A STUDY OF LIMITING CONDITIONS IN HYDRODYNAMIC LUBRICATION OF JOURNAL BEARINGS

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Abstract. *The present work deals with two simplified formulations for the Reynolds Equation, to study hydrodynamic lubrication of journal bearings. The first formulation considers long bearing (D/L $\rightarrow$ 0) with any eccentricity value. The second formulation is not restricted to the long bearing limitation, but considers only small eccentricity values. Thus, two semi-analytical solutions have been obtained for the problem. In the second case, the solution is carried out by Generalized Integral Transform Technique approach; the approach is applied to solve the partial differential equations. A comparative analysis of the results is realized to determine the domain of validity of each formulation.*

Keywords: *Reynolds Equation, hydrodynamic lubrication, journal bearings, GITT, tribology.*

1. INTRODUCTION

The analysis of journal bearings is probably the most important part of the classical hydrodynamic theory of lubrication. It is also most difficult and complex due to integration of the journal bearing equation (Reynolds equation). The journal bearings are mainly used for decrease the friction existing between solids parts of rotating machines and weaken the loads variations supported for these ones. The journal bearing must support the load carried with the energy lost minimal and low wear.

Various techniques for performance analysis of journal bearings are presented in the literature. Among these ones, interesting approximations consider infinitely long bearing, simplifying the solution of the Reynolds equation. However, Warner (1963) used a side flow leakage factor to improve the solution accuracy of long bearing approximation. In similar way Ritchie (1975), to improve the accuracy of short bearing approximation at high eccentricity, introduced the short bearing solution by Galerkin's method. A simple and precise solution for the infinitely long and infinitely narrow bearings is presented by Reason and Narang (1982). This technique shows good results and it is compared to the finites elements method (FEM). An analytic model of second order is presented by Capone (1994); this model reduces infinitely long and infinitely narrow bearings theory in limit cases characterized for a parameters pair (L/D, ε).

Williams *et al.* (1987) describe a procedure to solve Navier-Stokes equation for steady flow, in three dimensions of a non-Newtonian lubricant contained by finite journal bearing. The method uses an approximation by finites differences with a new technique of computational fluid dynamic knew as SIMPLES.

In this paper three simplifications are considered: (a) infinitely long bearing and small eccentricity, (b) infinitely long bearing and any eccentricity (except small) and (c) simplified formulation Reynolds equation (i.e. $\varepsilon \rightarrow 0$ where $\tilde{h}^3 \approx 1$). In case "c" the Integral Transform Technique approach (Mikhailov and Özisik, 1984) is used to transform the PDE in an infinity system of ODE that can be solved for analytically.

2. MATHEMATICS FORMULATIONS AND PROBLEMS SOLUTIONS

2.1 Reynolds Equation

The physical configuration of a journal bearing flow geometry is shown in Fig. 1. It consists of a cylindrical journal housed inside a cylindrical bearing. The journal is rotating at a given angular velocity relative to the bearing, while being supported by the lubricating action of the oil in the narrow clearance. The journal of radius R approaches the bearing surface at any circumferential section θ with velocity U. The film thickness h is a function of θ, i.e. h = c [1 + e Cos (θ)], where "c" is the radial clearance and "e" is the eccentricity of the journal center.

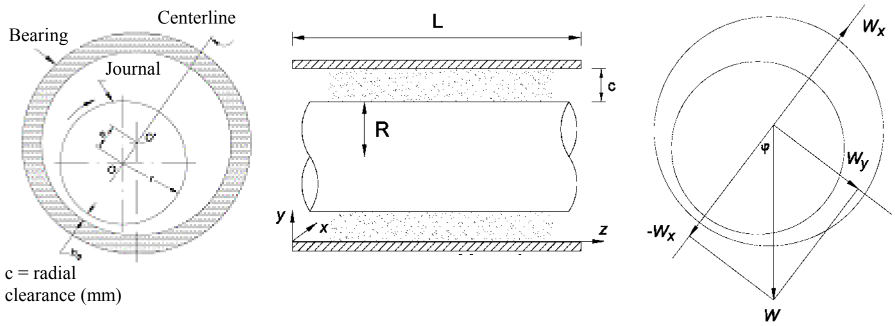

Figure 1. Journal bearing geometry, coordinates system and physical configuration.

