MODELING GAS TRANSMISSION NETWORKS BY USING OPTIMIZATION TECHNIQUES

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Abstract. *This paper presents a mathematical modeling to describe gas transmission networks involving the whole chain gas ranging from the gas production wells to the consumer market. Instead of considering the momentum conservation principle, a linear programming approach is employed to emulating the gas motion through the network. To achieve this goal, a constrained maximization of a suitable linear functional is taking into account. The mass conservation principle, along with compatibility conditions and some physical limitations of the components of the network, form the set of restrictions in terms of linear equalities and inequalities. This alternative approach simplifies considerable the numerical solution of the problem, rendering a simple, robust and quite genera model. A numerical example is presented in order to illustrate the capability of the modeling in properly predict important events in gas networks.*

Keywords*: gas networks, distribution systems, linear programming.*

1. INTRODUCTION

Gas transmission networks are systems that combined different ways of transporting gas through long distances, from production wells to consumer centers. As the consumer market begins to grow rapidly new production wells come into operation. As a result, an expressive increase in the gas network is experienced. To accommodate this new scenario, complex operational management of the whole system is required in order to ensure gas delivery to all markets. In a complex branched gas network, decision making regarding to gas deliverability to a market is in general not an easy task, since the cost associated with the gas production depends on each production well (Mokhatab *et al*., 2006). Moreover, the cost of the gas to a particular market depends on the distance traveled and the specific route the gas has traveled in the network (McAllister, 2005). As a first step to address this problem, it is presented in this paper a simple mathematical modeling which aims to forecasting the capability of an existing gas network in attending the gas demand of whole market. The model is based on the mass balance equation and takes into account, in a suitable fashion, pressure and flow rate as well as compressor stations restrictions of the gas lines. Given the production capacity, the infrastructure of the network and the gas demand of each consumer, the model predicts the behavior of the whole distribution system, verifying automatically all the associated constraints. Among others, important features such as packing and unpacking phenomena in the gas network are accurately described enabling the model as a promising tool in the task associated with operational management.

2. PROBLEM DESCRIPTION

A gas transmission system is a complex network which aims to delivering gas from the production wells to the consumer markets. To achieve this task, different infrastructures can be used by employing a variety of equipments, routes and transport media. If these items have been chosen, then the infrastructure is established and operational restrictions have automatically been imposed in the network. As a result, once the gas production capacity in a scenario has been specified, question arises as to the capability of the given infrastructure in attending the prescribed gas demand of the consumer market. When the gas demand is not fully attended, it also becomes important to precisely identify the consumers, the deficit and also the periods for the gas shortage.

No matter how complex the network can effectively be, the components of the infrastructure can be grouped into four distinct classes: production/processing, stock, transport and consumption. Typical components from the production/processing class are the associated or non-associated gas wells and the units of gas processing. As components belonging to the consumption class, one may cite the industries, the thermo-electrical power plants and the cities as a whole. Finally, the transport class may be subdivided into two categories: continuous and non-continuous transport. The continuum transport components are the gas pipelines and the non-continuous transport components encompass the transport of gas carried out by ships, trains and trucks.

To sum up, given the infrastructure of a gas network, its gas production capacity and its gas demand the problem in analysis consists in properly determine the temporal evolution of the effective production and consumption in the whole network.

3. MATHEMATICAL MODELING

The basic physical principle used to govern the gas motion through the distribution system is the mass conservation law, which must be stated not only globally (for the system as a whole) but also locally (for each component of the system). In fact, if the mass conservation principle is verified for each component of the system, then it is automatically satisfied for the system as a whole. As the reciprocal is not true, the starting point in the mathematical modeling process is to express the aforementioned principle for each component of the system. In the course of this process, the four classes of components (production/processing, stock, transport and consumption) mentioned in the past section are fully characterized.

3.1. Class description

Whatever the class the component belongs, every component in the system may be physically characterized by only one general equation which expresses the mass conservation principle. Thus, for each component n of the system consisting of N components we can write, for every time instant t:

$$\frac{d}{dt}V^{(n)} + \sum_{j=1}^{J^{(n)}} S_j^{(n)} - \sum_{i=1}^{I^{(n)}} E_i^{(n)} = P^{(n)} - C^{(n)} \quad for \quad n = 1, \ldots, N \tag{1}$$

in which $V^{(n)}$ stands for the quantity of gas mass expressed in Nm^3, whose definition is presented ahead, inside the *n*-th component at the instant t, $S_j^{(n)}$ and $E_i^{(n)}$ denote the instantaneous mass flow rate which, respectively, leaves and comes into the *n*-th component through the *j*-th inlet and the *i*-th outlet and, finally, $C^{(n)}$ and $P^{(n)}$ represent, respectively, the instantaneous time rate of gas consumption and production taking place in the *n*-th component at the time instant t. All the variables in Eq. (1) are functions of t, the only one independent variable in the problem formulation.

