INTEGRAL TRANSFORM ANALYSIS OF HYDRODYNAMIC JOURNAL BEARINGS

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Abstract. This paper presents a two-dimensional model for the hydrodynamic lubrication of journal bearings. The analysis considers incompressible laminar flow in the bearing with a Newtonian lubricant. The model is based on the Reynolds equation of hydrodynamic lubrication for the calculation of the pressure field in the fluid film. The partial differential equation is then solved through the application of the so-called Generalized Integral Transform Technique (GITT). Therefore, a critical comparison of the present results with those presented in the literature is performed. The major contribution of this paper consists in the validation of the GITT approach for the solution of problems in hydrodynamic lubrication of journal bearings.

Keywords: *hydrodynamic lubrication, journal bearings, lubrication theory, GITT approach, tribology.*

1. INTRODUCTION

The performance of the journals bearings has been studied for many researchers, which have used various numeric techniques to solve Reynolds equation. Chandrawat and Sinhasan (1987) presented a comparison between the Gauss-Seidel iterative method and a method with complements linear problems. Tayal et al. (1982) investigated the effect of the nonlinearity on the performance of the journals bearings with finite width, by using finites elements method. Williams and Symmons (1987) analyzed a procedure to solve Navier-Stokes equations for the steady flow, three-dimensional of a non-Newtonian fluid into the journal bearing with finite width. The procedure applied the finites differences method. Sivak and Sivak (1981) obtained a numeric solution of Reynolds equation by modified Ritz method.

In this paper a hybrid method numeric-analytic, the GITT; it is utilized to solve Reynolds equations for different specifics eccentricity and relationships D/L. the results are compared with those of literature. The GITT (Generalized Integral Transform Technique) is a method with analytics origins. It derives of the Classical Integral Transform Technique presented in the literature by Mikhailov and Özisik, 1984. The basic idea of the method is to transform original partial differential equation in an ordinaries differential equations system that can be promptly solved.

2. MATHEMATICAL FORMULATION

2.1. Modified Reynolds equation

For steady and incompressible flow, the Navier-Stokes equations in Cartesians coordinates are:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{1a}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{1b}$$

$$\rho\left(u\,\frac{\partial w}{\partial x}+v\,\frac{\partial w}{\partial y}+w\,\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{1c}$$

And the three-dimensional continuity equation is:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0 \tag{2}$$

Making the usual hypotheses of hydrodynamic lubrication, the Navier-Stokes equations become:

$$\frac{\partial p}{\partial x} = \mu\,\frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = \mu\,\frac{\partial^2 w}{\partial y^2} \tag{3a-c}$$

2.2. The boundary conditions

(a) For bearing surface.

$$u\,(x, 0, z) = 0 \qquad w\,(x, 0, z) = 0 \tag{4a-b}$$

(b) For journal surface.

$$u\,(x, h, z) = U \qquad w\,(x, h, z) = 0 \tag{5a-b}$$

Integrating the Eq. (3a) and (3c) with boundary conditions (4a), (4b), (5a) e (5c), the velocity components u and w are:

$$u = U\,\frac{y}{h}+\frac{1}{2\mu}\frac{\partial p}{\partial x}\,y\,(y-h) \qquad w = \frac{1}{2\mu}\frac{\partial p}{\partial z}\,y\,(y-h) \tag{6a-b}$$

Substituting the Eq. (6a) and (6b) into of the Eq. (2) and integrating through of the thickness of the fluid film with boundary conditions v (x, 0, z) = v (x, h, z) = 0, the Reynolds equations is:

$$\frac{\partial}{\partial x}\left(h^3\,\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\,\frac{\partial p}{\partial z}\right) = 6\,\mu\,U\,\frac{dh}{dx} \tag{7}$$