In this theoretical study, the lubricant in the system is considered as a Newtonian fluid. In the meanwhile, the fluid film is assumed to be thin, and the body force and body couple are not taken into account. Then, the Reynolds equation for Newtonian fluid in Cartesian coordinates (Cameron, 1966 and 1987; Chadan, 1982) is given by:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu U\frac{dh}{dx} \tag{1.a}$$

where h is a defined as,

$$h = c\,[1 + e\,\mathrm{Cos}\,(\theta)] \tag{1.b}$$

To obtain Reynolds equation in the dimensionless form the following groups are introduced:

$$\theta=\frac{x}{R};\quad \eta=\frac{z}{L};\quad \Phi=\frac{\theta}{\theta_L};\quad \tilde{h}=\frac{h}{c};\quad \varepsilon=\frac{e}{c};\quad \lambda=\frac{D}{L};\quad P=\frac{pc^2}{\mu UR};\quad \xi=\frac{y}{c};\quad \tilde{u}=\frac{u}{U};\quad \tilde{w}=\frac{w}{U} \tag{2.a-j}$$

where θ_L is the angular position at which cavitation starts (rad). Then, the Reynolds equation can be written in a no dimensional form as

$$\left(\frac{\theta_L\lambda}{2}\right)^2\frac{\partial}{\partial\eta}\left(\tilde{h}^3(\Phi)\frac{\partial P}{\partial\eta}\right)+\frac{\partial}{\partial\Phi}\left(\tilde{h}^3(\Phi)\frac{\partial P}{\partial\Phi}\right)=6\theta_L\tilde{h}'(\Phi) \tag{3.a}$$

where

$$\tilde{h}(\Phi)=1+\varepsilon\mathrm{Cos}(\theta_L\Phi);\qquad \tilde{h}'(\Phi)=-\varepsilon\theta_L\mathrm{Sin}(\theta_L\Phi) \tag{3.b, c}$$

The boundary conditions for the lubricant in the film region are the Reynolds conditions:

$$P=0\quad \text{at}\quad \eta=0\ \ \text{and}\ \ \eta=1 \tag{3.d, e}$$

$$P=0\quad \text{at}\quad \Phi=0\ \ \text{and}\ \ P=\frac{\partial P}{\partial\Phi}=0\quad \text{at}\quad \Phi=1 \tag{3.f-h}$$

2.2. Case A: Formulation for Infinitely Long Bearing and Small Eccentricity

An approximation considered in hydrodynamic lubrication is to assumer that journal bearing infinitely long (D/L $\to$ 0) and its eccentricity is small ($\varepsilon \to 0$). Thus, the term $\frac{\partial P}{\partial\eta}$ is neglected and the flow in the direction η is zero, consequently the problem develop into one-dimensional with Reynolds equation equal to,

$$\begin{cases} \dfrac{d^2P}{d\Phi^2} = 6\theta_L \tilde{h}'(\Phi) \\ P = 0 \quad \text{at} \quad \Phi = 0 \\ P = \dfrac{\partial P}{\partial \Phi} = 0 \quad \text{at} \quad \Phi = 1 \end{cases} \tag{4.a-d}$$

Integrating Eq. (4) and making use boundary conditions (4.b) and (4.d) we obtain

$$P(\Phi) = 6\varepsilon\left[\mathrm{Sin}(\theta_L\Phi) - \theta_L \mathrm{Cos}(\theta_L\Phi)\right] \tag{5.a}$$

to calculate the θ_L (angle that characterize the oil film length) we will use the boundary condition (4.c) and the result is given by:

$$\mathrm{Sin}(\theta_L\Phi) = \theta_L \mathrm{Cos}(\theta_L\Phi) \tag{5.b}$$

2.3. Case B: Formulation for Infinitely Long Bearing and any Eccentricity (except small)

In this case is considered an infinitely long bearing (D/L $\rightarrow$ 0) and any value eccentricity (except small). Thus, Reynolds equation reduces to

$$\frac{d}{d\Phi}\left(\tilde{h}^3(\Phi)\frac{dP}{d\Phi}\right) = 6\theta_L \tilde{h}'(\Phi) \tag{6}$$