Due to the form as Eq. (1) is written, it becomes evident that the following conditions set below must be satisfied:

$$V^{(n)} \geq 0, \quad S_j^{(n)} \geq 0, \quad E_i^{(n)} \geq 0, \quad P^{(n)} \geq 0, \quad C^{(n)} \geq 0 \quad for \quad n = 1, ..., N. \tag{2}$$

It is not difficult to realize that not all the components will have all the terms in Eq. (1). As a matter of fact, it is the identification of the non-null terms in Eq. (1) that will characterize not only the class the component belongs but also its peculiarities within each class. For instance, if the component in consideration belongs to the class production/processing, then $E_i^{(n)} \equiv 0,\ \forall\, i = 1, \ldots, I^{(n)},$ and $C^{(n)} \equiv 0.$ In a similar fashion, components which belong to the class consumption will be characterized by having the following non-null terms in Eq. (1) : $S_j^{(n)} \equiv 0,\ \forall\, j = 1, \ldots, J^{(n)},$ and $P^{(n)} \equiv 0.$

On the other hand, if the component belongs to the class stock or transport, then one must necessarily have $P^{(n)} \equiv 0$ and $C^{(n)} \equiv 0$ or $P^{(n)} \equiv 0$, respectively. Based on this classification, it is explicitly assumed that the components of the transport class may also stock as well as consume gas. By excluding the components used with the purpose of connecting the other components such as the junctions and the derivations, it is admitted as a basic assumption that the only difference between the components of the stock and transport class is that the components from this last class possess only one inlet and only one outlet, that is, $I^{(n)} = 1$ and $J^{(n)} = 1$. The junction and the derivation, which constitutes the exception to the rule of the transport class, give flexibility in mounting the system by allowing to considering branches of pipelines with multiples inlets and outlets, pipelines of varying diameters and so forth. The derivation component is characterized by assumption as having only one inlet and two outlets, i. e., $V^{(n)} \equiv 0$, $C^{(n)} \equiv 0$, $P^{(n)} \equiv 0$ with $I^{(n)} = 1$ e $J^{(n)} = 2$. Similarly, the junction component has only one outlet and two inlets, i. e. $V^{(n)} \equiv 0$, $C^{(n)} \equiv 0$, $P^{(n)} \equiv 0$ with $I^{(n)} = 2$ e $J^{(n)} = 1$.

To ensure that the mass conservation principle is satisfied globally (for the system as a whole), it is imposed compatibility conditions between the mass flow rates, at the inlet and at the outlet, of the components of the transport class and the mass flow rates of the components connected to them. Without loosing generality, if we admit that there exists *M* components of the transport class (with $M < N$), then there will be 2*M* compatibility equations as follows:

$$\left.\begin{array}{l} E_i^{(m)} - S_j^{(o)} = 0,\ for\ i = 1,2\ with\ j \in \left\{1,\ldots,J^{(o)}\right\} \\ S_j^{(m)} - E_i^{(o)} = 0,\ for\ j = 1,2\ with\ i \in \left\{1,\ldots,I^{(o)}\right\} \end{array}\right\}\ for\ m = 1,\ldots,M\ \ with\ o \in \{1,\ldots,N-M\},\ o \neq m \tag{3}$$

Every variable which appear in Eq. (1) are submitted, for every time instant t, to the additional conditions in the following form:

$$C^{(n)} \le {}^{max}C^{(n)},\ V^{(n)} \le {}^{max}V^{(n)},\ S_j^{(n)} \le {}^{max}S_j^{(n)},\ E_i^{(n)} \le {}^{max}E_i^{(n)},\ P^{(n)} \le {}^{max}P^{(n)} \tag{4}$$

