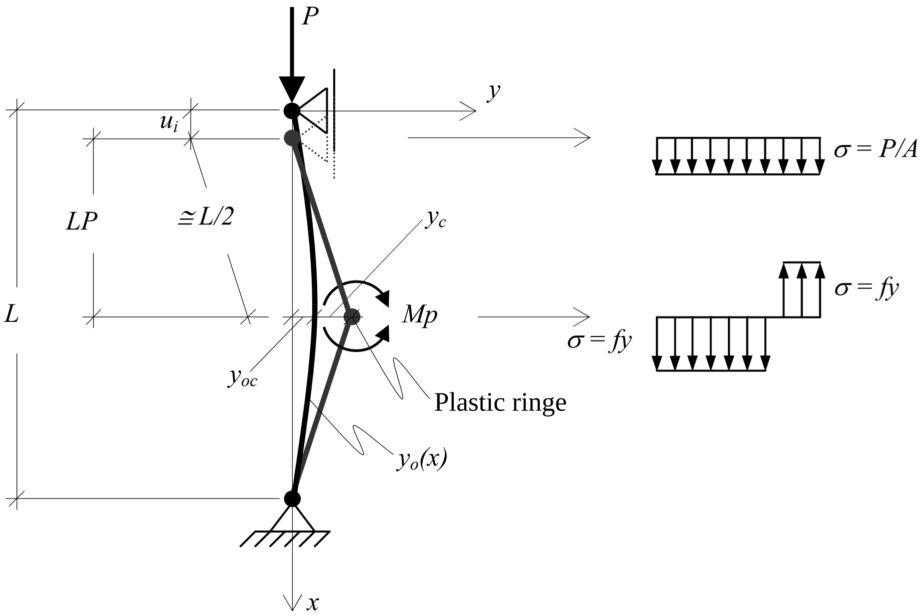



Figure 4.3. Pin-ended column with plastic hinge in the central section.

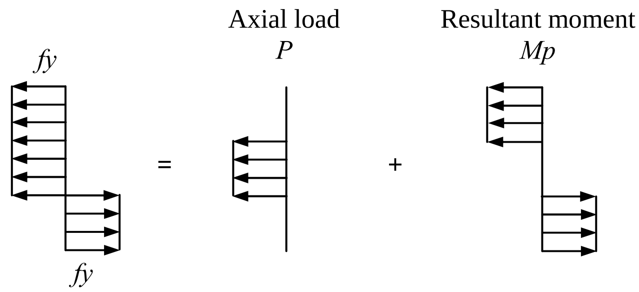



Figure 4.4. Stresses diagram in plastic hinge.

The strength of a column subjected to an axial load *P* and a minor-axis moment $Mp_z$ may be plotted in an interaction diagram. The interaction diagrams $P$ *vs* $Mp_z$ or $P$ *vs* $Mp_x$ of all angle bars of the tower "1" were determined.

Since:

$$P \; y_{tc} = P \; (y_{oc} + y_c) = Mp \tag{4.6}$$

in which *Mp* is a function of *P*, defined by the interaction diagram, considering that $L \cong u_i + 2\,LP$ (Fig. 4.3), the following relation between $y_c$ and $u_i$ can be found:

$$y_c = \sqrt{\left(\frac{L}{2}\right)^2 - \left(\frac{L - u_i}{2}\right)^2} \tag{4.7}$$

Substituting (4.7) in (4.6), a relation between $P$ and $u_i$ results. Adding the shortening due to the axial flexibility of the column $u_{as}$, the final relation is fitted by a potential function:

$$P = a \; u^b \tag{5.8}$$

for each member, crossing the point $P_2$ in the case of models with initial imperfection (Fig. 4.2).

The behavior in tension of steel angles members was assumed elasto-plastic, with the elastic range defined according to the NBR 8.850:1985 code. The maximum elongation before rupture $u_{t\,max}$ was admitted equal to 1.5% of the length $L$.

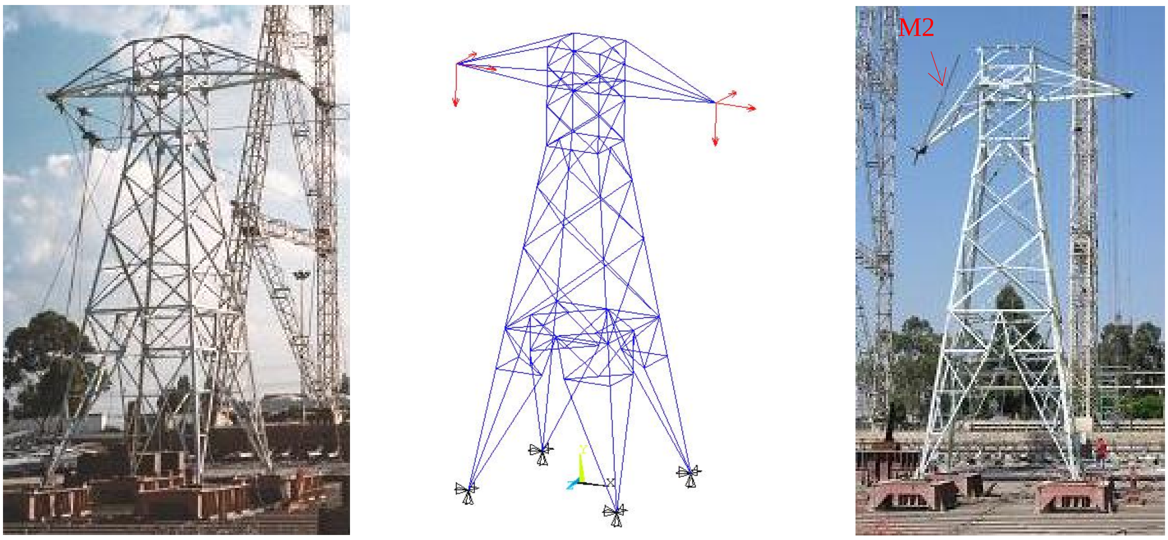
## 5. EXPERIMENTAL AND NUMERICAL RESULTS

In order to demonstrate the easy implementation of this numerical approach, a *TL* tower prototype proposed by CIGRÉ's Working Group 08, and used in a study of Silva *et al.* (2005), is analyzed. This prototype, named "tower 1" and shown in Fig 5.1 (a), was designed to eight loading cases.