after some manipulations we have the following solution,

$$P(\Phi) = \frac{6\varepsilon}{\tilde{h}^3(\Phi)}\left\{\mathrm{Sin}(\Phi\theta_L) + \frac{\varepsilon\theta_L\left[\mathrm{Cos}(2\theta_L) - 2\right] - \theta_L \mathrm{Cos}(\theta_L)}{1 + \varepsilon \mathrm{Cos}(2\theta_L) + 3\varepsilon\theta_L \mathrm{Sin}(\theta_L)}\Phi\right\} \tag{7.a}$$

and

$$\mathrm{Sin}(\theta_L\Phi) = \theta_L \mathrm{Cos}(\theta_L\Phi) \tag{7.b}$$

Can be observed of the Eq. (5.b) or (7.b) that θ_L is independent of eccentricity (ε).

2.4. Case C: Simplified Formulation Reynolds Equation

Now, is presented a simplified formulation for Reynolds equation, where is assumed that for small value of ε (i.e. $\varepsilon \rightarrow 0$) the term $\tilde{h}^3$, in Reynolds equation, is one (i.e. $\tilde{h}^3 \cong 1$). Thus, the simplified Reynolds equation is given by:

$$\begin{cases} \left(\dfrac{\theta_L\lambda}{2}\right)^2 \dfrac{\partial^2 P}{\partial \eta^2} + \dfrac{\partial^2 P}{\partial \Phi^2} = f(\Phi) \\ P = 0 \quad \text{at} \quad \Phi = 0 \\ P = \dfrac{\partial P}{\partial \Phi} = 0 \quad \text{at} \quad \Phi = 1 \end{cases} \tag{8.a-d}$$

where

$$f(\Phi) = -6\theta_L^2 \varepsilon \mathrm{Sin}(\theta_L\Phi) \tag{8.e}$$

The homogeneous formulation defined above by Eq. (8) can also be solved by the Classical Integral Transform Approach (Mikhailov and Özisik, 1984). Then, following the basic steps this technique, the appropriate eigenvalue problem needed for its solution is given by:

$$\begin{cases} \dfrac{d^2\psi_i(\eta)}{d\eta^2}+\mu_i^2\psi_i(\eta)=0 \\ \psi_i(0)=0; \qquad \psi_i(1)=1 \end{cases} \tag{9.a-c}$$

where $\psi_i(\eta)$ and μ_i are, respectively, the eigenfunctions and eigenvalues. This eigenvalue problem is solved to give:

$$\psi_i(\eta)=\mathrm{Sin}(\mu_i\eta); \qquad \mu_i=i\pi; \qquad N_i=\int_0^1 \psi_i^2(\eta)d\eta=1/2 \quad \text{to} \quad i=1,\,2,\,3... \tag{10}$$

The eigenvalue problem allows for the development of the following integral transform pair:

$$\underbrace{\tilde{P}_i(\Phi)=\int_0^1 \tilde{\psi}_i(\eta)P(\Phi,\eta)d\eta}_{\text{Transform}} \qquad \underbrace{P(\Phi,\eta)=\sum_{i=1}^{\infty}\tilde{\psi}_i(\eta)\tilde{P}_i(\Phi)}_{\text{Inverse}} \tag{11.a, b}$$

where $\tilde{\psi}_i(\eta)$ is the normalized eigenfunction, defined by

$$\tilde{\psi}_i(\eta)=\frac{\psi_i(\eta)}{N_i^{1/2}} \tag{12}$$

Now, following the basic steps of the methodology, the PDE (8.a) is multiplied by $\tilde{\psi}_i(\eta)$ and integrated in the finite region [0, 1] in η, which after to use the orthogonality property and the inversion formula (11.b), results the following decoupled system of ordinary differential equations for the transformed potentials, $\tilde{P}_i(\Phi)$:

$$\begin{cases} \dfrac{d^2\tilde{P}_i}{d\Phi^2}-m_i^2\tilde{P}_i=C_i f(\Phi) \\ \tilde{P}_i(0)=0; \qquad \dfrac{d\tilde{P}_i(1)}{d\Phi}=0 \end{cases} \tag{13.a-c}$$

where

$$m_i=\frac{\theta_L\lambda\mu_i}{2}; \qquad C_i=\int_0^1 \tilde{\psi}_i(\eta)d\eta \tag{13.d, e}$$