for $n = 1,...,N$, in which ${}^{max}C^{(n)}$, ${}^{max}V^{(n)}$, ${}^{max}S_j^{(n)}$, ${}^{max}E_i^{(n)}$ and ${}^{max}P^{(n)}$ are, for a given time instant, known positive constants which represent the maximum admissible values for $C^{(n)}$, $V^{(n)}$, $S_j^{(n)}$, $E_i^{(n)}$ and $P^{(n)}$, respectively. From the physical view point, these upper bounds stand for operational restrictions associated with each component. For instance, ${}^{max}V^{(n)}$ represents the maximum stock capacity of a component, ${}^{max}P^{(n)}$ the maximum time rate production of gas, ${}^{max}C^{(n)}$ the maximum time rate consumption of gas (also named demand) and, finally, ${}^{max}S_j^{(n)}$ and ${}^{max}E_i^{(n)}$ the maximum mass flow rates in the components of the transport class. If the component is a pipeline, then ${}^{max}S_1^{(n)} = {}^{max}E_1^{(n)} = {}^{max}Q^{(n)}$, in which ${}^{max}Q^{(n)}$ is the maximum mass flow rate in the pipeline. Besides the specification of the upper bounds involved in Eq. (1), initial conditions are required with respect to the initial quantities of gas (say at time $t = 0$) of each variable $V^{(n)}$,

$$V^{(n)} = V_0^{(n)} \quad at \quad t = 0 \quad for \quad n = 1,...,N. \tag{5}$$

With exception of the pipeline component, the initial quantity of gas in all components of the system is assumed to be an input. The approach used to estimate the initial quantity of gas in a pipeline based on the process variables will be described in section 3.3.

Once the basic features of each of the four classes have been defined, we are able to enumerate the set of fundamental assumptions which characterize the nature of the gas which is transported through the whole system.

3.2. Basic assumptions

To fully describe the proposed model, some fundamental assumptions enumerated ahead are assumed with regard to the constitutive nature of the gas:

1) The gas density, with respect to the air at the normal conditions of pressure and temperature, is for every time instant (including the initial at $t = 0$) and in every component of 0.725, which is equivalent to admit that the molecular weight is constant and equal to 21.

2) The compressibility factor of the gas can be approximated by the following expression of the CNGA – *California Natural Gasoline Association* (McAllister, 2005):

$$Z = \hat{Z}(p,T) := \frac{1}{1 + \dfrac{517060 \times 10^{1,294125}\, p}{T^{3,825}}} \tag{6}$$

with *T* expressed in degrees Kelvin and *p*, the gauge pressure, expressed in kgf/cm^2.

3) Whatever the system is, the gas temperature is assumed to be uniform and constant, being equal to 20^{o}C.

4) For all effects, it is assumed that the gas obeys the following equation of sate:

$$pV = nZRT \tag{7}$$

in which *V* represents the volume, n the number of moles and R the universal constant of the gases.

5) The unit of mass adopted for every component is the normal cubic meter, Nm^3. By definition, it is the mass of gas in a 1 m^3 at a reference temperature T_{ref} =20°C and at a reference absolute pressure p_{ref} =1 atm.

6) The superior caloric power of the gas, PC_S, is constant and equal to 9500 kcal/Nm^3. According to the classification of the Brazilian Agency of Petroleum ANP, the gas is considered a medium natural gas with PC_S within the interval 8800 to 10200 kcal/Nm^3. Moreover, the superior caloric power is assumed to be 90 % do PC_S.

Based upon the aforementioned assumptions, it can be verified that the compressibility factor in the reference state, $Z_{ref} = \hat{Z}(p = 0\text{kgf/cm}^2,\ T = 293.15\text{K})$, obtained from Eq. (6), is equal to 1.

With the assumptions set before one can estimate the initial quantity of gas inside the pipeline, i. e., that is its initial condition. In the next section we present a simple way to estimate the amount of gas inside the pipe according to its operational parameters.

3.3. Initial condition for the pipeline

The procedure used to estimate the initial quantity of gas inside the pipeline, as well as its minimum and maximum stock capacities, is presented next. The quantity of gas, which obeys the assumptions set before in the past section, that can be stored in a pipeline of internal diameter *D* and length *L* can be expressed according to:

$$V = \hat{V}\left(Z_M, p_M, T\right) := V_0 \frac{Z_{ref}}{Z_M} \frac{p_M}{p_{ref}} \frac{T_{ref}}{T} \tag{8}$$

in which $V_0 = \frac{\pi D^2 L}{4}$, $Z_M = \hat{Z}_M\left(p_M, T_{ref}\right)$ is the compressibility factor for the mean pressure p_M within the pipeline segment.

If the pipeline is in the shut-in condition (null flow rate or, equivalently, in static condition), the pressure p_M is the absolute and uniform pressure inside the pipeline. On the other hand, if it is running (or operates under dynamic condition), then the mean pressure p_M inside the pipeline can be estimated by the following expression (Mokhatab, 2006):

$$p_M = \frac{2}{3}\left(p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2}\right) \tag{9}$$

in which p_1 e p_2 represent the absolute pressures at the inlet and at the outlet of the pipeline segment, respectively.