where

$$h = c\,(1+\varepsilon\,\mathrm{Cos}\theta) \tag{8}$$

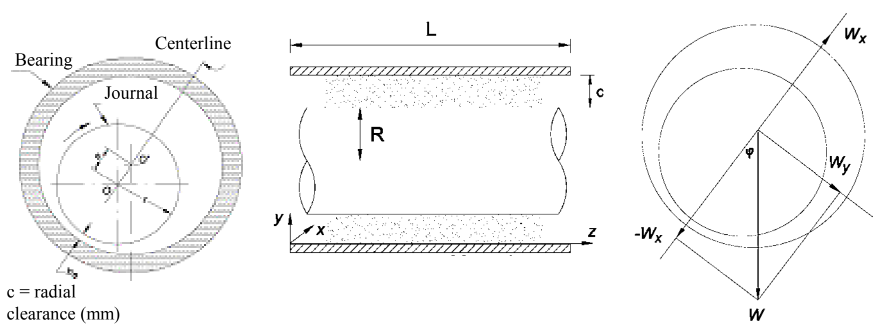

Figure 1. Journal bearing geometry coordinates system and schema of the load components

In this study, it is necessary to obtain dimensionless Reynolds equation. Thus, the following dimensionless variables are introduced:

$$\theta = x / R \; ; \; \eta = z / L \; ; \; \tilde{h} = h / c \; ; \; \phi = \theta / \theta_L \; ; \; \lambda = 2R / L \; ; \; P = pc^2 / \mu UR ; \; \xi = y / c \; ; \; \tilde{u} = u / U \; ; \; \tilde{w} = w / U \, .$$

Thus, the dimensionless velocity components are:

$$\tilde{u}(\xi) = \frac{\xi}{\tilde{h}} + \frac{1}{2\,\theta_L}\frac{\partial P}{\partial \phi}\left(\xi^2 - \tilde{h}\,\xi\right) \qquad \tilde{w}(\xi) = \frac{\lambda}{4}\frac{\partial P}{\partial \eta}\left(\xi^2 - \tilde{h}\,\xi\right) \tag{9a-b}$$

and Reynolds equation is:

$$\left(\frac{\theta_L \lambda}{2}\right)^2 \frac{\partial}{\partial \eta}\left(\tilde{h}^3(\phi)\frac{\partial P}{\partial \eta}\right) + \frac{\partial}{\partial \phi}\left(\tilde{h}^3(\phi)\frac{\partial P}{\partial \phi}\right) = 6\,\theta_L\,\tilde{h}'(\phi) \tag{10}$$

where

$$\tilde{h}(\phi) = 1 + \varepsilon \cos(\phi\,\theta_L) \qquad \tilde{h}'(\phi) = -\varepsilon\,\theta_L\,\mathrm{sen}(\theta_L\,\phi) \tag{11a-b}$$

2.3. Pressure boundary conditions

$$P = 0 \text{ for } \eta = 0 \qquad P = 0 \text{ for } \eta = 1 \qquad P = 0 \text{ for } \phi = 0 \qquad P = \frac{\partial P}{\partial \phi} = 0 \text{ for } \phi = 1 \tag{12a-d}$$

The Equations (12a) and (12b) are obtained by application of the atmospheric pressure in the journal bearing extremities, and the Eqs. (12c) and (12e) are Reynolds conditions.

2.4. Solution methodology

For application of the GITT, the following steps are developed:

2.4.1. Eigenvalue problem determination

The procedure of integral transformation demands the definition of an eigenvalue:

$$\frac{d^2\psi_i}{d\eta^2} + \mu_i \psi_i = 0 \tag{13a}$$

$$\psi_i = 0 \text{ for } \eta = 0; \quad \psi_i = 0 \text{ for } \eta = 1 \tag{13b-c}$$

where $\psi_i(\eta)$ e μ_i are the eigenfunctions and eigenvalues of problem (13) respectively.

Problem (13) is promptly solved analytically and it gives,

$$\psi_i(\eta) = \mathrm{Sin}(\mu_i\,\eta) \tag{14}$$

where the eigenvalues are roots of the following equation:

$$\mathrm{Sin}(\mu_i) = 0 \Rightarrow \mu_i = i\pi \quad , \quad i = 1,2,3,... \tag{15}$$

which satisfy the orthogonality:

$$\int_0^1 \psi_i(\eta)\,\psi_j(\eta)\,d\eta \quad , \quad \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \tag{16}$$

