ANALYSIS OF TRANSIENTS IN RIGID PIPELINES CONVEYING LIQUID-GAS MIXTURES

Felipe Bastos de Freitas Rachid, rachid@vm.uff.br
Laboratory of Liquid & Gas Transport - PGMEC/TEM - Department of Mechanical Engineering - Universidade Federal Fluminense

Abstract. *This work presents a numerical model for predicting the isothermal transient two-phase flow of liquid-gas homogeneous mixtures in rigid pipelines. The resulting mathematical problem is governed by a system of non-linear hyperbolic partial differential equations which is solved by means of an operator splitting technique, combined with the Glimm's method. To implement Glimm's method, it is presented the closed-form analytical solution of the associated Riemann problem. Preliminary numerical results are presented in order to illustrated the model performance.*

Keywords: *liquid-gas homogeneous flow , unsteady state , Riemann problem .*

1. INTRODUCTION

Unsteady homogeneous flows of liquid-gas mixtures in pipelines have been extensively investigated from the theoretical and experimental points of view in the past decades due to its significant importance in industrial applications [Bergant et al., 2006]. Fluid flows in conduits of hydroelectric and nuclear power plants, water supply networks, petroleum and sewage pipelines are some of a vast universe of practical engineering problems. In these fluid flows, small concentrations of gas by volume are known to drastically alter the dynamic response of the system [Wylie and Streeter, 1993]. Due to the presence of gas in the mixture, the wave propagation velocity in the medium becomes highly dependent on the pressure and the system of partial differential hyperbolic equations describing the unsteady two-phase flow becomes strongly non-linear. Since analytical solutions are virtually impossible, numerical techniques must be considered when predictive behaviors are required. The task of finding appropriate numerical solutions for this problem has also been the subject of intense research due to its inherent complexity [Kessal and Amaouche, 2001] and [Bergant et al., 2006] (and references therein). The wave front spreads during rarefaction waves and steeps during the passage of compression waves with such an intensity that shock waves may appear. Finite difference and finite element schemes as well as techniques having a stem on the method of characteristics have already been proposed and proved not to be adequate because of the excess of numerical dissipation.

To overcome such a difficulty, it is presented in this paper the complete solution of the Riemann problem for the homogeneous and isothermal unsteady two-phase flow. The mass of gas per volume of mixture is not treated as being constant as usual it is, so that the governing equations are formed by a system of three non-linear hyperbolic partial differential equations. The solution of the Riemann problem is then used with the Glimm's scheme and the operator splitting technique to generate numerical solutions which do not dissipate nor change the true magnitude and position of the wave fronts. Numerical solutions for a particular problem are presented in order to highlight the aforementioned features.

2. GOVERNING EQUATIONS

Because piping systems used for liquid transmission are composed of slender members, pressure transients in fluid-filled pipes are commonly described by means of one-dimensional models. By assuming that the pipe is rigid, has an internal diameter D and a length L, the mass and moemtum balance equations describing the isothermal fluid flow of liquid-gas mixtures are, in Eulerian coordinates:

$$\partial_t \mathbf{u} + \partial_s \mathbf{F}(\mathbf{u}) = \mathbf{S}(\mathbf{u}) \quad \text{in} \quad (0, L) \times (0, \infty) \tag{1}$$

in which $\mathbf{u}(s,t) = \mathbf{u} \in I\!R^3$ is the conserved quantity, s is the spatial coordinate along the pipe centerline and t is the time instant. The symbols $\partial_t \chi$ and $\partial_s \chi$ are used to designate partial derivative of a general dependent variable χ with respect to t and s, respectively. The vector-valued functions $\mathbf{F}(\mathbf{u}) = \mathbf{F} : I\!R^3 \to I\!R^3$ and $\mathbf{S}(\mathbf{u}) = \mathbf{S} : I\!R^3 \to I\!R^3$ are the flux and the source/sink terms, respectively. The particular form of these vector quantities are:

$$\mathbf{u} \;:=\; (u_1, u_2, u_3)^T := (\alpha\rho_g, \rho, \rho v)^T \tag{2}$$

$$\mathbf{F} \;:=\; \big((u_1 u_2)/u_3, u_2, u_3^2 u_2 + p\big)^T \tag{3}$$

$$\mathbf{S} \;:=\; (\Gamma, -u_2 g sin\theta, -(u_2 f(u_3/u_2)|(u_3/u_2)|)/(2D))^T \tag{4}$$

in which

$$\rho \;:=\; \alpha\rho_g + (1-\alpha)\rho_l \tag{5}$$

$$\Gamma \;:=\; \beta max\{0, (p_s - p)\}. \tag{6}$$

In the above equations, p, v, ρ and α are functions of the time t and the spatial position s along the pipe. They represent, respectively, the pressure, the axial velocity, the mass density and the gas volume fraction of the liquid-gas mixture. The mass densities of the liquid and the gas are denoted by ρ_l and ρ_g, respectively. The angle formed between the pipe centerline and the horizontal is designated by θ, whereas g and f stand for the local gravitational acceleration and the Darcy-Weisbach friction factor. The term Γ stands for the time rate of mass of gas release per unit volume, p_s is the saturation pressure of the dissolved gas (assumed constant) and β is a constant associated with the solubility coefficient. Equation (6) establishes that gas will evolve from the mixture as long as $p < p_s$.

To complete the model, equations of state are required to specify the thermodynamic behavior of the liquid and gaseous phases. In the context of homogeneous flows, the liquid and the gas temperatures are assume to be equal $T_l = T_g = T$. Moreover, the process is considered to be isothermal, so that we can express the liquid and gas pressures as a functions of its mass densities,

$$p_i := \widehat{p}_i(\rho_i), \quad \text{with} \quad a_i^2 := \widehat{p}_i^{\,'}(\rho_i) > 0 \quad \text{for} \quad i \in \{l, g\} \tag{7}$$

in which the a_i is the isothermal speed of the sound in the liquid ($i = l$) and in the gas ($i = g$) and $\widehat{p}_i^{\,'}$ stands for the derivative of the function $\widehat{p}_i$ with respect to the mass density ρ_i, for $i \in \{l, g\}$.

By neglecting surface tension effects, the liquid pressure becomes equal to the gas pressure [Freitas Rachid, 2006], $p_l = p_g = p$, and the following relationships may be written by virtue of (7),

$$\rho_i = \widehat{p}_i^{\,-1}(p) \quad \text{for} \quad i \in \{l, g\}. \tag{8}$$

in which $\widehat{p}_i^{\,-1}$ denotes the inverse of the function $\widehat{p}_i(\rho_i)$. Substituting (8) into (5) by taking into account (2) allows one to write,

$$u_2 - u_1 = \widehat{p}_l^{\,-1}(p) - u_1 \frac{\widehat{p}_l^{\,-1}(p)}{\widehat{p}_g^{\,-1}(p)} \tag{9}$$

No matter the equations of state (7) for the liquid and the gas look like, Eq. (9) shows that one can (implicitly or explicitly) write the pressure of the mixture in terms of the conserved quantities u_1 and u_2, such as:

$$p = \widehat{p}(u_1, u_2) \tag{10}$$

Equation (1), along with (2-4) and (10), form a non-linear hyperbolic system of partial differential equations, whose eigenvalues given by,

$$det\left(d\mathbf{F}/d\mathbf{u} - \lambda \mathbf{1}\right) = 0, \tag{11}$$

being $\mathbf{1}$ the identity matrix, are (in crescent order),

$$\lambda_1 = v - a < \lambda_2 = v < \lambda_3 = v + a \quad \text{with} \tag{12}$$

$$a^2 = (u_1/u_2)\partial_{u_1}\widehat{p}(u_1, u_2) + \partial_{u_2}\widehat{p}(u_1, u_2) \tag{13}$$

in which a is the isothermal wave velocity with which disturbances propagate in the mixture liquid-gas. Straightforward calculations show that the eigenvalues in (12) are all real numbers, since

$$a^2 = \frac{a_l^2 \rho_l a_g^2 \rho_g}{\rho\left(\alpha a_l^2 \rho_l + (1-\alpha) a_g^2 \rho_g\right)} > 0. \tag{14}$$