By knowing the pipeline geometrical dimensions, the inlet pressure and also the mass flow rate *Q*, then the outlet pressure can be estimated based on the Weymouth formulae (Mokhatab et al., 2006):

$$Q = 453 \frac{D^{8/3}}{L^{0.51}}\left[p_1^2 - p_2^2\right]^{0.51} \tag{10}$$

in which *Q* is given in Nm^3, *D* stands for the inside diameter (in inches), *L* is the pipeline length expressed in km and p_1 and p_2 are the pressures in kgf/cm^2.

By admitting that *Q*, *D*, *L* and p_1 are taken as input data for the pipeline, then Eq. (10) can be used to estimate p_2. It is worthwhile noting that one must have $p_2 \le p_1$. If $p_2 < p_1$ then, the pipeline segment operates dynamically. If, on the other hand, $p_2 = p_1$ is in shut-in condition. With p_1 and p_2 we compute via Eq. (9) the pressure p_M and, in the sequence, $Z_M = \hat{Z}\left(p_M, T_{ref}\right)$ through Eq. (6). Finally, with p_M and Z_M we compute the initial quantity of gas in the pipeline segment $V = \hat{V}\left(Z_M, p_M, T_{ref}\right)$ by using Eq. (8).

The minimum and maximum quantities of gas that can be stored in the pipeline segment can be estimated based upon the minimum p_{min} and maximum p_{max} operational pressures. These data are also assumed to be input data for each pipeline segment. The minimum gas quantity in the pipeline is $V^{min} = \hat{V}\left(Z_{min}, p_M = p_{min}, T_{ref}\right)$ in which $Z_{min} = \hat{Z}\left(p_{min}, T_{ref}\right)$. Analogously, the maximum quantity of gas in the pipeline segment is $V^{max} = \hat{V}\left(Z_{max}, p_M = p_{max}, T_{ref}\right)$ in which $Z_{max} = \hat{Z}\left(p_{max}, T_{ref}\right)$.

3.4. Mathematical formulation

The fundamental background required by the proposed model to describe gas transmission networks have been presented in the past sections. However, the use of the mass conservation principle itself is not sufficient to describe the gas motion throughout the network. From the mechanical viewpoint, it would be necessary to consider additionally the momentum conservation principle for each component. However, the use of this principle not only requires the knowledge of a large amount of operational data, in general not easily available, but also is itself not sufficient to fully describe the operandi modus of such networks. To overcome such difficulty, we proposed an alternative and simple way to describe the network operation, without appealing to the momentum conservation principle. As we shall see next, the motion of gas throughout the network is emulated by maximizing a linear functional suitably postulated, subjected to the restrictions imposed by the mass conservation principle Eq.(1-2), by the compatibility equations Eq.(3), by the effective capacities given by Eq.(4) and also the initial conditions expressed by Eq.(5).

Formally, by considering that ${}^{max}V^{(n)}$, ${}^{max}S_j^{(n)}$, ${}^{max}E_i^{(n)}$ are known quantities and that ${}^{max}C^{(n)}$ and ${}^{max}P^{(n)}$ are prescribed for each time instant, for $n = 1,...,N$, $i = 1,...,I^{(n)}$ and $j = 1,...,J^{(n)}$ (when pertinent), then the mathematical problem which describes the gas motion in the system in consideration consists to find $V^{(n)}$, $S_j^{(n)}$, $E_i^{(n)}$, $C^{(n)}$ and $P^{(n)}$ for $n = 1,...,N$, $i = 1,...,I^{(n)}$ and $j = 1,...,J^{(n)}$ (when pertinent), subjected to Eq.(1-5).

By inspecting the system of equations formed by Eq. (1) and Eq. (3), along with Eq. (5), we can see that it is undetermined since there are more unknowns than equations. To allow that this initial value problem has a solution which consistently represents an actual operation of the gas network, it becomes necessary to impose additional condition(s). This is done by choosing a suitable linear functional of some variables of the problem, which has to be maximized.

Thus, the mathematical problem can now be formally formulated as follows:

Given ${}^{max}C^{(n)}$ e ${}^{max}P^{(n)}$ for all time instant t, find $V^{(n)}$, $S_j^{(n)}$, $E_i^{(n)}$, $C^{(n)}$ e $P^{(n)}$ for $n = 1,...,N$, $i = 1,...,I^{(n)}$ and $j = 1,...,J^{(n)}$ (when pertinent) which satisfy Eqs. (1) to (5) and that maximize the linear function $f\left(V^{(n)}, S_j^{(n)}, E_i^{(n)}, C^{(n)}, P^{(n)}\right)$.