2.4.2. Transform-inverse pair determination

Eigenvalue problem represented by Eq. (13a) allow to define the transform-inverse pair as follow:

$$\tilde{P}_i(\phi) = \int_0^1 \tilde{\psi}_i(\eta)\, P(\phi,\eta)\, d\eta \text{, transform} \qquad P(\phi,\eta) = \sum_{i=1}^{\infty} \tilde{\psi}_i(\eta)\, \tilde{P}_i(\phi) \text{, inverse} \qquad (17a\text{-}b)$$

where is the eigenfunction normalized,

$$\tilde{\psi}_i(\eta) = \frac{\psi_i(\eta)}{N_i^{1/2}} = \frac{\psi_i(\eta)}{\sqrt{N_i}} \qquad N_i = \int_0^1 \psi_i^2(\eta)\, d\eta = \frac{1}{2} \qquad (18a\text{-}b)$$

2.4.3. Transformation integral of differential equation

It is executed, multiplying the Eq. (10) by $\tilde{\psi}_i(\eta)$, the result is integrated in the domain [0,1] in function of η, which after the employ of the orthogonality (13) and inversion formula (17), gives:

$$\frac{d^2\tilde{P}_i}{d\phi^2} + g(\phi)\frac{d\tilde{P}_i}{d\phi} - mi^2\, \tilde{P}_i = C_i\, \tilde{f}(\phi) \qquad \phi = 0 \text{ for } \tilde{P}_i = 0\,;\quad \phi = 1 \Rightarrow \frac{d\tilde{P}_i}{d\phi} = 0 \qquad (19a\text{-}c)$$

where

$$f(\phi) = -\,6\,\theta_L^2\, \varepsilon\, \mathrm{sen}(\theta_L\, \phi) \qquad g(\phi) = \frac{3\,\tilde{h}'(\phi)}{\tilde{h}(\phi)} \qquad \tilde{f}(\phi) = \frac{f(\phi)}{\tilde{h}^3(\phi)} \qquad (20a\text{-}c)$$

$$C_i = \frac{1 - \cos(\mu_i)}{\mu_i \sqrt{Ni}} \qquad mi = \frac{\theta_L\, \lambda\, \mu_i}{2} \qquad (21a\text{-}c)$$

2.4.4. θ_L calculation

The angle θ_L characterizes the film fluid length and it depends of cavitation boundary. Thus, if there is not cavitation $\theta_L = \theta$, if there is cavitation its value is $\theta_L = \theta + \alpha$, where α is cavitation angle. It is calculated with boundary condition (Eq. 12d), that is, P (ϕ, η) = 0 for $\phi = 0$. The problem defined by Eq. (19a) depends of θ_L and this one made part of the problem itself. By using the additional condition (Eq. 12.d) one obtain transcendental equation for θ_L calculation,

$$\sum_{i=1}^{\infty} \tilde{\psi}_i(\eta)\, \tilde{P}_i \Big|_{\phi=1} = 0 \qquad (22)$$

Eq. (22) is solved through the ZREAL routine (IMSL, 1987), with prescript error of 10^{-9}.

With θ_L determined, the transformed potentials can be obtained, $\tilde{P}_i(\phi,\eta)$. They are used to build the original potential original P (ϕ, η) by inversion formula (Eq.17b), in everyplace of interest.

2.4.5. Load capacity and action angle calculation

Axial load component:

$$\tilde{W}_1 = -\,\theta_L \sum_{i=1}^{\infty} A_i\, C_i \qquad (23)$$

Normal load component to the axis:

$$\tilde{W}_2 = \theta_L \sum_{i=1}^{\infty} B_i\, C_i \tag{24}$$

where the coefficients above are determined through of the following integrals:

$$A_i = \int_0^1 \tilde{P}_i(\phi)\cos(\theta_L\,\phi)\,d\phi \qquad B_i = \int_0^1 \tilde{P}_i(\phi)\,\mathrm{sen}(\theta_L\phi)\,d\phi \qquad C_i = \int_0^1 \tilde{\psi}_i(\eta)\,d\eta \tag{25a-c}$$

2.4.6. Friction coefficient calculation

For this calculation it is necessary to obtain the shear stress, defined by:

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=h} = \mu\left(\frac{U}{h} + \frac{h}{2\mu}\frac{\partial p}{\partial x}\right) \tag{26}$$