The solution for the transformed potential given by Eq. (13) is readily obtained in the form:

$$\tilde{P}_i(\Phi)=\frac{6\theta_L^2 C_i}{m_i\left(m_i^2+\theta_L^2\right)}\left[m_i\mathrm{Sin}(\theta_L\Phi)-\frac{\theta_L\mathrm{Cos}(\theta_L)}{\mathrm{Cosh}(m_i)}\mathrm{Sinh}(m_i\Phi)\right] \tag{14}$$

The angle θ_L depends of the cavitation frontier, if there is not cavitation $\theta_L = \theta$ and if there is cavitation its value is $\theta_L = \theta + \alpha$, where α is the cavitation angle. The problem defined for the Eq. (8) depends of θ_L and it makes part of the problem solution. Then, θ_L is calculated using the boundary condition (8.c) (i.e. $P(\Phi,\eta=1)=0$), of the following form

$$\sum_{i=1}^{\infty} \tilde{\psi}_i(\eta)\,\tilde{P}_i(\Phi)\Big|_{\Phi=1} = 0 \tag{15}$$

The non-linear Eq. (15) is truncated in a sufficiently high order N to determine θ_L. In the solution of such equation an appropriate subroutine must be employed, such as the subroutine ZREAL from IMSL Library (1989). After solution of the non-linear Eq. (15), the inversion formula, Eq. (11.b), is recalled to provide the pressure field.

2.5. Determination of load carried and attitude angle

The load components (axial W1 and normal W2) in the dimensionless form are defined by:

$$\tilde{W}_1 = -\theta_L \int_0^1 P(\phi)\cos(\phi\,\theta_L)\,d\phi, \qquad \tilde{W}_2 = \theta_L \int_0^1 P(\phi)\ \mathrm{sen}(\phi\,\theta_L)\,d\phi \tag{13, 14}$$

The dimensionless load carrying capacity *W* and the attitude angle φ are:

$$\tilde{W} = \sqrt{\tilde{W}_1^2 + \tilde{W}_2^2}, \qquad \varphi = \mathrm{tg}^{-1}\left(\frac{\tilde{W}_2}{\tilde{W}_1}\right) \tag{15, 16}$$

2.6. Determination of the friction coefficient

The friction force can be obtained by integrating the shear stress around the journal surface. The shear stress at the journal surface:

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=h} = \mu\left(\frac{U}{h} + \frac{h}{2\mu}\frac{\partial p}{\partial x}\right) \tag{17}$$

The friction force can be obtained by integrating the shear stress around the journal surface and written as:

$$\tilde{F} = \theta_L \int_0^1 \left(\frac{1}{\tilde{h}(\phi)} + \frac{\tilde{h}(\phi)}{2\theta_L}\frac{dP}{d\phi}\right) d\phi \tag{18}$$

The coefficient of friction can be obtained by dividing friction force by the film force:

$$C_f = \frac{\tilde{F}}{\tilde{W}} \tag{19}$$

2.7. Determination of side leakage flow

The dimensionless side leakage flow for a journal bearing is defined by:

$$\tilde{Q}_S = -\frac{\theta_L}{6}\int_0^1 \tilde{h}^3(\phi)\frac{\partial P}{\partial \eta}\bigg|_{\eta=0} d\phi \tag{20}$$

For the cases where P is function of ϕ only, the flow axial for journal bearing is,

$$\tilde{Q}_S = 0 \tag{21}$$

However when P is function of ϕ and η, we have:

$$\tilde{Q}_S = -\frac{\theta_L}{6}\sum_{i=1}^{\infty}\frac{\partial \tilde{\psi}_i}{\partial \eta}\bigg|_{\eta=0} E_i \tag{22}$$

where the coefficient E_i is obtained from following integral:

$$E_i = \int_0^1 \tilde{h}^3(\phi)\,\tilde{P}_i(\phi)\,d\phi \qquad (23)$$

3. RESULTS AND DISCUSSION

To solve the non-linear equations above, can be use the Mathematica 5.0 and. Therefore, also, to solve the Eq. (14) and (15) a computational program has been developed in FORTRAN 90/95, and it uses the subroutine ZREAL of library IMSL (1989).