Naturally, the choice of f is crucial in order to ensure uniqueness of solution (if it exists!) and, at the same time, to properly describe the actual operation of a gas network. To achieve these goals, we proposed based, on some the practical information available in (Nayyar, 2000), that the linear functional have the following form:

$$f\left(V^{(n)}, S_j^{(n)}, E_i^{(n)}, C^{(n)}, P^{(n)}\right) := \sum_{n=1}^{N}\left[p_c{}^{(n)}C^{(n)} + p_p{}^{(n)}P^{(n)} + \Omega^{(n)}V^{(n)} + V_2^{(n)}\right] \tag{11}$$

in which $p_c{}^{(n)}$ and $p_p{}^{(n)}$ are prescribed functions of the time, with image within the real interval [1,10], and represent the priorities associated with the variables ${}^{max}C^{(n)}$ and ${}^{max}P^{(n)}$ of the component n of the consumption and production/processing classes, respectively.

The variable $\Omega^{(n)}$ which appears in Eq. (11) is a function of the time and is defined as:

$$\Omega^{(n)} = \begin{cases} 1, \text{ if there exists, at the current time instant, a discoontious component of the transport class connected} \\ \quad \text{to the } n \text{ component;} \\ 0, \text{ otherwise.} \end{cases} \tag{12}$$

The third term at the right-hand side of Eq. (11) is taken into account in the linear functional in order to emulate the loading and unloading of the discontinuous components of the transport class, such as train, truck and ship. In this term the variable $V^{(n)}$ stands for the quantity of gas in the tank of these components for the loading process or the quantity of gas in the tank of the component which receives the gas in the case of the unloading process.

To assign the model a more realistic behavior, we decompose the variable which represents the quantity of gas $V^{(n)}$ inside the pipeline into two distinct additive parcels, that is $V^{(n)} = V_1^{(n)} + V_2^{(n)}$, with $0 \le V_1^{(n)} \le 0.2\ {}^{max}V^{(n)}$ .and

$0 < {}^{min}V \le V_2^{(n)} \le 0.8\ {}^{max}V^{(n)}$, being ${}^{min}V^{(n)}$ and ${}^{max}V^{(n)}$ the minimum and maximum capacities of storing gas inside the pipeline. To emulate the packing and unpacking effects in the pipeline the variable $V_2^{(n)}$ is included in the linear functional given by Eq. (11). Besides the packing/unpacking behavior, this strategy implicitly imposes a mean operational pressure in steady-state around 80% of the maximum allowable operational pressure in the pipeline.

Finally, it is possible to prove that if the totally implicit Euler method is employed to approximate the time derivative in Eq.(1), then the mathematical formulation presented in this section forms a typical problem of linear programming (Luenberger, 1973). To numerically solve this problem we use the well-known SIMPLEX method (Luenberger, 1973).

4. NUMERICAL EXAMPLE

To illustrate the capability of the proposed model in properly describe the gas transmission network a representative numerical example is shown in this section The example presented ahead aims to illustrating the following features:

a) operational situations in which the effective production becomes inferior to its maximum capacity;
b) gas shortage; a situation in which at least one component of the consumption class is forced to present an effective consume that is inferior to its demand;
c) time variation of the gas stock in the network when the gas production supplants the gas demand or is less than to it;
d) the packing/unpacking effect in the gas pipeline, highlighting its ability to operate as regulator element of the gas supply in the network.

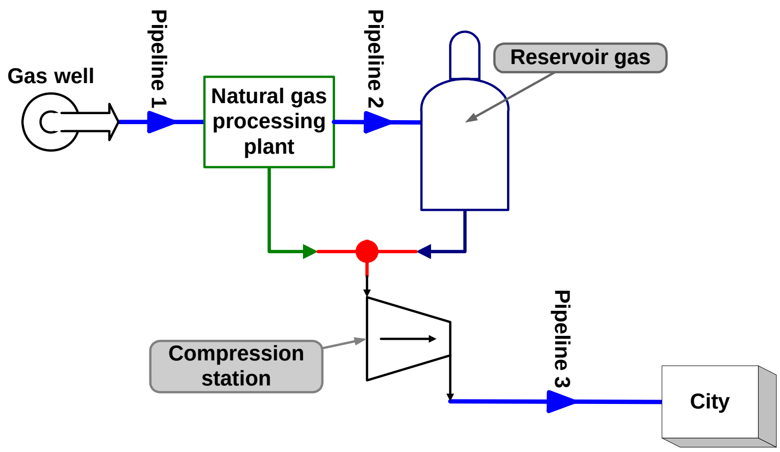

Figure 1. Schematic representation of the gas network analyzed.