Thus, the dimensionless friction force can be obtained by multiplying of shear stress τ_w by journal bearing surface:

$$\tilde{F} = \theta_L \int_0^1 \frac{1}{\tilde{h}(\phi)}\,d\phi + \frac{1}{2}\sum_{i=1}^{\infty} D_i C_i \tag{27}$$

where

$$D_i = \int_0^1 \tilde{h}(\phi)\frac{d\tilde{P}_i}{d\phi}\,d\phi \tag{28}$$

Therefore, the friction coefficient is calculated by equation:

$$C_f = \frac{\tilde{F}}{\tilde{W}} \tag{29}$$

2.4.7. Axial flow rate calculation

It is calculated by following equation:

$$\tilde{Q}_S = -\frac{\theta_L}{6}\sum_{i=1}^{\infty}\frac{\partial\tilde{\psi}_i(\eta=0)}{\partial\eta}\,E_i \tag{30}$$

where coefficient E_i is obtained by:

$$E_i = \int_0^1 \tilde{h}^3(\phi)\,\tilde{P}_i(\phi)\,d\phi \tag{31}$$

And the eigenfunction derived is given by:

$$\tilde{\psi}_i(\eta=0) = \frac{\mu_i}{\sqrt{N_i}} \tag{32}$$

3. RESULTS AND DISCUSSION

Generalized Integral Technique Transformed was used for solution mathematical model given by Eq. (19a). The technique implementation was made via a program in language computational FORTRAN 90/95, where was utilized the subroutine BVPFD of the library IMSL (1987). Tab. 1, 2, 3 and 4 show the θ_L, φ, $\widetilde{W}$ and P_{max} (maximum pressure) convergence of the problem. The convergence has been investigated for different truncation orders (NT) in the solution of pressure field. It is observed that a convergence excellent rate is obtained for the parameters analyzed in the journal bearing medium plan.

Table 1. Convergence of θ_L, φ, $\widetilde{w}$ and P_{max} for journal bearing medium plan for ε = 10-5 and D/L = 10-5.

NT	θ_L	φ	$\widetilde{w} \times 10^3$	$P_{max} \times 10^4$
10	**257,453**	**70,911**	0,130	0,824
22	**257,453**	**70,911**	0,133	0,825
30	257,453	70,911	0,134	0,878
42	257,453	70,911	**0,135**	0,878
50	257,453	70,911	**0,135**	0,829
62	257,453	70,911	0,135	0,827
70	257,453	70,911	0,135	0,825
82	257,453	70,911	0,135	0,825
90	257,453	70,911	0,135	0,814
102	257,453	70,911	0,135	**0,817**
110	257,453	70,911	0,135	**0,818**
122	257,453	70,911	0,135	0,818

In the Tab. 1, for ε = 10-5 e D/L = 10-5, is verified that few terms are necessaries to converge θ_L and φ, that is, less of 10 series terms, while $\widetilde{w}$ and P_{max} have a convergence more slow, starting to converge with 42 and 102 terms respectively.

Table 2. θ_L, φ, $\widetilde{w}$ and P_{max} convergence for journal bearing medium plan with $\varepsilon = 10^{-5}$ and D/L = 0,5.

NT	θ_L	φ	$\widetilde{w} \times 10^4$	$P_{max} \times 10^4$
10	232,71	80,37	**0,5604**	**0,490**
22	**232,67**	**80,34**	**0,5607**	**0,489**
30	**232,67**	**80,35**	0,5607	0,489
42	232,67	80,35	0,5607	0,489
50	232,67	80,35	0,5607	0,489

In the Tab. 2, as in the Table 1 is observed a good convergence, with few terms, for the analyzed parameters. It is noted that θ_L and φ start to converge for 22 and 30 series terms; while $\widetilde{w}$ and P_{max} converge with less than 10 series terms.

Table 3. θ_L, φ, $\widetilde{w}$ and P_{max} convergence for journal bearing medium plan with ε = 0,5 and $D/L = 10^{-5}$.