In addition, it can be proved that the eigenvectors associated with λ_k, for $k \in \{1, 2, 3\}$, form a set of linearly independent vectors spanning the space $I\!R^3$. These two last conditions ensure the system (2) is strictly hyperbolic, no matter the specific forms of (7) are. It can also be demonstrated that the characteristics field associated to the eigenvalue λ_2 is linearly degenerated, whereas the ones related to the eigenvalues λ_k, for $k \in \{1, 3\}$, are genuinely nonlinear provided the following condition holds,

$$\widehat{p}_i^{\,''}(\rho_i) \geq 0 \quad \text{for} \quad i \in \{l, g\}. \tag{15}$$

where $\widehat{p}_i^{\,''}$ stands for the second order derivative of the function $\widehat{p}_i$ with respect to the mass density ρ_i. If (7) and (15) hold, then the system (1), along with (2-4) and (10), is a genuinely nonlinear hyperbolic system of equations. Hereto after, we tacitly assumed that the condition (15) holds. However, it is worth noting that this condition is only a sufficient condition in order to ensure that (1) is a genuinely nonlinear hyperbolic system.

3. THE ASSOCIATED RIEMANN PROBLEM

The Riemann problem associated to the homogeneous (with $\mathbf{S}(\mathbf{u}) = \mathbf{0}$) system of equations (1), is an initial-value problem of with discontinuous data at the left and at the right of an arbitrary position s_o, given at an arbitrary time instant t_o:

$$\partial_t \mathbf{u} + \partial_s \mathbf{F}(\mathbf{u}) = \mathbf{0}, -\infty < s < +\infty, t > t_o, \tag{16}$$

$$\mathbf{u}(s, t = t_o) = \begin{cases} \mathbf{u}_L, & \text{for } s < s_o \\ \mathbf{u}_R, & \text{for } s > s_o \end{cases} \tag{17}$$

in which $\mathbf{u}_L = (\alpha\rho_g, \rho, \rho v)_L^T$ and $\mathbf{u}_R = (\alpha\rho_g, \rho, \rho v)_R^T$ are two arbitrary constant states. The generalized solution of this Riemann problem for $t > t_o$ and $-\infty < s < +\infty$ depends only on the ratio $\xi = (s - s_o)/(t - t_o)$. It is constructed by connecting the left state $\mathbf{u}_L$ to the right state $\mathbf{u}_R$ through two intermediates constant states $\mathbf{u}_L^*$ and $\mathbf{u}_R^*$, as follows:

$$\mathbf{u}_L \overset{1-\text{wave}}{\longleftarrow - \longrightarrow} \mathbf{u}_L^* \overset{2-\text{wave}}{\longleftarrow - \longrightarrow} \mathbf{u}_R^* \overset{3-\text{wave}}{\longleftarrow - \longrightarrow} \mathbf{u}_R \tag{18}$$

To connect these states the k-waves, centered at (s_o, t_o), associated with the eigenvalues λ_k, for $k \in \{1, 2, 3\}$, are used. The 2-wave is a contact discontinuity and is used to connect the states $\mathbf{u}_L^*$ and $\mathbf{u}_R^*$. Across the 2-wave the generalized Riemann invariants $p = constant$ and $v = constant$ hold, so that:

$$p_L^* = p_R^* = p^* \tag{19}$$

$$v_L^* = v_R^* = v^* \tag{20}$$

The k-waves, for $k \in \{1, 3\}$, can be either a k-rarefaction wave or a k-shock wave. The 1-wave will be a 1-rarefaction wave iff $p^* < p_L$ and it will be a 1-shock wave iff $p^* > p_L$. Similarly, the 3-wave will be a 3-rarefaction wave iff $p^* < p_R$ and it will be a 1-shock wave iff $p^* > p_R$. Whatsoever the type of the k-wave is, u_1/u_2 is constant across these waves, giving rise to the following relationships:

$$x_L := (u_1/u_2)_L^* = (u_1/u_2)_L = \text{constant} \tag{21}$$

$$x_R := (u_1/u_2)_R^* = (u_1/u_2)_R = \text{constant} \tag{22}$$

Expressions (21) and (22) allows one to rewrite the pressure (Eq. (10)) and the wave speed (Eq. (13)), at the left (1-wave) and at the right (3-wave) waves, in terms of $u_2 := \rho$ only:

$$\widehat{\widehat{p}}_i(\rho) = \widehat{p}(x_i\rho, \rho), \quad \text{for} \quad i \in \{L, R\} \tag{23}$$

$$\widehat{\widehat{a}}_i(\rho) = \sqrt{\widehat{\widehat{p}}_i^{\,'}(\rho)}, \quad \text{for} \quad i \in \{L, R\} \tag{24}$$

If the k-wave, with $k \in \{1, 3\}$, is a rarefaction wave ($p^* < p_i$), for $i \in \{L, R\}$, the generalized Riemann invariant is used to connect the states $\mathbf{u}_i$ and $\mathbf{u}_i^*$;

$$v^* = v_i + (k-2)\int_{\rho_i}^{\rho_i^*} \frac{\widehat{\widehat{a}}_i(\rho)}{\rho} d\rho \tag{25}$$

Otherwise, if the k-wave, with $k \in \{1, 3\}$, is a shock wave ($p^* > p_i$), for $i \in \{L, R\}$, the Rankine-Hugoniot jump conditions are used to connect the states $\mathbf{u}_i$ and $\mathbf{u}_i^*$ and also compute the shock speed S_i;

$$v^* = v_i + (k-2)\sqrt{\frac{(p^* - p_i)(\rho_i^* - \rho_i)}{\rho_i \rho_i^*}} \tag{26}$$

$$S_i = \frac{\rho_i v_i - \rho_i^* v^*}{\rho_i - \rho_i^*} \tag{27}$$

Whatever the type of the k-wave is, equations (25) and (26) can be combined and the following transcendental equation can be written in terms of p^* by noticing that $\rho_i^* = \widehat{\widehat{p}}_i^{\,-1}(p^*)$, for $i \in \{L, R\}$;

$$\psi(p^*) = v_R - v_L + \phi_L(p^*; \mathbf{u}_L) + \phi_R(p^*; \mathbf{u}_R) = 0 \tag{28}$$

in which

$$\phi_i(p^*; \mathbf{u}_i) = \begin{cases} \displaystyle\int_{\rho_i}^{\rho_i^*} \frac{\widehat{\widehat{a}}_i(\rho)}{\rho} d\rho & \text{, if } p^* \le p_i \\ \sqrt{\frac{(p^* - p_i)(\rho_i^* - \rho_i)}{\rho_i^* \rho_i}} & \text{, if } p^* > p_i \end{cases} \tag{29}$$

for $i \in \{L, R\}$. Once the root of (28) has been found, the intermediate states can all be computed as well as the type (rarefaction or shock) of the k-wave, for $k \in \{1, 3\}$. The complete solution of the associated Riemann problem involving all the four cases is summarized below:

3.1 Solution of the type 1-rarefaction ↔ 2-wave ↔ 3-rarefaction

This type of solution will occur if $p^* < p_L$ and $p^* < p_R$ ($v_L < v^* < v_R$) and the generalized solution of the problem is given as follows:

$$\mathbf{u}(\xi) = \begin{cases} \mathbf{u}_L & \text{, if } -\infty < \xi < v_L - a_L \\ \mathbf{u}_{LF} & \text{, if } v_L - a_L < \xi < v^* - a_L^* \\ \mathbf{u}_L^* & \text{, if } v^* - a_L^* < \xi < v^* \\ \mathbf{u}_R^* & \text{, if } v^* < \xi < v^* + a_R^* \\ \mathbf{u}_{RF} & \text{, if } v^* + a_R^* < \xi < v_R + a_R \\ \mathbf{u}_R & \text{, if } v_R + a_R < \xi < +\infty \end{cases} \tag{30}$$

3.2 Solution of the type 1-rarefaction ↔ 2-wave ↔ 3-shock

This type of solution will occur if $p_L > p^* > p_R$ ($v^* > v_L$ and $v^* > v_R$) and the generalized solution of the problem is given as follows:

$$\mathbf{u}(\xi) = \begin{cases} \mathbf{u}_L & \text{, if } -\infty < \xi < v_L - a_L \\ \mathbf{u}_{LF} & \text{, if } v_L - a_L < \xi < v^* - a_L^* \\ \mathbf{u}_L^* & \text{, if } v^* - a_L^* < \xi < v^* \\ \mathbf{u}_R^* & \text{, if } v^* < \xi < S_R \\ \mathbf{u}_R & \text{, if } S_R < \xi < +\infty \end{cases} \tag{31}$$

3.3 Solution of the type 1-shock ↔ 2-wave ↔ 3-rarefaction

This type of solution will occur if $p_L < p^* < p_R$ ($v^* < v_L$ and $v^* < v_R$) and the generalized solution of the problem is given as follows:

$$\mathbf{u}(\xi) = \begin{cases} \mathbf{u}_L & \text{, if } -\infty < \xi < S_L \\ \mathbf{u}_L^* & \text{, if } S_L < \xi < v^* \\ \mathbf{u}_R^* & \text{, if } v^* < \xi < v^* + a_R^* \\ \mathbf{u}_{RF} & \text{, if } v^* + a_R^* < \xi < v_R + a_R \\ \mathbf{u}_R & \text{, if } v_R + a_R < \xi < +\infty \end{cases} \tag{32}$$

3.4 Solution of the type 1-shock ↔ 2-wave ↔ 3-shock

This type of solution will occur if $p^* > p_L$ and $p^* > p_R$ ($v_L > v^* > v_R$) and the generalized solution of the problem is given as follows:

$$\mathbf{u}(\xi) = \begin{cases} \mathbf{u}_L & \text{, if } -\infty < \xi < S_L \\ \mathbf{u}_L^* & \text{, if } S_L < \xi < v^* \\ \mathbf{u}_R^* & \text{, if } v^* < \xi < S_R \\ \mathbf{u}_R & \text{, if } S_R < \xi < +\infty \end{cases} \tag{33}$$

In the above expressions, $\mathbf{u}_{LF} = \widehat{\mathbf{u}}_{LF}(\xi)$ and $\mathbf{u}_{RF} = \widehat{\mathbf{u}}_{RF}(\xi)$ are increasing functions of ξ which represent, respectively, the left and the right rarefaction fans and $a_i^* = \widehat{a}_i(\rho_i^*)$, for $i \in \{L, R\}$.

4. NUMERICAL PROCEDURE

One way to obtain a numerical approximation to the governing equations is based on operator splitting technique. The objective of this technique is to take advantage of the additive decomposition property of the mathematical operator to solve a sequence of simpler problems instead of an unique complex problem. The main attraction of splitting schemes is in the fact that one can deploy the optimal, existing numerical methods for each subproblem.