The network simulated in this example has nine components and is illustrated in Figure 1. The gas from the well is transported to the unit of gas processing through a pipeline of short extension (Pipe1). From that point, the gas can flow through two different routes. It can be directed to a gas reservoir, by means of a short pipeline (Pipe 2), which aims to attending the demand in critical operational situations. Alternatively, it can be conveyed to the gas compression station which is responsible by the gas delivery to the city (the unique consumer in the network) through a pipeline 150 km long (Pipe 3). Since in this example there exists only one component of the production class and only one component of the consumption class there is no need to specify the priorities associated to them.

The maximum production capacity of the well varies along the time and is specified in the input data as being of 3000kNm^3/day until the end of the 11th day. At the end of the 12th day, the maximum production is reduced to 25000 kNm^3/day, raised to 35000kNm^3/day at the end of the 13th day and, finally, elevated to 40000 kNm^3/day at the end of the 14th and then kept constant from that day on.

The gas demanded by the city is also an increasing function of time, being of the order of 15000 kNm^3/day in the 1st day, 20000 kNm^3/day at the end of the 2nd day, 30000 kNm^3/day at the end of the 3rd day and, finally, 35000 kNm^3/day at the end of the 4th day. From that day on, the gas demand remains constant. The complete specification of input data associated with these and other components are presented in the Appendix.

Figure 2 shows the gas demand curve of the city as a function of the time as described in the preceding paragraph for a twenty-day period of simulation. Also represented in this figure are the effective gas consumption and gas production of the city and of the well, respectively. Although the city consumption is of only 15000 kNm^3/day in the first day, the well production totalizes 30000 kNm^3/day, resulting an excess of 15000 kNm^3/day. This amount of gas is stored in distinct regions of the network. More specifically, it is retained in the gas reservoir, inside pipe 3 and in a small capacity reservoir that exists in the city (see Figure 4).

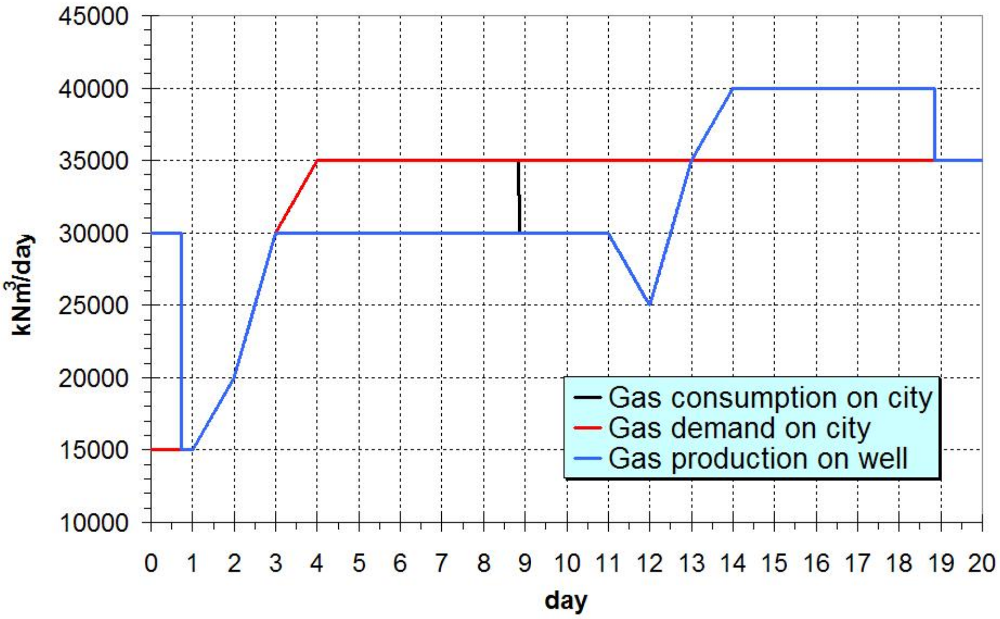

Figure 2. Gas consumption, gas demand and gas production as a function of time.