In tables 1, 2 and 3 shown the convergence results for θ_L, φ, $\tilde{W}$, P_{max} (maximum pressure) of the problem given by Eq. (14) and (15) for small eccentricity. The convergence has been investigated with different truncation orders (NT) in the solution of the pressure equation. An excellent convergence rate is evaluated for the parameters studied in the medium plan of the journal bearing.

Table 1. Convergence de θ_L, φ, $\tilde{W}$ e P_{max} with $\varepsilon = 10^{-5}$ e $D/L = 10^{-5}$.

NT	θ_L	φ	$\tilde{W} \times 10^3$	$P_{máx} \times 10^4$
10	**257.453**	**70.9107**	0.130	0.871
20	**257.453**	**70.9107**	0.133	0.793
30	257.453	70.9107	0.134	0.836
40	257.453	70.9107	0.135	0.806
50	257.453	70.9107	0.135	0.829
100	257.453	70.9107	**0.135**	0.814
200	257.453	70.9107	**0.136**	**0.816**
400	257.453	70.9106	0.136	**0.818**
600	257.453	70.9108	0.136	0.818
#	**257.453**	**78.790**	**0.136**	**0.818**

\# Results obtained by Eq. (5.b) for $D/L \rightarrow 0$ and $\varepsilon \rightarrow 0$ (Case A).

In table 1 is presented the convergence behaviors θ_L (angle that characterize the oil film length), φ (attitude angle), $\tilde{W}$ (load carried) and P_{max} (maximum pressure) with $\varepsilon = 10^{-5}$ e $D/L = 10^{-5}$ (i.e. long journal bearing and small eccentricity). It is noted a good convergence rate for these parameters. The convergence is established with less of 10 terms in series solution for θ_L and φ, but $\tilde{W}$ converges with NT between 100 and 200 terms and P_{max} with NT between 200 and 400 terms. Also, it is observed in this table an excellent agreement with the analytic solution for journal bearing infinitely long ($D/L \rightarrow 0$) and its eccentricity is small ($\varepsilon \rightarrow 0$).

In tables 2 and 3 are presented the convergence behaviors form small eccentricity ($\varepsilon = 10^{-5}$) and short journal bearing (Table 2 - $D/L = 0.1$ and Table 3 - $D/L = 0.5$). It is observed that the parameters are converged with three digits significant for the low truncation orders, no more that 200 terms. But, how much lesser the journal bearing (i. e. $D/L = 0.1$) more terms in series solution are necessary to reach the convergence of θ_L.

Table 2. Convergence of θ_L, φ, $\tilde{W}$ and e P_{max} with $\varepsilon = 10^{-5}$ and $D/L = 0.1$.

NT	θ_L	φ	$\tilde{W} \times 10^3$	$P_{max} \times 10^4$
10	256.719	76.145	**0.116**	0.826
20	256.542	75.899	**0.117**	0.810
30	256.547	75.861	0.117	0.813
40	256.510	75.851	0.117	0.812
50	256.507	**75.847**	0.117	**0.813**
100	**256.505**	**75.843**	0.117	**0.812**
200	**256.504**	75.843	0.117	0.812
400	256.504	75.843	0.117	0.812

Table 3. Convergence of θ_L, φ, $\widetilde{W}$ and P_{max} with $\varepsilon = 10^{-5}$ e D/L = 0.5.

NT	θ_L	φ	$\widetilde{w} \times 10^4$	$P_{máx} \times 10^4$
10	232.708	80.366	**0.560**	**0.490**
20	232.678	80.350	**0.561**	**0.489**
30	**232.674**	**80.349**	0.561	0.489
40	**232.673**	**80.348**	0.561	0.489
50	232.673	80.348	0.561	0.489

The behavior of pressure field, $\widetilde{W}$, Q_S and C_f field are analyzed in graphic form. The results obtained in each formulation are compared using the following parameters $\varepsilon = 10^{-5}$ and D/L = 10^{-5}, 0.1 and 0.5.

Initially is analyzed (Figure 2), in solution obtained with CITT approach (Case C), that for an infinitely long bearing (D/L $\to$ 0) the pressure field is independent of the axial position (η).