To obtain a numerical solution for (8) consider a uniform partition $0 = s_1 < \ldots s_\imath < s_{\imath+1} < \ldots < s_{N+1} = L$ of the spatial domain $[0, L]$ such that $\Delta s = s_{\imath+1} - s_\imath$. The procedure to advance the solution from time t^n to time $t^{n+1} = t^n + \Delta t$ is based on the operator splitting technique. With this technique, the approximation for $\mathbf{u}(s,t)$ at time $t = t^{n+1}$ and $s = s_\imath$, $\mathbf{u}_\imath^{n+1}$, is obtained by solving the following problem;

$$\partial_t \mathbf{u} = \mathbf{S}(\mathbf{u}) \tag{34}$$

$$\mathbf{u} = \widetilde{\mathbf{u}}^{n+1}(s) \qquad \text{at} \qquad t = t^n \tag{35}$$

as follows:

$$\mathbf{u}_\imath^{n+1} = \widetilde{\mathbf{u}}_\imath^{n+1} + \Delta t \mathbf{S}(\widetilde{\mathbf{u}}_\imath^{n+1}) \tag{36}$$

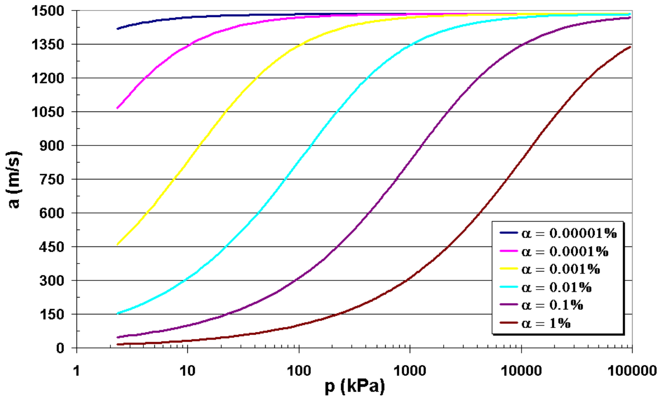

Figure 1. Wave speed as a fucntion of the pressure and gas volume fraction.

In the expression above, $\mathbf{u}_\imath^{n+1}$ and $\widetilde{\mathbf{u}}_\imath^{n+1}$ stand for the approximations of $\mathbf{u}(s = s_\imath, t = t^{n+1})$ and $\widetilde{\mathbf{u}}(s = s_\imath, t = t^{n+1})$, respectively.

The field $\widetilde{\mathbf{u}}^{n+1}(s) = \widetilde{\mathbf{u}}(s, t = t^{n+1})$ used as initial condition in (9) is obtained from the homogeneous hyperbolic problem:

$$\partial_t \widetilde{\mathbf{u}} + \partial_s (\mathbf{F}(\widetilde{\mathbf{u}})) = \mathbf{0} \tag{37}$$

$$\widetilde{\mathbf{u}} = \mathbf{u}^n(s) \qquad \text{at} \qquad t = t^n \tag{38}$$

In other words, $\widetilde{\mathbf{u}}^{n+1}(s)$ is the solution of (11) evaluated at time $t = t^{n+1}$.

The problem characterized by (37) and (38) is solved numerically by using the Glimm's scheme [Smoller, 1983]. Glimm's scheme has been used to solve one-dimensional non-linear hyperbolic problems because of its already proved efficiency in not only treating discontinuous initial data but also capturing solutions which present first or zeroth order discontinuous solutions [Marchesin and Paes-Leme, 1983, Freitas Rachid and Costa Mattos, 1998a, Freitas Rachid et al., 1994, Freitas Rachid, 2005]. This numerical method preserves the shock waves magnitude and position, within an uncertainty of Δs (width of each step). Such features are not found in the usual numerical procedures (e.g. finite elements and finite differences).

In order to employ the Glimm's scheme, the initial data at the time instant t^n are approximated by piecewise constant functions as follows:

$$\mathbf{u}(s, t^n) \simeq \mathbf{u}_\imath^n = \mathbf{u}(s_\imath + \theta_n \Delta s, t^n) \tag{39}$$

for $\quad s_\imath - \frac{\Delta s}{2} < s < s_\imath + \frac{\Delta s}{2}$ with where θ_n is a number randomly chosen in the open interval $(-0.5, 0.5)$.