NT	θ_L	φ	$\widetilde{w}$	P_{max}
10	**219,69**	**58,30**	6,19	4,757
22	**219,69**	**58,30**	6,34	4,604
30	219,69	58,30	6,37	4,570
42	219,69	58,30	6,39	4,543
50	219,69	58,30	6,40	4,531
62	219,69	58,30	6,41	4,521
70	219,69	58,30	6,42	4,516
82	219,69	58,30	**6,42**	4,510
90	219,69	58,30	**6,43**	**4,507**
102	219,69	58,30	6,43	**4,503**
110	219,69	58,30	6,43	4,503

Table 4. θ_L, φ, $\widetilde{w}$ and P_{max} convergence for journal bearing medium plan with ε = 0,5 e D/L = 0,1.

NT	θ_L	φ	$\widetilde{w}$	P_{max}
10	219,89	60,68	5,694	4,588
22	**219,63**	60,13	**5,756**	4,493
30	**219,68**	60,26	**5,751**	4,482
42	219,66	**60,21**	5,756	4,477
50	219,67	**60,23**	5,754	4,476
62	219,66	60,23	5,755	4,475
70	219,66	60,23	5,755	**4,475**
82	219,66	60,23	5,755	**4,474**
90	219,66	60,23	5,755	4,474

In the Tab. 3 is verified that θ_L and φ convergence occur with less than 10 series terms, while $\widetilde{w}$ and P_{max} converge with 90 and 102 series terms respectively. In the Table 4 is noted that θ_L converge with a series of 30 terms, φ between 62 and 70 terms, $\widetilde{w}$ with 22 and P_{max} about 70 terms.

Table 5. Load capacity - present work and available literature.

L/D	Methods	Eccentricity Specific (ε)				
		0,1	0,4	0,5	0,6	0,8
1	Reason and Narang[1](1982)	0,228	-	1,722	-	6,924
	FEM[1]	0,228	-	1,584	-	5,964
	Hirani *et al*. (1997)	0,228	-	1,62	-	6,204
	D. Sharma *et al*. (1991)	-	1,2386	-	2,7206	7,578
	Present work	0,239	1,2154	1,767	3,9064	8,082
0,25	Reason and Narang[1](1982)	0,0192	-	0,1782	-	1,2216
	FEM [1]	0,0198	-	0,1740	-	1,1304
	Hirani *et al*. (1997)	0,0192	-	0,1770	-	1,1616
	D. Sharma *et al*. (1991)	-	0,1123	-	0,2967	1,1855
	Present work	0,0196	0,1959	0,2868	0,4287	1,2410

Table 5 illustrate the obtained results in the present work and those available in the literature for the load capacity with L/D = 0,25 and 1 and ε = 0,1, 0,4, 0,5, 0,6 and 0,8. It is observed that the results of the present work have a concordance acceptable with the results of the literature for L/D = 1 and eccentricity $\varepsilon \leq 0,5$. When L/D = 0,25, the good results only there are for ε = 0,1 and 0,8.

Graphics results are obtained for dimensionless pressure field in the circumferential direction, load capacity, friction coefficient and axial flow rate for different values of ε, λ and η in the journal bearing medium plan. Which are compared with results obtained by Mokhiamer et al. (1999), Sharma et al. (1991) and Williams and Symmons (1987). The Fig. 2 shows an excellent concordance of the present work results with the literature results.

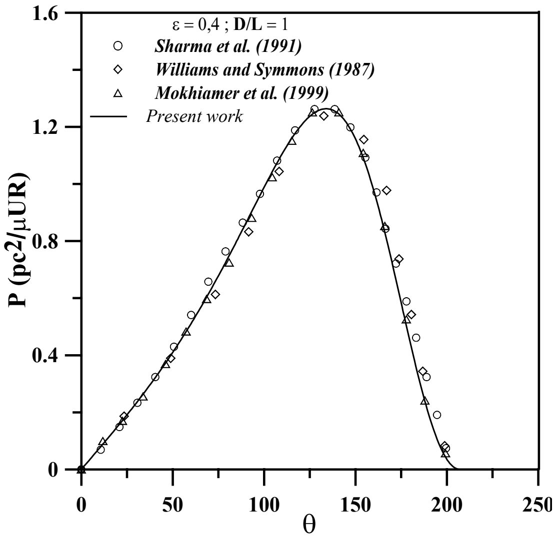

Figure 2. Comparison of pressure field in the circumferential direction.