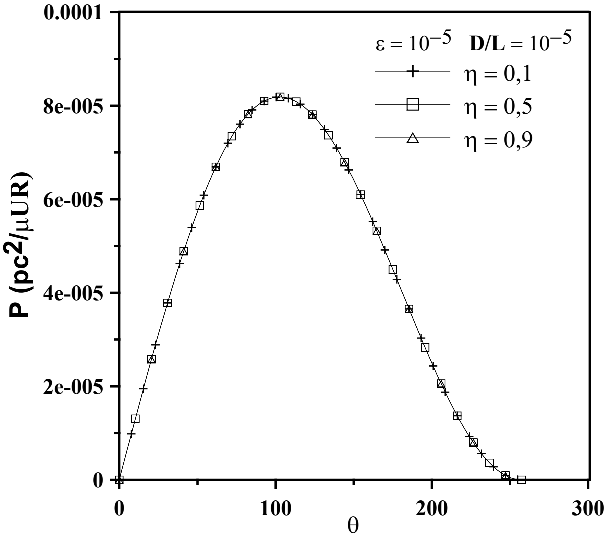

Figure 2 –Pressure field versus θ for different axial positions (η) with $\varepsilon = 10^{-5}$ e D/L = 10^{-5} (CITT Approach).

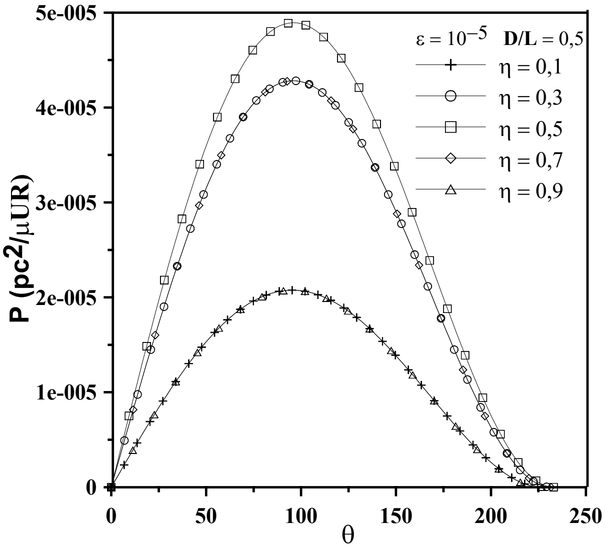

Figure 3 –Pressure field versus θ for different axial positions (η) with $\varepsilon = 10^{-5}$ e D/L = 0.5 (CITT Approach).

In Figure 3 is observed the same tendency of the curves of the figure 2. However, in this situation D/L = 0.5 characterize a narrow bearing and it is verified that pressure field depends of the axial positions η. Others facts are the coincidence between symmetric positions curves and the maximum pressure is obtained in the medium plan of bearing. From figures 2 and 3, is noted that the maximum pressure is inversely proportional to D/L relation.

In Figures 4 and 5 are shown the influence of the eccentricity in axial flow for D/L = 0.1 and 0.5, respectively. It is possible to observe in the Figures 4 and 5 that the curves are analogous with relation to the positions analyzed (η = 0.1, 0.5 and 0.9); given that the formulation is only valued for small ε, conclude that in despite of the curves have the tendency waited, the flow increase with raise of ε (for small ε). These results are not realists, because when ε raise, the results should to diverge of the results waited.

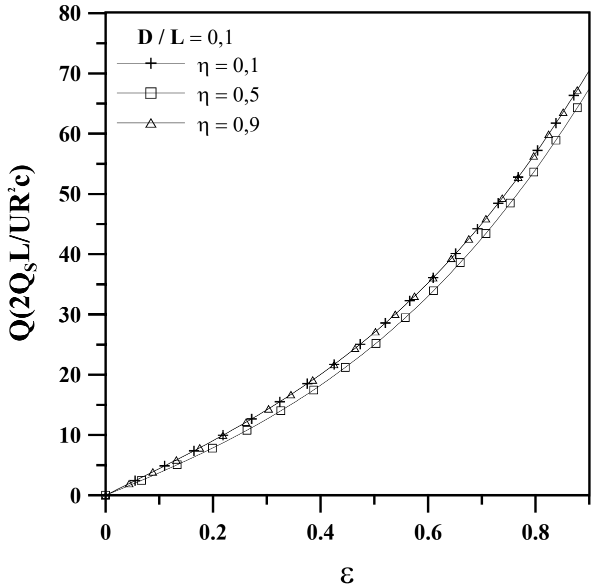

Figure 4 – Axial flow variation versus ε for different axial positions and D/L = 0.5 (CITT Approach).