The above approximations give rise, for each two consecutive steps i and $i + 1$, to an initial-value problem, known as Riemann problem, given by:

$$\partial_t \widetilde{\mathbf{u}} + \partial_s \big(\mathbf{F}(\widetilde{\mathbf{u}})\big) = \mathbf{0} \tag{40}$$

$$\widetilde{\mathbf{u}}(s, t^n) = \begin{cases} \mathbf{u}_\imath^n & \text{for } -\infty < s < s_\imath + \frac{\Delta x}{2} \\ \mathbf{u}_{\imath+1}^n & \text{for } s_{\imath+1} - \frac{\Delta s}{2} < s < \infty \end{cases} \tag{41}$$

Denoting by $\widehat{\mathbf{u}}(s, t) = \mathbf{u}(\xi)$, with $\xi = (s - s_o)/(t - t_o)$ being $s_o = s_\imath + \frac{\Delta x}{2}$ and $t_o = t^n$, the generalized solution of (40) and (41), which has been given by (30) or (31) or (32) or (33), the approximation for the solution of (37) with (39) at time t^{n+1} is given as follows:

$$\widetilde{\mathbf{u}}^{n+1}(s) \simeq \widetilde{\mathbf{u}}_\imath^{n+1} = \widehat{\mathbf{u}}(s, t = t^{n+1}) \tag{42}$$

for $\quad s_\imath < s < s_{\imath+1}$. In the procedure, the time instant t^{n+1} must be such that the Courant-Friedrichs-Levy condition is satisfied:

$$t^{n+1} - t^n = \Delta t \leq \frac{\Delta s}{2|\lambda|_{max}} \tag{43}$$

where $|\lambda|_{max}$ is the maximum (in absolute value) propagation speed, taking into account the N Riemann problems at time t^n. Boundary conditions are properly imposed at the ends of the pipe, after the second step for each advance in time.

Figure 2. Hydraulic installation considered in the numerical example.

5. NUMERICAL SIMULATIONS

To illustrate the application of the model described in the past sections, some preliminary numerical results are presented herein. We consider a two-phase flow in which the liquid is water and the gas is air. For the sake of simplicity, we admit as equations of state for the liquid and gas $p = a_l^2(\rho_l - \rho_{lo})$ and $p = a_g^2 \rho_g$, respectively, in which $a_l = 1485m/s$, $a_g = 290m/s$ and $\rho_{lo} = 998.20kg/m^3$ are all constants. For this specific set of parameters, the wave speed in the mixture a is depicted in Fig. 1 as a function of the pressure p for different values of the gas volume fraction, in accordance with (14). As it can be seen, small values of gas concentration by volume are sufficient to generate a highly non-linear pressure-dependent wave speed relationship.

As hydraulic system for the numerical simulations, we consider an hypothetical horizontal line transmission whose length is $L = 20km$, having a inside diameter $D = 2m$ as depicted in Fig. 2. At time $t = 0$ the flow takes place in steady-state from $s = 0$ to $s = L$. The mixture velocity and pressure (at time $t = 0$) at the pipe entrance ($s = 0$) is $v_0 = 2.32m/s$ and $p_0 = 4MPa$. To highlight the influence of the gas volume fraction on the dynamic behavior of the system two different values of the gas volume fraction are considered at $s = 0$; $\alpha_0 = 0.02\%$ and $\alpha_0 = 0.02\%$. For simplicity we admit that $\beta = 0$, so that there is no gas release and that the friction factor is constant and equal to $f = 0.025$. The steady-state solutions for $\alpha_0 = 0.02\%$ and $\alpha_0 = 0.2\%$ are shown in Fig. 3 in terms of the dimensionless parameters $\Phi = p/p_0$, $\Phi = a/a_L$, $\Phi = \alpha/\alpha_0$ as a function of the s/L. As the pressure in the mixture decreases due to friction along the flow, the gas volume fraction increases by promoting a reduction in the wave speed in the mixture towards the pipeline exit. The greater the gas volume fraction at the entrance is, the more stringent the reduction in the wave speed is.