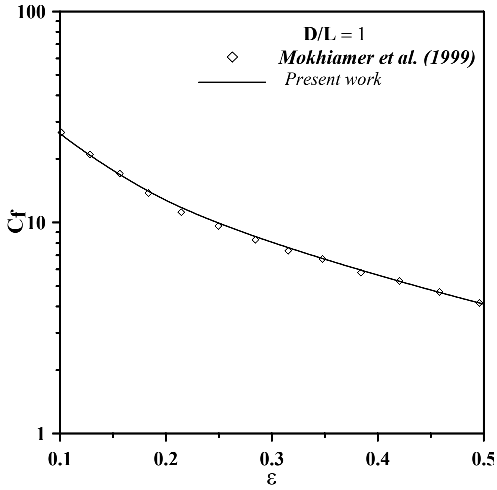

Figure 3. Comparison of the friction coefficient in function of the specific eccentricity.

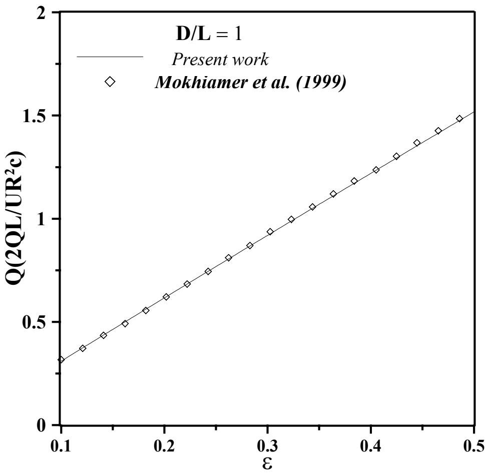

Figure 4. Comparison of the axial flow rate in function of the specific eccentricity.

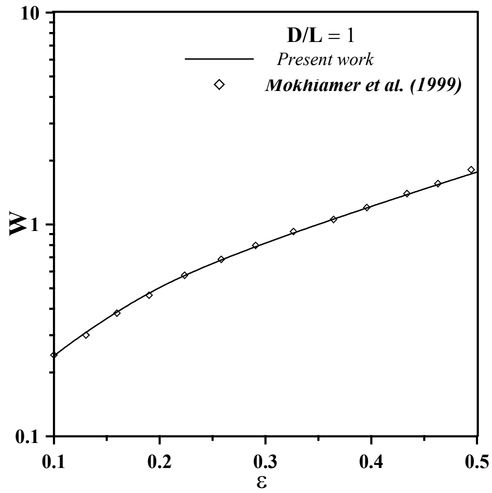

Figure 5. Comparison of the capacity load in function of the specific eccentricity.

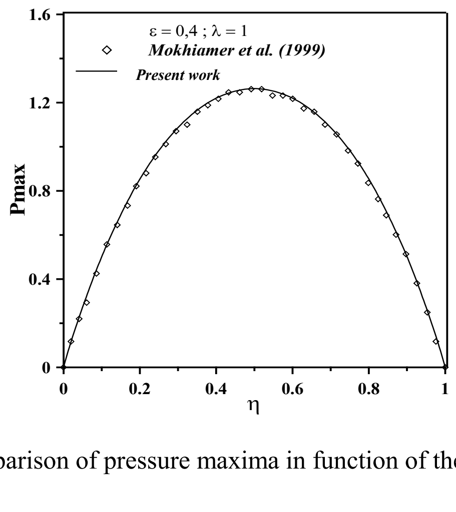

Figure 6. Comparison of pressure maxima in function of the axial position

In the Figs. 2 to 6 it is noted an excellent concordance between the results of the present work and those of the literature. This fact validates, one more time, the computational program developed in this study.

The pressure field depends of specific eccentricity and relation ship D/L. Thus, it is interesting to simulate the pressure field in function of ε and D/L. The simulation was executed for specific eccentricity equal to 10^{-5} and 0,5 and D/L equal to 10^{-5} and 1, as showed in the Figs. 7 to 10.