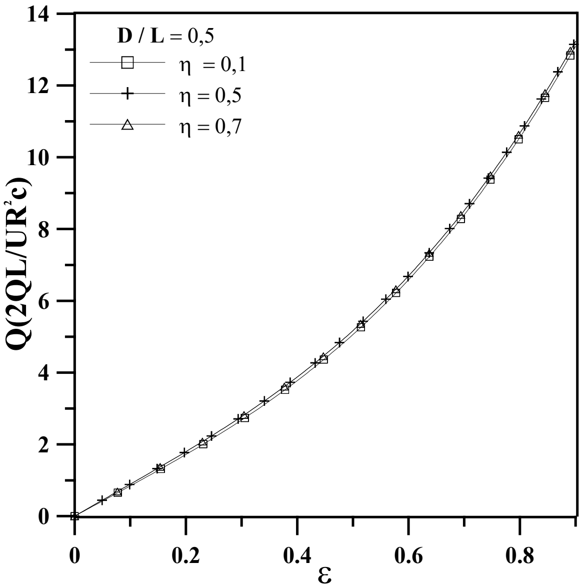

Figure 5 – Axial flow variation versus ε for different axial positions and D/L = 0.5 (CITT Approach).

In the figures 6 and 7 as in the figures 4 and 5 (axial flow), even that the curves have the comportment waited, W should increase with augment of ε. This results they are only valid for small ε, therefore this was the hypothesis used for this formulation.

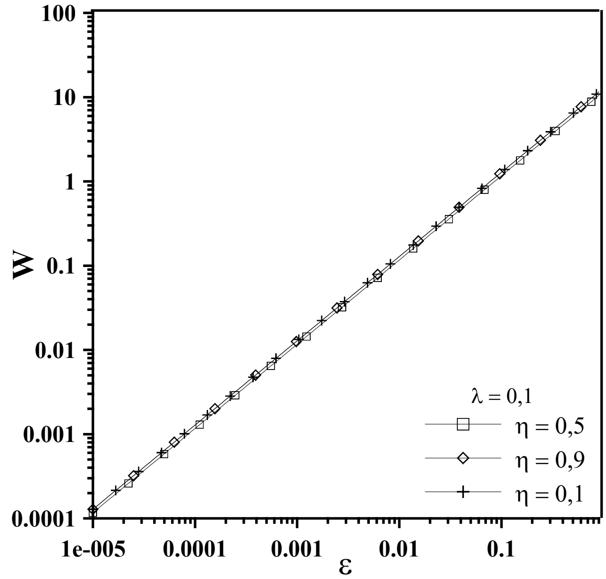

Figure 6 – Load carried variation versus ε for different axial positions and D/L = 0.1 (CITT Approach).

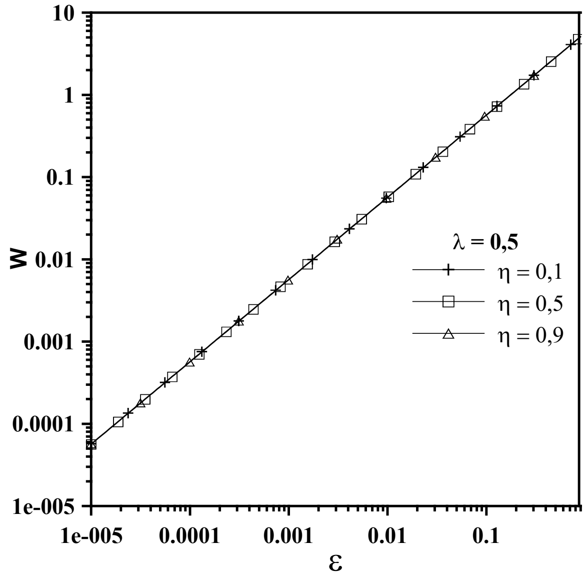

Figure 7 – Load carried variation versus ε for different axial positions and D/L = 0.1 (CITT Approach).