The transient flow in the system is induced by the rapid closure of the valve positioned at $s = 0$, so that $v_0 = 0m/s$ for $t > 0$. Sine at the downstream end of the pipe it is located a reservoir, the pressure is kept constant at $s = L$. The pressure histories at the valve $s = 0$ for $\alpha_0 = 0.02\%$ and $\alpha_0 = 0.2\%$ are illustrated in Fig. 4. The ordinate of Fig. 4 has been made dimensionless by taking the ratio $p(s = 0, t)/p_0$, whereas the abscissa has been dimensionalized by using the time $t_r = L/a_l$ required for the disturbances to travel the distance L with the liquid wave speed. The results shown in Figure 4 were simulated by using typical mesh sizes of the order of $\Delta s \simeq 500m$. As it can be seen in Figure 4, the numerical method is capable of capturing the severe discontinuities properly, without neither dissipating it nor presenting numerical oscillations.

6. FINAL REMARKS

A model, along with a detailed numerical procedure to approximate its solution, have been proposed to predict the transient isothermal two-phase flow of liquid-gas mixtures flowing in rigid pipelines. Numerical simulations have been

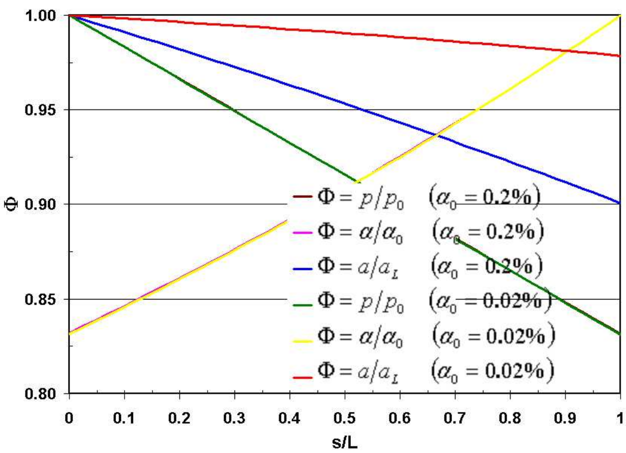

Figure 3. Steady-state solution in terms of Φ against s/L for $\alpha_0 = 0.02\%$ and $\alpha_0 = 0.02\%$.

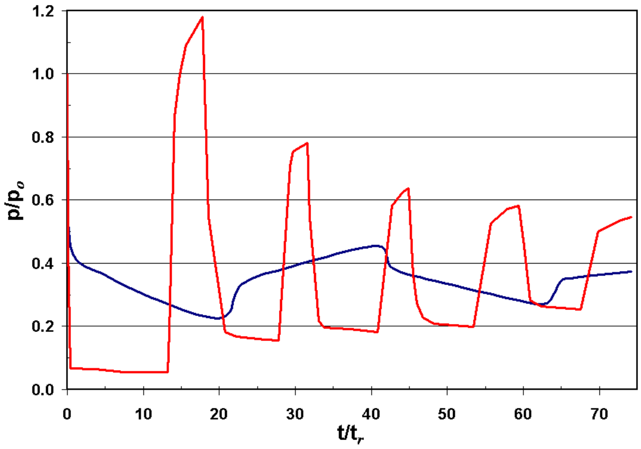

Figure 4. Pressure history at $s = 0$ for $\alpha_0 = 0.02\%$ (red line) and $\alpha_0 = 0.2\%$ (blue line).

carried out to illustrate the model performance. The preliminary results obtained seems to express correctly the physical phenomena involved in. Comparisons with other existing numerical methodologies as well as with experimental data are under way to definitively validate the model.

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