In Fig 5.1 (b) is shown the model of tower "1" with the loading case "4D", which is the more several and consists of six nodal forces of 49.05 kN (5000 kgf) applied in *x*, *y* and *z* directions, in two nodes.

For the loading case "4D", the prototype was destructively tested, *i.e.*, the forces were gradually applied above the design loading, until the collapse.

During the destructive test, the rupture occurred on the member "M2", by tensile breaking / shearing of the bolts as shown in Fig. 5.1 (c), at stage of 101.4% of the design loading.




(a) (b) (c)

Figure 5.1: (a) Tower "1" – Prototype testing; (b) Model of tower "1" with loading case "4D"; (c) Destructive testing of tower "1".

In the numerical example to find the collapse loading, the nodal forces were gradually applied according to the function:

$$F(t) = F_o \frac{t}{t_R} \Rightarrow for\ t \le t_R \tag{5.1}$$

in which $t_R$ is the duration of the loading process. In this example, $t_R$ was taken as 1 s in order to cover approximately 15 vibration periods, thus avoiding dynamic amplifications. This value was found in a test with $F_o$ set as 24.525 kN in Eq. 5.1 (for $t \le t_R$), applied in *x*, *y* and *z* directions, in two nodes (half of the design loading "4D"). When $t > t_R$: $F(t) = F_o$. The structural response (displacement) at the top of the tower is plotted in Fig. 5.2.

After defining the time for loading application as 1 s, an evaluation to find the collapse load was performed, with $F_o$ = 98.1 kN (two times the design load) in Eq. 5.1. Further parameters are:

$$c_m = 0.5\ s^{-1} \tag{5.2}$$

$$\Delta t = 2 \times 10^{-5}\ s \tag{5.3}$$

The integration time interval $\Delta t$ was chosen according Eq. 2.4, in which the initial length of the smallest truss element of the tower "1" $L_{min}(0)$ is equal to 0.5 m, resulting a $\Delta t_{crit} = 1 \times 10^{-4}$. Tests with different $\Delta t \leq \Delta t_{crit}$ were performed to assure the precision of the results.

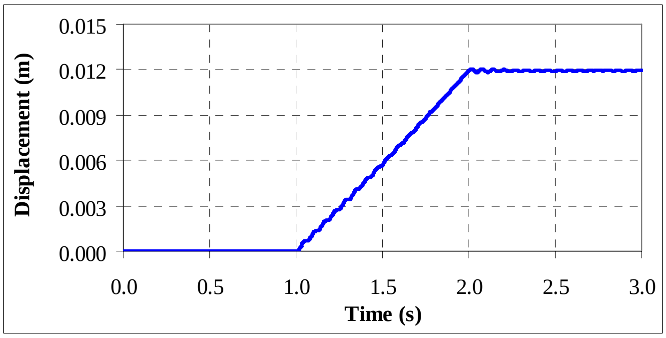



Figure 5.2. Structural response for loading case "4D"

The result in terms of displacement at the top of the tower is shown in Fig. 5.3, in which can be observed that the tower collapsed at t = 1.49 s, being the components of nodal forces $F(t)$ = 48.07 kN or 98% of the design load (49.05 kN). This load occurred for a displacement at the top of 0.025 m. This critical load is quite close of the collapse load found in the destructive testing (101.4%).

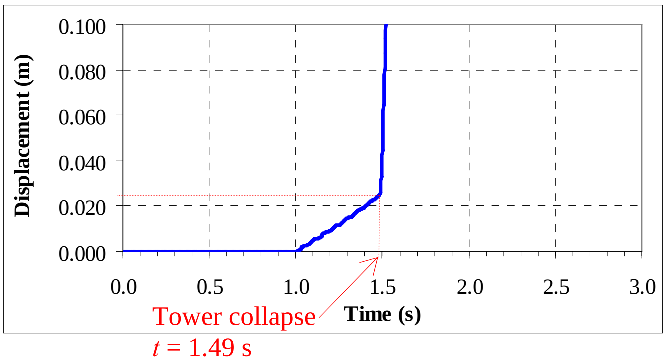



Figure 5.3. Displacement at top of the tower in the numerical example to find the collapse loading.

In Fig. 5.2 and 5.3 the time interval between 0 and 1 s was used for application of the tower dead weight. The application of the nodal forces started at 1 s and finished in 2 s.

## 6. SUMMARY AND CONCLUSIONS

The methodology presented for evaluating the collapse loads of transmission line (*TL*) latticed steel towers allows considering physical and geometrical nonlinearities without the need of developing complex computer program. The FORTRAN code used in the example has around 1000 lines, including comments, input and output data.

The total integration time of 3 s in the test of Fig. 5.2 was reached after 150000 integration steps, performed by an Athlon 64 (3200+) processor in less than 7 seconds.

The efficiency of this approach is proven by the good agreement between numerical and experimental results, showing that explicit integration methods are a good option to predict the collapse load of this kind of structure.

## 7. ACKNOWLEDGEMENTS

The authors acknowledge the financial support of CNPq and CAPES, Brazil.

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