At the end of 0.7 days approximately, there is no more room in these components to store the gas, forcing the gas well production to decrease form 30000 kNm^3/day to 15000 kNm^3/day – the same amount demanded by the city – as it can be observed in Figure 2. From this time instant to the end of the 3rd day, the progressive increase in the city demand is followed by an effective increase of the well production until it reaches 30000 kNm^3/day – the maximum production capacity specified in the input data. From this time instant to the end of the 9th day approximately, although the gas demand in the city is greater than the maximum production capacity, there is no gas shortage in the city since the gas initially stored in the components of the network is enough to attend the deficit of 5000 kNm^3/day. When the gas stock in the network is run off, being the gas demand (of 35000 kNm^3/day) greater than the maximum production capacity, it is observed a gas shortage in the city from the 9th day until the end of the 13th day. This period is identified in the graph by the elapsed time in which the gas consumption becomes inferior to the gas demand, as it can be seen in Figure 2 and highlighted in Figure 3.

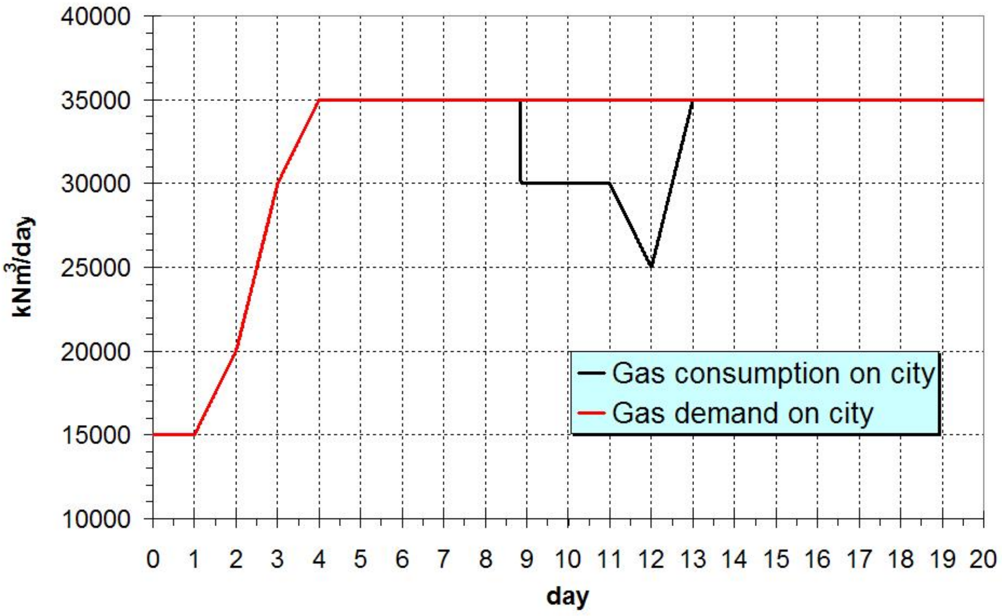

Figure 3. Gas consumption and gas demand as a function of time.

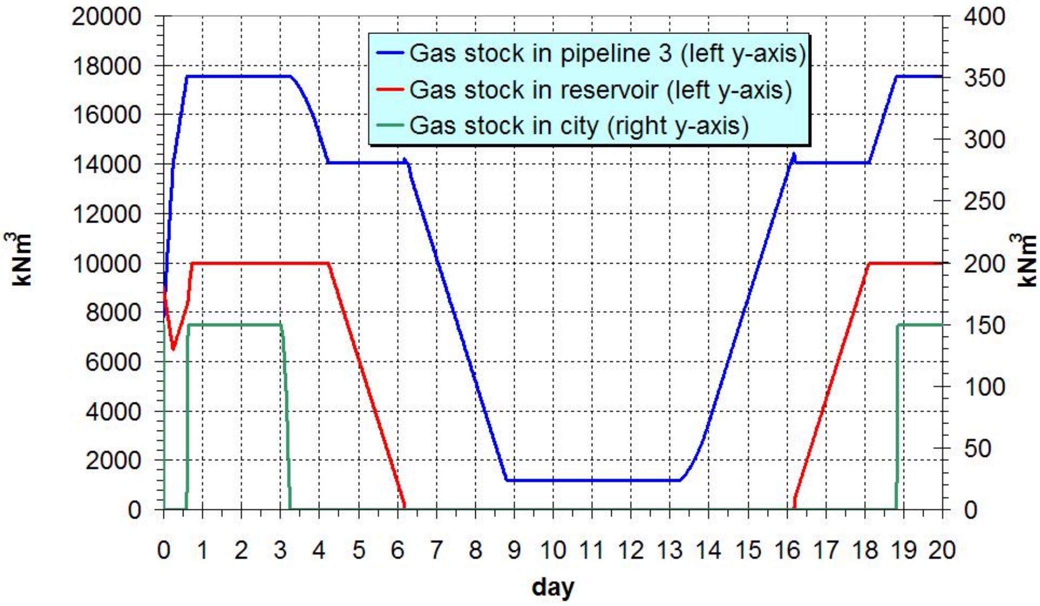

Figure 4. Gas stock as a function of time.