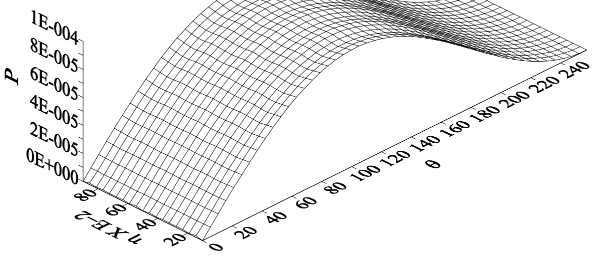

Figure 7. Pressure field in function of θ for $\varepsilon = 10^{-5}$, D/L = 10^{-5} in different positions of the journal bearing.

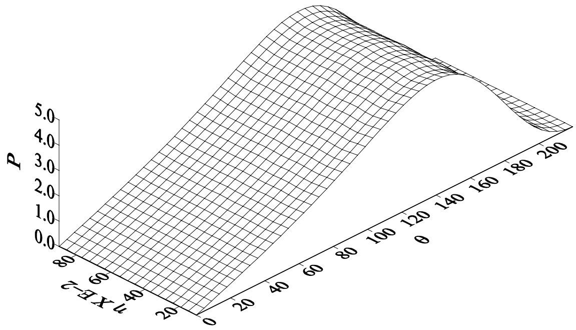

Figure 8. Pressure field in function of θ for ε = 0,5, D/L = 10^{-5} in different positions of the journal bearing.

In the Figs. 7 and 8 can be observed that for longs journals bearing (D/L → 0), the circumferential pressure field does not depend of the axial position. It is explained for the pressure gradient in the direction η to be negligible in relation to the gradient in the direction θ ($\partial p/\partial\theta >> \partial p/\partial\eta$). For longs journals bearing, the flow in the direction θ is very more important that those in the directionη. It is also verified that the pressure in the journal bearing is proportional to ε. The variation in the pressure field is due to eccentricity (ε) to regulate the wedge effect. In other words, this effect is major when major is the eccentricity and consequently major is the lubricant shear, which is proportional to hydrodynamic pressure, as can be observed in the Eq. (26).

In the Fig. 9, it is observed that pressure field vary in the direction η and it is due to the journal bearing not be long (D/L = 1). As in the Figures 7 and 8, the pressure is proportional to ε. Comparing the Figures 7 and 9 and the Figures 8 and 10, it is noted that the pressure and θ are inversely proportional to D/L. This demonstrates that there is a dependence of the pressure in relation to the parameter D/L, which determines if a journal bearing is long or short.

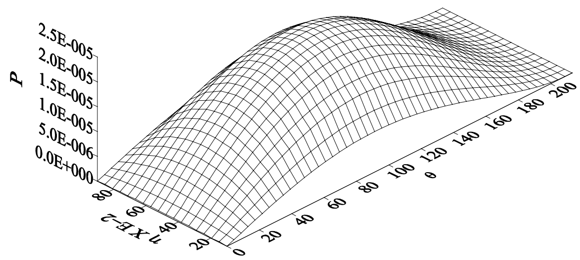

Figure 9. Pressure field in function of θ in different positions of the journal bearing for $\varepsilon = 10^{-5}$, D/L = 1.

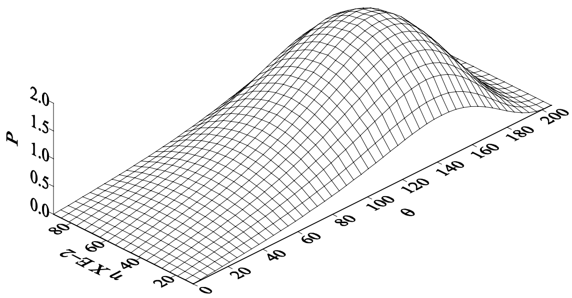

Figure 10. Pressure field in function of θ and in different positions of the journal bearing for ε = 0,5, D/L = 1.

4. CONCLUSIONS

This paper demonstrate the applicability of the GITT, as a method able to provide good results for problems of hydrodynamic lubrication The model is based in the Navier-Stokes equations, by reproducing results with good concordance in relation to those of the literature. This fact validates the computational programs developed in this analysis.

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6. RESPONSIBILITY NOTICE

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