In the figures 8 and 9 are presented a comparison of the results, of the **W** (load carried) and $\mathbf{C_f}$ friction coefficient in function of the relative eccentricity **ε**, for limit cases presented in this paper (i. e. **A:** $\lambda \to 0$ and $\varepsilon \to 0$; **B:** $\lambda \to 0$ and any ε, except $\varepsilon \to 0$; and **C:** CITT Approach – $\varepsilon \to 0$). Can be observed in these figures an excellent agreement between the results, where if it verifies that the trend of the curves is kept until $\varepsilon \approx 0.1$. But, for $\varepsilon > 0.1$ the cases A and C ($\varepsilon \to 0$) are in perfect agreement and they disagree of the Case B. This results already waited since the formulations A and C have as hypotheses $\varepsilon \to 0$.

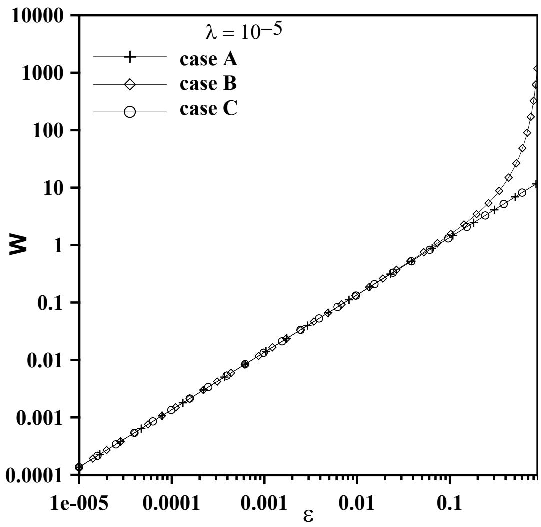

Figure 8 – Figure 8 – Comparison of the Load carried variation versus ε for case **A** ($\lambda \to 0$ and $\varepsilon \to 0$), case **B** ($\lambda \to 0$ and any ε, except $\varepsilon \to 0$) and case **C** (CITT Approach – small ε).

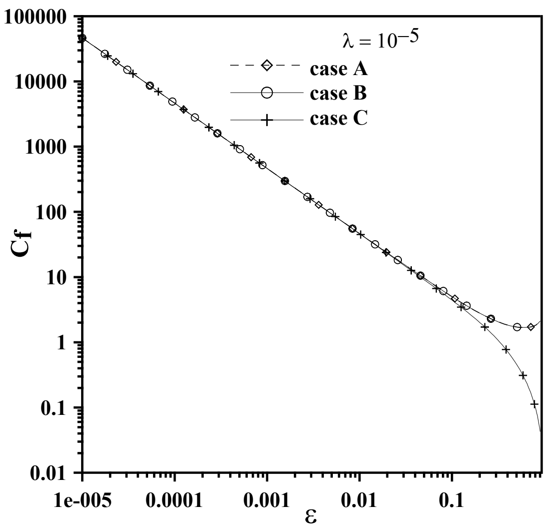

Figure 9 – Comparison of the friction coefficient variation versus ε for case **A** ($\lambda \to 0$ and $\varepsilon \to 0$), case **B** ($\lambda \to 0$ and any ε, except $\varepsilon \to 0$) and case **C** (CITT Approach – small ε).

4. CONCLUSIONS

In this work are presented three limit solutions for the Reynolds Equation, to study hydrodynamic lubrication of journal bearings, are they: Case **A** for $\lambda \to 0$ and $\varepsilon \to 0$, case **B** for $\lambda \to 0$ and any ε (except $\varepsilon \to 0$) and **C** for $\varepsilon \to 0$. In case **C** was applied the Classical Integral Transform Technique to get the analytical solution of the EDP, where was observed an excellent convergence rate in the series solution. In the condition of small eccentricity and for journal bearing infinitely long these solutions tend to converge to one same value. Also, in the cases **A** and **B** the transcendental equation to calculate the θ_L (angle that characterize the oil film length) is the same and do not depend of ε and λ.

To the journal bearing infinitely long the pressure field do not depend of axial coordinate. But, short journal bearing the pressure field starts to depend on the axial coordinate. For the short journal bearings (case C) symmetry is observed in pressure field and the maximum pressure is reached in medium plan of bearing in an angular position θ. This, however, does not depend of ε for short bearing, but it depends of the D/L ratio.

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