Due to the increase in the gas production as defined in the input data, from the 13th day to approximately to the end of the 19th day, the maximum production capacity supplants the gas demand in the city. Once again, the excess of the gas production is stored in the city, in the gas reservoir and in pipe 3, as illustrated in Figure 4. Ceased the time variation imposed in the gas production in the well and in the demand in the city, the system reaches its steady-state regime at the end of the 19th day, being the demand of 35000 kNm^3/day less than the maximum gas production capacity of 40000 kNm^3/day.

Finally, it is presented in Figure 4 the temporal evolution of the gas stock in the components of the network. These curves are characterized by periods in which the gas stock is raised until it reaches its maximum capacity, reduced to its minimum capacity and, finally, kept constant in their limit levels in situations the gas becomes equal to or less than to the maximum production capacity. It should be noticed in particular in Figure 4 the packing/unpacking phenomenon of gas inside the pipeline 3, which is in fact present in actual installations. It is also worth mentioning that in the context of the proposed modeling the unique effect of the presence of the gas compression station is the one associated with the self consume, when it exists. This explains the reasons for not including the gas compression stations at the entrance of pipe 1 and pipe 2.

6. CONCLUDING REMARKS

It has been presented in this paper a mathematical modeling to describe gas transmission networks involving the whole chain gas, ranging from the gas production wells to the consumer markets. Although pipelines are the most common way of transporting gas through a network, the proposed model is also capable to deal with discontinuous means of carrying gas, such as ships, trains and trucks.

Mass quantities and mass flow rates are taken as being the dependent variables only. The time instant is the unique independent variable. Instead of considering the momentum conservation principle, a linear programming approach is employed to emulating the gas motion through the network. To achieve this goal, a constrained maximization of a suitable linear functional is taking into account. The mass conservation principle, along with compatibility conditions and some physical limitations of the components, form the set of restrictions in terms linear equalities and inequalities. This alternative approach simplifies considerable the numerical solution of the problem, rendering a simple, robust and quite general model.

The versatility and capability of the model in emulating several actual features in the gas transmission networks are illustrated in a very elucidative and interesting numerical example.

7. ACKNOWLEDGEMENTS

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9. APPENDIX

Table 1. Gas well parameters and conditions.

Gas well	
^{max}P	40000 kNm^3/day
$P(t=11$ day$)$	30000 kNm^3/day
$P(t=12$ day$)$	25000 kNm^3/day
$P(t=13$ day$)$	35000 kNm^3/day
$P(t=14$ day$)$	40000 kNm^3/day

Table 2. Natural gas processing plant parameters and conditions.

Natural gas processing plant	
^{max}P	50000 kNm^3/day
^{max}C	0 kNm^3
^{max}V	0 kNm^3
$^{max}S_j$	20000 kNm^3/day

Table 3. Reservoir gas parameters and conditions.

Reservoir gas	
^{max}V	10000 kNm^3
$^{max}S_j$	20000 kNm^3/day
$V(t=0$ day$)$	90% of ^{max}V

Table 4. City parameters and conditions.

City	
^{max}V	150 kNm^3
$V(t=0$ day$)$	100% of ^{max}V
$C(t=1$ day$)$	15000 kNm^3/day
$C(t=2$ day$)$	20000 kNm^3/day
$C(t=3$ day$)$	30000 kNm^3/day
$C(t=4$ day$)$	35000 kNm^3/day

Table 5. Compression station parameters and conditions.

Compression station	
${}^{max}C$	0 kNm^3
${}^{max}S_j$	20000 kNm^3/day

Table 6. Pipeline 1 and 2 parameters and conditions.

Pipeline 1 and 2	
L	1 km
D	32”
p_1	80 kgf/cm^2
Q	20000 kNm^3/day
$p_{\min}$	15 kgf/cm^2
$p_{\max}$	150 kgf/cm^2
${}^{max}S$	40000 kNm3/day

Table 7. Pipeline 3 parameters and conditions.

Pipeline 3	
L	150 km
D	32”
p_1	80 kgf/cm^2
Q	20000 kNm^3/day
$p_{\min}$	15 kgf/cm^2
$p_{\max}$	150 kgf/cm^2
${}^{max}S$	50000 kNm^3/day

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.